

 **MATHS METHODS 3 & 4**

 **TRIAL** **EXAMINATION 2**

 **SOLUTIONS**

 **2021**

# SECTION A – Multiple-choice answers

1. **B 9. A 17. B**
2. **D 10. D 18. B**
3. **E 11. E 19. E**
4. **C 12. C 20. E**
5. **B 13. B**
6. **D 14. C**
7. **B 15. A**
8. **A 16. C**

# SECTION A – Multiple-choice solutions

# Question 1

Define  on your CAS.



The answer is B.

**Question 2**

For option A, maximal domain is .

For option B, maximal domain is .

For option C, maximal domain is .

For option D, maximal domain is .

For option E, maximal domain is .

The answer is D.

**Question 3**

Method 1 – using CAS

Solve .



The answer is E.

**Question 3** (cont’d)

Method 2 – by hand.





 

The answer is E.

**Question 4**

For option A, , we have a dilation by a factor of  from the *x*-axis so  becomes  then a translation 2 units to the left, which gives  which is not point *Q*. Reject option A.

For option B, , we have a dilation by a factor of 2 from the *x*-axis so  becomes  then a translation 3 units to the right which gives . Reject option B.

For option C, , we have a dilation by a factor of 3 from the *x*-axis so  becomes  then a translation 2 units to the right which gives  which is the image point *Q* that is required.

Option D gives an image point of  and option E gives an image point of .

The answer is C.

**Question 5**



Do a quick sketch of .

For  to exist, then *h* has to be .

Since the local maximum and local minimum

of the graph of occurs at

 respectively, then *h* will be

.

The answer is B.

**Question 6**

The average rate of change of *f* between  is given by



The instantaneous rate of change of *f* at  is given by .



The answer is D.

**Question 7**

The graph of  must have

* a negative gradient for 
* a zero gradient for 
* a positive gradient for 

Option A has a positive gradient for .

Reject option A.

Option B has a negative gradient for , a zero gradient at  and a positive gradient for .

Also, as  and as .

Option B could represent .

Option C can’t because it has a sharp point at  and therefore the gradient at  is not defined.

Option D does not have a zero gradient at .

Option E has a positive gradient for  and a negative gradient for .

The answer is B.

**Question 8**

Since *A* and *B* are independent,



The answer is A.

**Question 9**

Draw a diagram.





Since this answer is not offered,

look for an equivalent area under the

bell curve, remembering that it is

symmetrical about the line .

So .

This is offered in option A.

The answer is A.

**Question 10**

We are sampling from a small population of 10 balls.

If , then we are interested in one of the four balls in the sample being white.



The answer is D.

**Question 11**

****

Reject option A.



Reject option B.



Reject option C.



Reject option D.



The answer is E.

# Question 12



The answer is C.

**Question 13**



The answer is B.

**Question 14**



Method 1 – using area of triangles

 



The answer is C.

Method 2 – using formula



Note that the left hand branch of the graph has the rule  and the right hand branch has the rule . (Use the gradient formula  and the straight line formula  to find these.)

The answer is C.

Just out of interest, what the answer means graphically is that the area enclosed above the line  between function *f* and between  is equal to the area enclosed below the line  between the function *f* and between .

In other words in the diagram below, the big triangle above the line  is equal in area to the sum of the areas of the two smaller triangles below the line .



**Question 15**



So mean and standard deviation.

The answer is A.

**Question 16**

Since *m* is the median, 

 



The answer is C.

**Question 17**



The graph of *f* is shown.

The point  lies on this graph.



A couple of tangents are drawn to the graph as shown.

The *y*-intercept of these two tangents is less than –1.



The horizontal tangent shown, has a *y*-intercept

of –1.

This is the maximum possible value of the *y*-intercept

i.e. of *c*.

The answer is B.

**Question 18**



Note that  is defined because







Do a quick sketch of  to find the range.



Option B offers this combination.

The answer is B.

**Question 19**

A possible sketch is shown.





So the left endpoint of the left hand branch of the graph of *g* occurs at .



So the right endpoint of the right hand branch of the graph of *g* occurs at .



The answer is E.

**Question 20**



The first three possible graphs of *f* shown above (on the left hand side) are all when *f* is a positive cubic i.e. the minimum turning point lies to the right of the maximum turning point.

The transformed graphs for each of these graphs of *f* are shown to their right i.e. the graph of *g*.

The transformation from *f* to *g* involves

* a dilation by a factor of 2 from the *y*-axis
* a reflection in the *x*-axis
* a translation 1 unit to the right.

One of these graphs of *g* has a minimum turning point which occurs at a point where

.

These values are not offered in the answers.

The other possible graph of *f* is shown below.



This graph shows the graph of a negative cubic i.e. the minimum turning point lies to the left of the maximum turning point.

The transformed graph, *g*, has a minimum turning point occurring at a point where .

The answer is E.

**SECTION B**

**Question 1 (10 marks)**

1. Find the *x*-coordinate of the maximum turning point.

 

**(1 mark)**

The graph of *f*  is strictly decreasing for . **(1 mark)**

1.  **(1 mark)**



1. 

 

So the tangent makes an angle of  (to the nearest degree) with the positive direction of the *x*-axis. **(1 mark)**

1. The tangent at  has a slope of .

So the tangent at  has a slope of 2 (ie the negative reciprocal).



**(1 mark)**

**(1 mark)**

1. *P* is the point .

Equation of tangent at *P* is  using CAS. **(1 mark)**

The *y*-intercept of this tangent is *p.* **(1 mark)**

1. From part **e.**, the *y* intercept of the tangent at .

So the *y*-intercept of the tangent at .

The tangent perpendicular to this tangent occurs at  from part **d.**

So the *y*-intercept of this perpendicular tangent is . **(1 mark)**



**(1 mark)**

**Question 2** (11 marks)

1. The right endpoint of both the function *n* and the function *s* occurs at .

So  **(1 mark)**

1. Define *n* and *s* on your CAS.



So the length of the dam wall is 1.1 km. **(1 mark)**

1. 

 

 **(1 mark)**

1. 

 

**(1 mark)**

1. The graph of  is symmetrical about the line .

So a line running in an east-west direction that passes through  would also pass through the point . So the length of barrier required is 0.6 km.

**(1 mark)**

1. We want the shaded area east of the barrier to equal the shaded area to the west.



 

**(1 mark)**

**(1 mark)**



**(1 mark)**

1. Let 

Solve  **(1 mark)**

 **(1 mark)**

Make sure that you have limited the domain of *D* to  if you hadn’t initially put the domains of *n* and *s* into your CAS.



**(1 mark)**

**Question 3** (10 marks)

1. The graph of  needs to be translated 1 unit to the right and 1 unit up to become the graph of .

So .

**(1 mark)**

1. The endpoint of the graph of *f* is .

The endpoint of the image graph is .

The graph of *f* has been dilated by a factor of 2 from the *y*-axis.

So .

**(1 mark)**

1. 

Since .

So .

 **(1 mark)**

1. 

 

Swap *x* and *y* for inverse.

 

Solve for *y* by hand or CAS.

 

So  **(1 mark)**



**(1 mark)**

Method 2

Solve **** for *x*

 ****

1. Method 1

Solve **** for *x*

 ****

Since *h* and  intersect on the line , the points of intersection are

 

 **(1 mark) (1 mark)**

1. Area enclosed by graphs of 



So area enclosed is a constant i.e. . **(1 mark)**

1. Solve . **(1 mark)**



**(1 mark)**

**Question 4** (17 marks)

1. **i.** 

**(1 mark)**

1. 

 

**(1 mark)**

1. 



**(1 mark)**

1. Let the random variable *R* represent the distribution of the amount of the next four fishing licences purchased.



Method 1 using CAS

 (using CAS binomPdf(4,0.5,3))

Method 2 by hand



 

 **(1 mark)**

1. **i.** Let the random variable *Y* represent the distribution of the weight, in kg, of

Golden Perch fish that have been inspected.





 

**(1 mark)**

1. 

 

**(1 mark)**

1. 



Maximum weight is 4.8634 kg (to 4 decimal places).

**(1 mark)**

1. **i.** Define  on your CAS.



**(1 mark)**

 **(1 mark)**

1. 

 

**(1 mark)**

1. 



**(1 mark)**

**(1 mark)**

1. **i.** Let *W* represent the distribution of the number of anglers questioned who

 didn’t have a licence.



 **(1 mark)**

* 1. 

**(1 mark)**

 

**(1 mark)**

1. **i.** approximate confidence interval 

 (from the formula sheet)

Now .

Sample proportion is 0.1298.

 **(1 mark)**

* 1. The 95% confidence interval for this other state, (0.1092, 0.1504) does not contain the proportion of the original state which was 0.08.

**(1 mark)**

**Question 5** (12 marks)

1. Define *f* on your CAS.



*x*-intercepts are 

 **(1 mark)**

1. Solve .

  **(1 mark)**

Since the graph of *f* is a negative cubic, the local maximum occurs to the right of the local minimum, and since *a* is a positive constant, then, and so the local maximum occurs at .

 

Local maximum is .

**(1 mark)**

1. **i.** The graphs intersect when



There is a solution when .

Consider the quadratic equation.



**(1 mark)**

If there is just one point of intersection i.e. at , then we want

the quadratic equation to have no solutions i.e. .



So  for one point of intersection.

 **(1 mark)**

* 1. Method 1

From part **i.**, there is one point of intersection at .

For one more point of intersection, which would give a total of exactly two points of intersection, we require .



**(1 mark)**

but *a* is a positive constant so reject .

For exactly two points of intersection .

**(1 mark)**

 Method 2

 

**(1 mark)**

**(1 mark)**

1. Both graphs pass through the origin. The graph of *g* will be a tangent to the graph of *f* when it touches the graph of *f* at one other point.

In total, the graphs will intersect exactly twice.

So from part **c. ii**, .





solve these two equations simultaneously for *x*,



Point of tangency is .

**(1 mark)**

1. **i.** Solve  for *x* using CAS.

Given that 



 **(1 mark)** – one correct value

**(1 mark)** – all 3 correct values

1. Draw a graph.



Solve . **(1 mark)**



**(1 mark)**