

Student Name……………………………………

### MATHEMATICAL METHODS UNITS 3 & 4

### TRIAL EXAMINATION 1

**2021**

#### Reading Time: 15 minutes

Writing time: 1 hour

###### Instructions to students

This exam consists of 9 questions.

All questions should be answered in the spaces provided.

There is a total of 40 marks available.

The marks allocated to each of the questions are indicated throughout.

Students may **not** bring any calculators or notes into the exam.

Where a numerical answer is required, an exact value must be given unless otherwise directed.

Where more than one mark is allocated to a question, appropriate working must be shown.

Diagrams in this trial exam are not drawn to scale.

A formula sheet can be found on pages 13 and 14 of this exam.

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**Question 1** (3 marks)

1. Let .

Find . 1 mark

1. Let .

Evaluate . 2 marks

**Question 2** (3 marks)

Let .

**a.** Find the rule for , the inverse function of *f*. 1 mark

**b.** State the domain of . 1 mark

**c.** Show that . 1 mark

**Question 3** (3 marks)

Julian randomly selects a die and throws it.

The two dice he selects from, are an unbiased six-sided black die and a biased six sided red die.

With the red die, the probability of throwing a six is  and there is equal probability of throwing a 1, 2, 3, 4 or 5.

1. What is the probability that Julian throws a six? 2 marks
2. What is the probability that Julian selected the black die given that he threw a six? 1 mark

**Question 4** (4 marks)

Let .

Part of the graph of *f* is shown below.



1. Find the average rate of change of *f* between .

Give your answer in the form  where *a*, *b*, *c* and *d* are constants. 2 marks

1. The graph of *f*  undergoes a transformation defined by

,

to become the graph of *h*.

On the set of axes above, sketch the graph of *h*.

Indicate clearly on the graph the coordinates of any axis intercepts. 2 marks

**Question 5** (5 marks)

Voters in a national referendum can cast a “for” vote or an “against” vote.

In a random sample taken of 100 voters, ten cast an “against” vote.

1. What is the proportion of “against” voters in this sample? 1 mark
2. Thirty of the voters in this sample are randomly selected to complete an additional survey.

The probability that at least two of them are “against” voters is equal to , where *a* and *n* are positive integers and *b* and *c* are positive rational numbers.

Find *a*, *b*, *c* and *n*. 2 marks

1. Based on the original sample of 100 voters, find the approximate 95% confidence interval for the proportion of “against” voters in the referendum.

Use the *z* value . 2 marks

**Question 6** (3 marks)

The area of the region bounded by the curve with equation , where *k* is a positive number, and the *x* and *y* axes is  square units. Find the value of *k*.

**Question 7** (4 marks)

Part of the graph of is shown below.



1. Find the minimum value of *f*. 2 marks
2. Find the average value of *f* between . 2 marks

**Question 8** (7 marks)

The points *P*, *Q*, *R* and *S* lie on the graph of the function . These four points form a regular trapezium which is shaded below.



1. Show that the area *A*, in square units, of the trapezium *PQRS* is given by . 1 mark
2. Find the coordinates of point *P* in terms of *a*, when *A* is a maximum. 3 marks

**c.** Find the angle *PQR* when *A* is a maximum. 1 mark

* 1. Find the equation of the tangent to the graph of *f* that is parallel to the line segment *PQ* when *A* is a maximum. 2 marks

**Question 9** (8 marks)

Let .

1. Show that the point  lies on the graph of *g*. 1 mark
2. Find. 1 mark

Give your answer in terms of .

* 1. Hence find the area enclosed by *g*, the line  and the *y*-axis. 3 marks

The graph of *g* is dilated by a factor of  from the *y*-axis and then reflected in the *y*-axis to become the graph of the function *h*.

* 1. Write down, in terms of *a*,
     1. the rule for *h*. 1 mark
     2. the domain of *h*. 1 mark

* 1. Let *p* be the *x*-coordinate of the left endpoint of the graph of *h*.

Given that , find the possible values of *a.* 1 mark

**Mathematical Methods formulas**

## Mensuration

|  |  |  |  |
| --- | --- | --- | --- |
| area of a trapezium |  | volume of a pyramid |  |
| curved surface area of a cylinder |  | volume of a sphere |  |
| volume of a cylinder |  | area of a triangle |  |
| volume of a cone |  |  | |

## Calculus

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
| product rule |  | quotient rule |  |
| chain rule |  |  | |

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## Probability

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
| mean |  | variance |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **Probability distribution** | | **Mean** | **Variance** |
| discrete | = |  |  |
| continuous |  |  |  |

## Sample proportions

|  |  |  |  |
| --- | --- | --- | --- |
|  | | mean |  |
| standard  deviation |  | approximate  confidence  interval |  |