TSSM
Creating VCE Success

MATHEMATICAL METHODS 2020

Unit 3
Key Topic Test 6 – Exponential & Logarithmic Functions
Technology Active

Recommended writing time*: 45 minutes

Total number of marks available: 30 marks

SOLUTIONS

Section A: Multiple Choice

Question 1

Answer: B

$(3, \infty) \cap (-\infty, 10)$
$(3, 10)$

Question 2

Answer: C

As $x \to \infty, f(x) \to 1$

Question 3

Answer: B

$81^m \times 9^n$
$= 3^{4m} \times 3^{2n}$
$= 3^{4m+2n}$

Question 4

Answer: A

$2log_b(x^2) = log_b 9 + 4$
$log_b x^4 = log_b 9 + log_b b^4$
$log_b x^4 = log_b 9b^4$
$x^4 = 9b^4$
$x = \sqrt{3}b$ (Assume b must be positive.)

Question 5

Answer: D

Domain $f(x)$ $(-\infty, 2)$
$f^{-1}(x) = 2 - e^{-x}$

Question 6

Answer: C

$10^2 = mx - 5$
$105 = mx$
$x = \frac{105}{m}$
$2 = \frac{105}{m}$
$m = \frac{105}{2}$

Question 7

Answer: B

CAS solve

Question 8

Answer: D

$2log_2 m + log_2 n - log_2(m - n)$
$= log_2 m^2 + log_2 n - log_2(m - n)$
$= log_2 \frac{m^2 n}{(m-n)}$

Section B: Extended Response

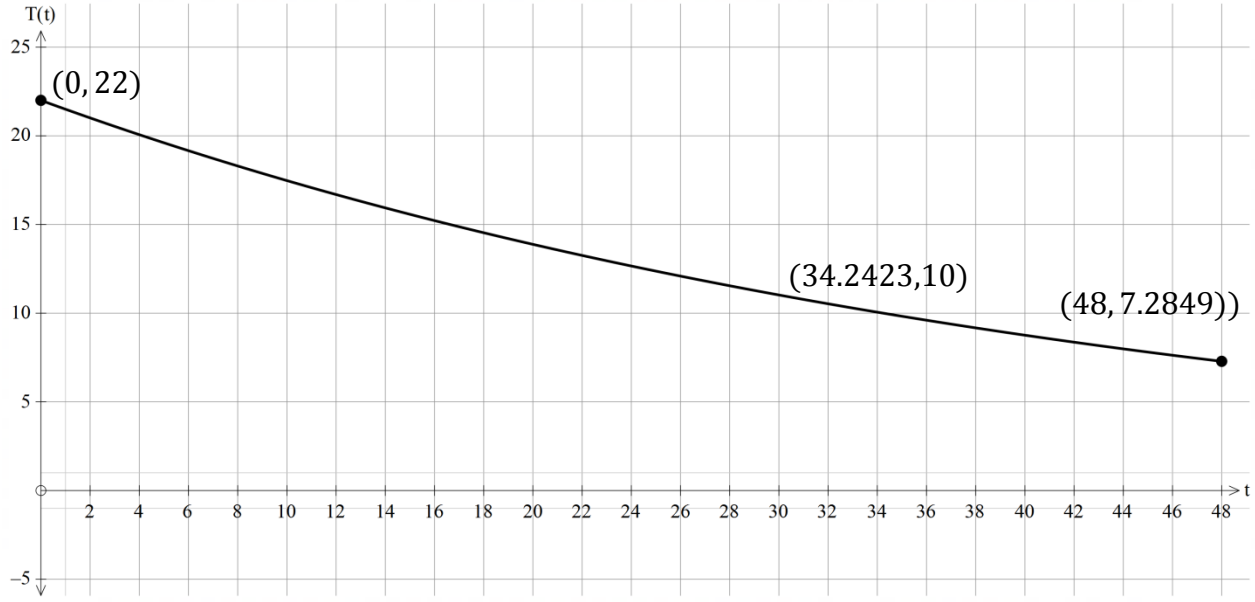
Question 1

a. T(0) = 22°C 1 mark

b. $10 = 22 \times 10^{-0.01t}$

$t \approx 34\ hours\ 15\ minutes$ 1 mark

c.



1 mark endpoints
1 mark shape

d. $T(20) = \frac{11}{5} \times 10^{\frac{4}{5}}$ 1 mark

$T_1(20) = \frac{11}{5} \times 10^{\frac{4}{5}}$ also

$T_1(t) = a \times 10^{\frac{4}{5}+k(t-20)}$

$T_1(20) = a \times 10^{\frac{4}{5}}$

$a = \frac{11}{5}$ 1 mark

e. $T_2(24) = 1$ 1 mark

$k = -0.2856$ 1 mark

Question 2

a. $1000 = A(1 - e^{-k})$ (1)
$120\,000 = A(1 - e^{-2k})$ (2) 1 mark

$\frac{(2)}{(1)}$ $120 = \frac{1-e^{-2k}}{1-e^{-k}}$ 1 mark

$120 - 120e^{-k} = 1 - e^{-2k}$
$119 - 120e^{-k} + e^{-2k} = 0$ 1 mark

b. $119 - 120e^{-k} + e^{-2k} = 0$
$Let\ a = e^{-k}$ 1 mark

$119 - 120a + a^2 = 0$
$a = 1, 119$ 1 mark
$e^{-k} = 1, k = 0$, disregard as $k > 0$
$e^{-k} = 119$
$k = -log_e(119)$ 1 mark

c. $1000 = A(1 - e^{log_e(119)})$
$1000 = A(1 - 119)$
$A = -\frac{500}{59}$ 1 mark

d. $2\,000\,000 = -\frac{500}{59}(1 - e^{log_e 119 \times t})$
$t = 2.5887$ hours 1 mark

Question 3

a. $C(0) = 20$
$S(0) = 22.5 \approx 23$ 1 mark

b. $C(1.5) \approx 40$
$S(1.5) \approx 27$ 1 mark

c. $20 \times 10^{0.2t} = 22.5 \times 10^{0.05t}$
$10^{0.15t} = \frac{9}{8}$ 1 mark
$0.15t = log_{10}(\frac{9}{8})$ 1 mark

$t = \frac{20}{3} log_{10}(\frac{9}{8})$ 1 mark

d. $d(t) = |C(t) - S(t)|$ 1 mark
$d(0) = 2.5$
$d(2) \approx 21.91$ ...

$d(t)$ is greatest after 2 years 1 mark