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YEAR 12 *Trial Exam Paper*

2021
MATHEMATICAL METHODS
Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 15 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

Let $y = e^{\sin(x)}$.

a. Find $\frac{dy}{dx}$.

1 mark

b. Let $f(x) = \log_e(x^2 + x + 1)$.

Evaluate $f'(1)$.

2 marks

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Question 2 (2 marks)

Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} -12x^3 + 12x^2 & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected value of X.

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Question 3 (4 marks)

Let $f(x) = 2x(x-1)$ and $g(x) = \log_2(x+1)$.

a. Find $f\left(g\left(-\frac{1}{2}\right)\right)$.

2 marks

b. Find the minimum value of $g\left(f\left(x\right)\right)$.

2 marks

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Question 4 (3 marks)

Solve the equation $2\log_4(x+6) - \log_4(x+3) = 2$.

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Question 5 (4 marks)

In a particular game, a bag contains 10 cubes. The cubes are red or black, but it is not known how many there are of each colour.

a. Justin draws a cube from the bag, observes and then replaces it. This is done 12 times. A red cube is observed three of these times.

Using this sample, find the approximate 95% confidence interval for the proportion of red cubes in the bag. Use the *z*-value $\frac{49}{25}$.

2 marks

b. Let *p* be the true proportion of red cubes in the bag.

Uvini draws three cubes from the bag, without replacement. The first cube drawn is black. Find the probability that Uvini draws exactly one red cube, in terms of *p.* Express your answer in the form $ap^2 + bp$, where $a, b \in Q$.

2 marks

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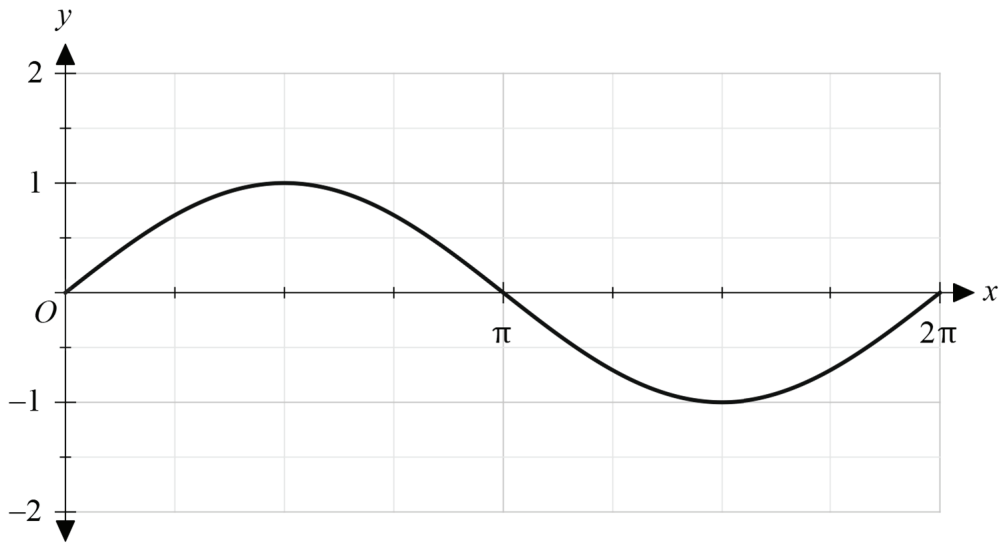
Question 6 (8 marks)

Let $f: R \to R,\ f(x) = \sin(x)$.

a. The graph of $y = f(x)$ is shown on the axes below.



Let $g: [0, 2\pi] \to R,$ where $g(x) = f\left(x - \frac{\pi}{2}\right) + 1$.

On the axes above, sketch the graph of g. Label all points of intersection of the graphs of f and g and label all the coordinates of the axes intercepts of g.

2 marks

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b. Find the total area of the regions enclosed by the graphs of f and g for $x \in [0, 2\pi]$.

3 marks

c. Let $h:[0, 2\pi] \to R,\ h(x) = f\left(x - \frac{\pi}{2}\right) + k$, where $k \in R$.

Find the values of k for which f and h intersect at least once.

3 marks

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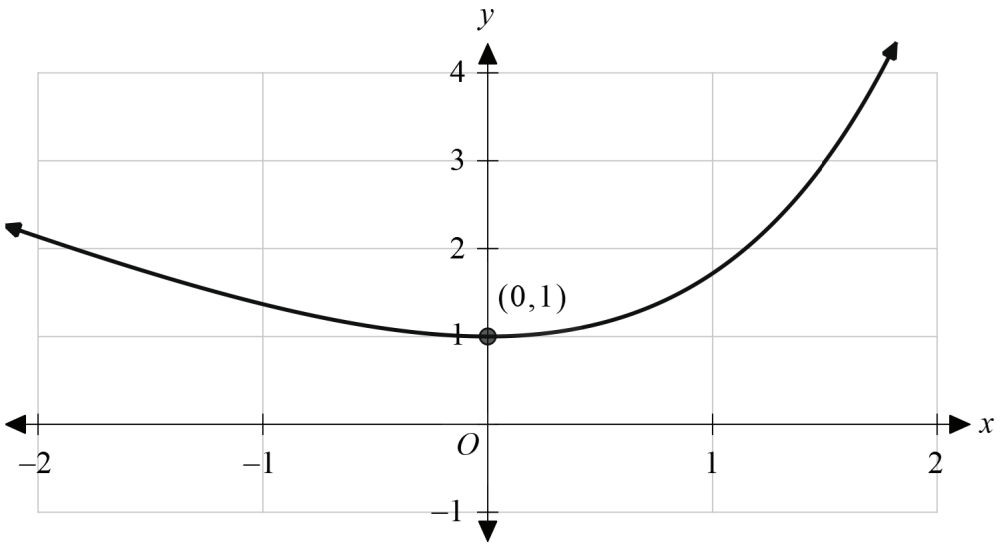
Question 7 (6 marks)

Let $f: R \to R,\ f(x) = ae^x - x,$ where $a \in R^+$. A graph of $y = f(x)$ for $a = 1$ is shown below.



a. Show that the equation of the tangent of f at $x = 1$ is given by the equation $y = (e \times a - 1)x$.

2 marks

b. For what value of a is $y = x$ a tangent of f?

1 mark

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c. Find the x-coordinate of the stationary point of f in terms of a.

1 mark

d. Hence, or otherwise, find the values of a for which the graph of f has two x-axis intercepts.

2 marks

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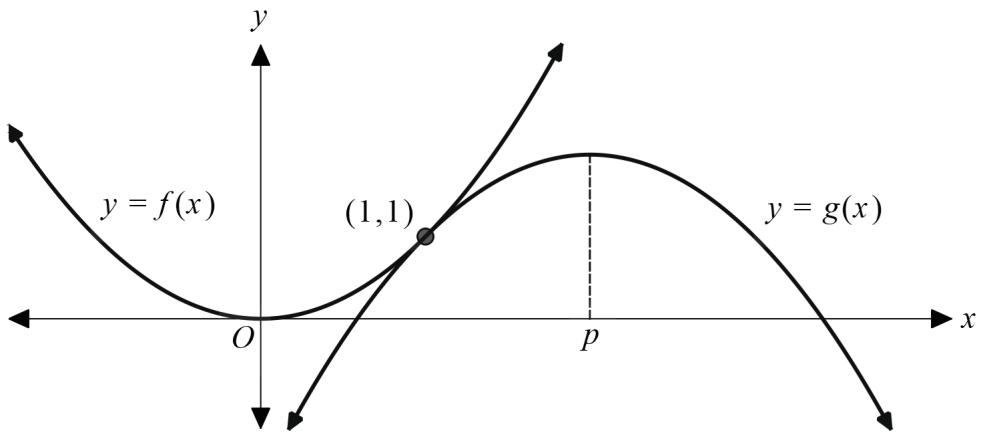
Question 8 (10 marks)

The graph below shows the functions f and g, where $f: R \to R,\ f(x) = x^2$ and $g: R \to R,\ g(x) = a(x-p)^2 + p,$ and where a and p are real numbers and $p \neq 1$.



At $x = 1,$ the graphs of $y = f(x)$ and $y = g(x)$ intersect and have an equal gradient.

a. **i.** Show that $a = \dfrac{1}{1-p}$.

1 mark

ii. For what values of p is $g(x) \geq f(x)$ for $x \in R$?

2 marks

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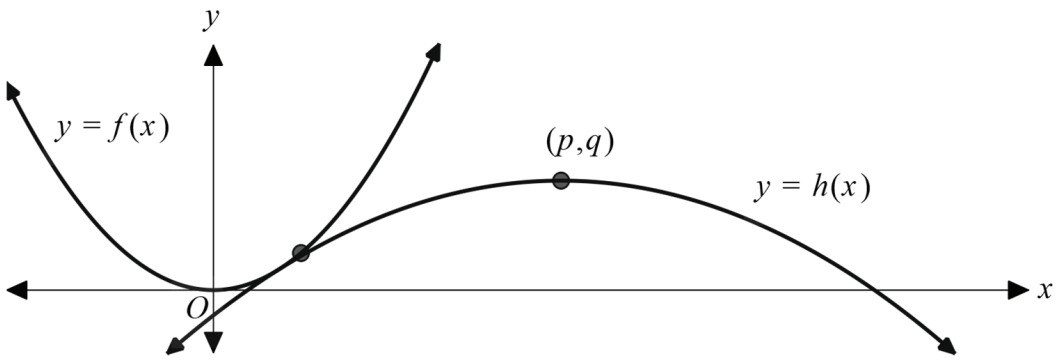
The graph below shows the functions f and h, where $f: R \to R, f(x) = x^2$ and $h: R \to R, h(x) = \frac{q}{q - p^2}(x - p)^2 + q$, where p and q are real numbers and $p, q \neq 0$.



The graphs of $f(x)$ and $h(x)$ intersect at only one point.

b. **i.** Show that the point of intersection of $f(x)$ and $h(x)$ is at $\left(\frac{q}{p}, \frac{q^2}{p^2}\right)$.

2 marks

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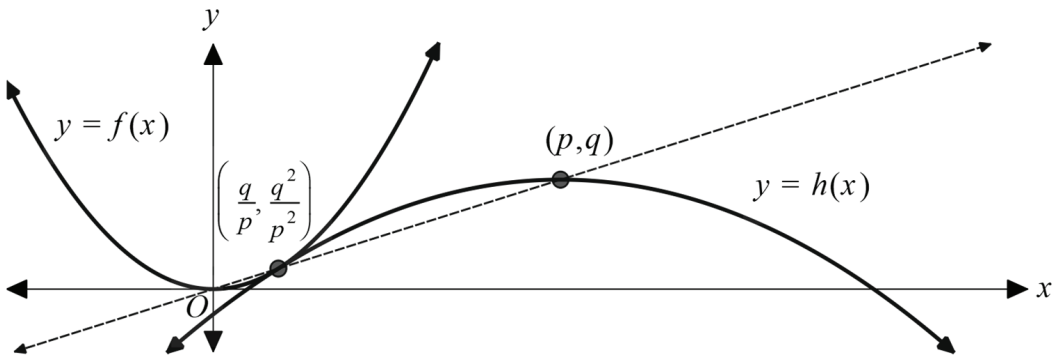
ii. Find the equation of the line passing through the vertices of both parabolas and their point of intersection.



1 mark

iii. For what values of q, in terms of p, is the point of intersection closer to the vertex of $f(x)$ than the vertex of $h(x)$?

2 marks

iv. Two parabolas have their vertices at the coordinates $(2000, 21)$ and $(2021, 2121)$ and have a single point of intersection at $x = 2100$.

Find the y-coordinate of the point of intersection.

2 marks

END OF QUESTION AND ANSWER BOOK

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