The Mathematical Association of Victoria

Trial Examination 2021

MATHEMATICAL METHODS

Trial Written Examination 1 - SOLUTIONS

Question 1

$y = x^2 \log_e\left(\frac{x}{3}\right)$ Use the product rule.

$$\frac{dy}{dx} = 2x\log_e\left(\frac{x}{3}\right) + x^2 \frac{\frac{1}{3}}{\frac{x}{3}} \quad \textbf{1M}$$

$$= 2x\log_e\left(\frac{x}{3}\right) + x \quad \textbf{1A}$$

Question 2

$f(x) = \tan(2x)$ Use the chain rule.

$$f'(x) = 2\sec^2(2x) \quad \textbf{1M}$$

$$f'\left(\frac{\pi}{3}\right) = 2\sec^2\left(\frac{2\pi}{3}\right)$$

$$= 2\times(-2)^2$$

$$= 8 \quad \textbf{1A}$$

Question 3

$$3e^x + 4 = e^{-x}$$

$$3e^x + 4 = \frac{1}{e^x}$$

Multiply by e^x.

$$3e^{2x} + 4e^x - 1 = 0 \quad \textbf{1M}$$

Let $a = e^x$

$$3a^2 + 4a - 1 = 0$$

$$a = \frac{-4 \pm \sqrt{16+12}}{6} \quad \textbf{1M}$$

$$= \frac{-4 \pm 2\sqrt{7}}{6}$$

$$= \frac{-2 \pm \sqrt{7}}{3}$$

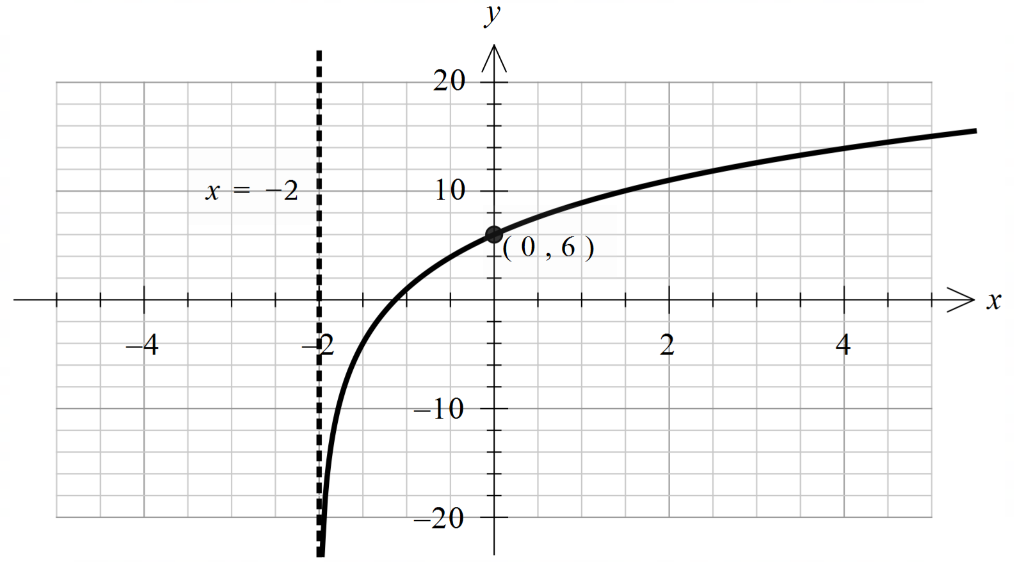
$$e^x \neq \frac{-2-\sqrt{7}}{3}$$

$$e^x = \frac{-2+\sqrt{7}}{3}$$

$$x = \log_e\left(\frac{-2+\sqrt{7}}{3}\right) \quad \textbf{1A}$$

Question 4

a. $x = -2$ labelled on the graph. **1A**



The graph is of the form $y = a\log_2(x-b)+c$ where a, b and c are real constants.
The equation of the vertical asymptote is $x = b$ and so $b = -2$

b. Using $y = a\log_2(x+2)+c$ **1A**

and points $(0,6)$ and $\left(\frac{1}{2^{\frac{1}{5}}}-2, 0\right)$

$6 = a\log_2(2)+c$ simplifies to $6 = a + c$

$0 = a\log_2\left(\frac{1}{2^{\frac{1}{5}}}-2+2\right)+c$ simplifies to $0 = a\log_2\left(2^{-\frac{1}{5}}\right)+c$ **1M**

Solving $a + c = 6$ and $-\frac{1}{5}a + c = 0$

Gives $\frac{6}{5}a = 6,\ a = 5$

$a = 5,\ b = -2,\ c = 1$ **1A**

Question 5

a. $g(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$

Test ran $g \subseteq$ dom f

ran$[0,\infty) \not\subset R \setminus \{0\}$ **1A**

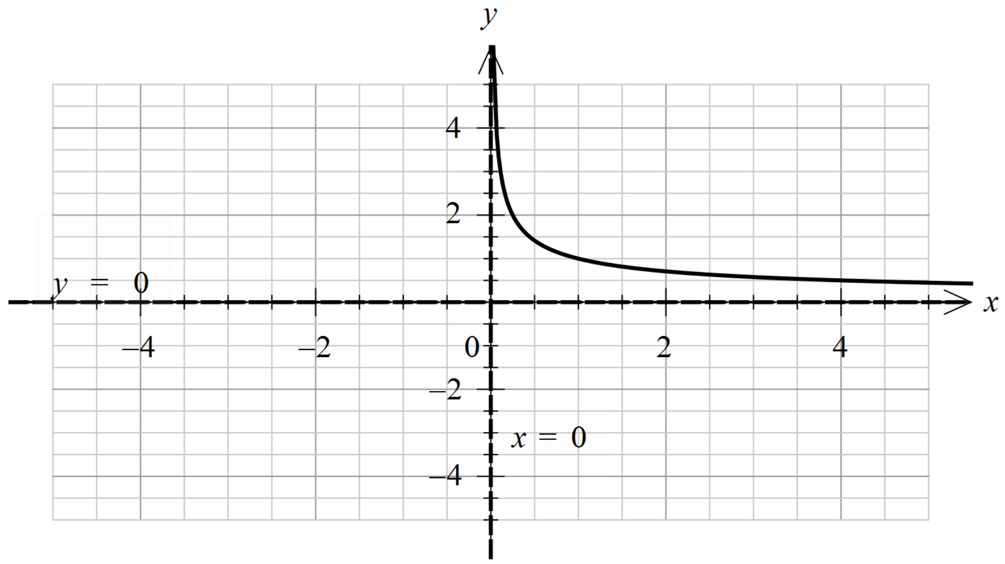
The function $h(x) = f(g(x))$ does not exist.

b. $g_1(x) = \sqrt{x}$ and $h_1 : D \to R,\ h_1(x) = f(g_1(x))$

$D = (0,\infty)$ **1A**

Rule $h_1(x) = \frac{1}{\sqrt{x}}$ **1A**

c. shape **1A**
asymptotes **1A**



Question 6

a. $\int (3x+1)^{-3}\,dx = \frac{(3x+1)^{-2}}{3\times -2}+c$ **1M**

An antiderivative is $\frac{1}{-6(3x+1)^2}$ **1A**

b. Let $f(x)=\frac{1}{-6(3x+1)^2}+c$ and $f(-1)=2$

$2=\frac{1}{-6(-2)^2}+c$

$c=\frac{49}{24}$

$f(x)=\frac{1}{-6(3x+1)^2}+\frac{49}{24}$ **1A**

Question 7

a. $f:[0,2\pi]\to R, f(x)=-2\sin(2x)\sin\left(x-\frac{\pi}{3}\right)$

Solve $-2\sin(2x)=0$ for x.

$2x=0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi$

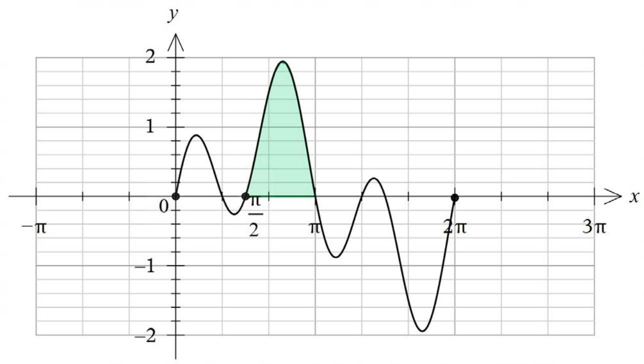
$x=0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2},\ 2\pi$ **1A**

Solve $\sin\left(x-\frac{\pi}{3}\right)=0$ for x.

$x-\frac{\pi}{3}=0,\ \pi$

$x=\frac{\pi}{3},\ \frac{4\pi}{3}$ **1A**

b.



$$\int_{\frac{\pi}{2}}^{\pi}\left(\sin\left(3x+\frac{\pi}{6}\right)-\cos\left(x+\frac{\pi}{3}\right)\right)dx$$ **1A**

$$=\left[-\frac{1}{3}\cos\left(3x+\frac{\pi}{6}\right)-\sin\left(x+\frac{\pi}{3}\right)\right]_{\frac{\pi}{2}}^{\pi}$$ **1M**

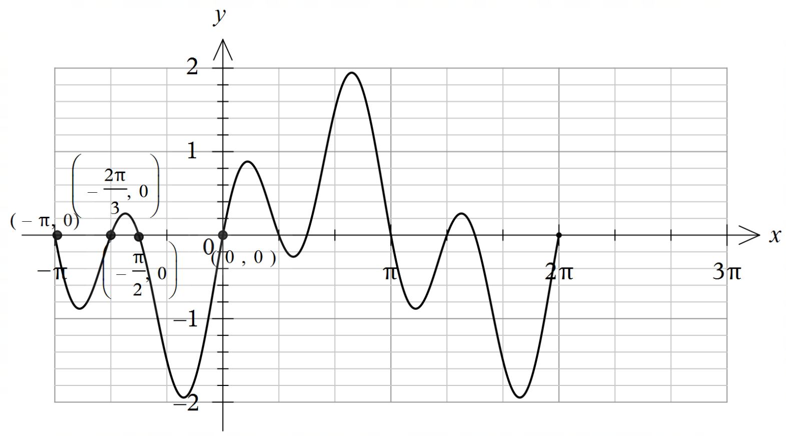
$$=\left(-\frac{1}{3}\cos\left(3\pi+\frac{\pi}{6}\right)-\sin\left(\pi+\frac{\pi}{3}\right)\right)-\left(-\frac{1}{3}\cos\left(\frac{3\pi}{2}+\frac{\pi}{6}\right)-\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)\right)$$

$$=\left(-\frac{1}{3}\cos\left(\frac{19\pi}{6}\right)-\sin\left(\frac{4\pi}{3}\right)\right)-\left(-\frac{1}{3}\cos\left(\frac{5\pi}{3}\right)-\sin\left(\frac{5\pi}{6}\right)\right)$$

$$=\frac{\sqrt{3}}{6}+\frac{\sqrt{3}}{2}+\frac{1}{6}+\frac{1}{2}$$

$$=\frac{2\sqrt{3}+2}{3}$$ **1A**

c. Shape **1A**
Coordinates **1A**



Question 8

a. $\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2=\frac{9}{256}$ **1A**

b. $S\sim \text{Bi}\left(4,\frac{1}{4}\right)$

$\Pr(S>1)=\Pr(S\geq 2)=1-\left({}^4C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^4+{}^4C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^3\right)$ **1M**

$\Pr(S>1)=1-\left(\frac{81}{256}+4\times\left(\frac{1}{4}\right)\left(\frac{27}{64}\right)\right)$

$\Pr(S>1)=1-\left(\frac{81}{256}+\frac{27}{64}\right)=\frac{67}{256}$ **1A**

c. $0.1+0.25+0.05+0.5+k=1$
Giving $k=0.1$ **1A**
Mean = $(1\times 0.1)+(2\times 0.25)+(3\times 0.05)+(4\times 0.5)+(5\times 0.1)$
Giving mean = $\mu=3.25$

$\frac{\Pr(X\leq 2)}{\Pr(X<3.25)}=\frac{0.35}{0.4}=\frac{35}{40}$ **1M**

Answer $=\frac{7}{8}$ **1A**

Question 9

Let $y=g(x)=\frac{x+b}{x+a}=1+\frac{b-a}{x+a}$
Inverse swap *x* and *y*

$x=1+\frac{b-a}{y+a}$ **1M**

$(x-1)(y+a)=b-a$

$y+a=\frac{b-a}{x-1}$

$f^{-1}(x)=\frac{b-a}{x-1}-a$ **1A**

$a=-1,\ b\in R\setminus\{-1\}$ **1A**

Question 10

a. $h(x) = -28x^3 + 4x^2 + 7x - 1$

Rational Root Theorem

Factors of 28: $\pm 1, \ \pm 2, \ \pm 4 \ldots$ **1M**

Factors of -1: ± 1

Try $h\left(\frac{1}{2}\right) = 0$

$(2x-1)$ is a factor

$h(x) = (2x-1)(-14x^2 - 5x + 1)$

$\quad = -(2x-1)(2x+1)(7x-1)$ **1A**

OR

$h(x) = -28x^3 + 7x + 4x^2 - 1$

$= -7x(4x^2 - 1) + 4x^2 - 1$ **1M** (grouping)

$= (-7x + 1)(4x^2 - 1)$

$= -(7x-1)(2x-1)(2x+1)$ **1A**

b. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$x' = mx + 2$ **1M**

$y' = ny$

$p(x) = \frac{1}{2}(x-3)(x-1)(7x-16)$

$ny = \frac{1}{2}(mx + 2 - 3)(mx + 2 - 1)(7(mx+2) - 16)$

$ny = \frac{1}{2}(mx-1)(mx+1)(7mx-2)$

$y = \frac{1}{2n}(mx-1)(mx+1)(7mx-2)$

Equate coefficients of x^3, $h(x) = -28x^3 + 4x^2 + 7x - 1$ **1M**

$\frac{7m^3}{2n} = -28$, $n = -\frac{m^3}{8}$

Equate the constant term.

$\frac{2}{2n} = -1$

$n = -1, \ m = 2$ **1A**

END OF SOLUTIONS