

The Mathematical Association of Victoria

Trial Exam 2021

# MATHEMATICAL METHODS

## WRITTEN EXAMINATION 1

STUDENT NAME \_\_\_\_\_

Reading time: 15 minutes

Writing time: 1 hour

### QUESTION AND ANSWER BOOK

#### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room : any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

#### Materials supplied

- Question and answer book of 12 pages,
- Formula sheet
- Working space is provided throughout the book.

#### Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (2 marks)

If  $y = x^2 \log_e \left( \frac{x}{3} \right)$ , find  $\frac{dy}{dx}$ .

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**Question 2** (2 marks)

Let  $f(x) = \tan(2x)$ . Evaluate  $f' \left( \frac{\pi}{3} \right)$ .

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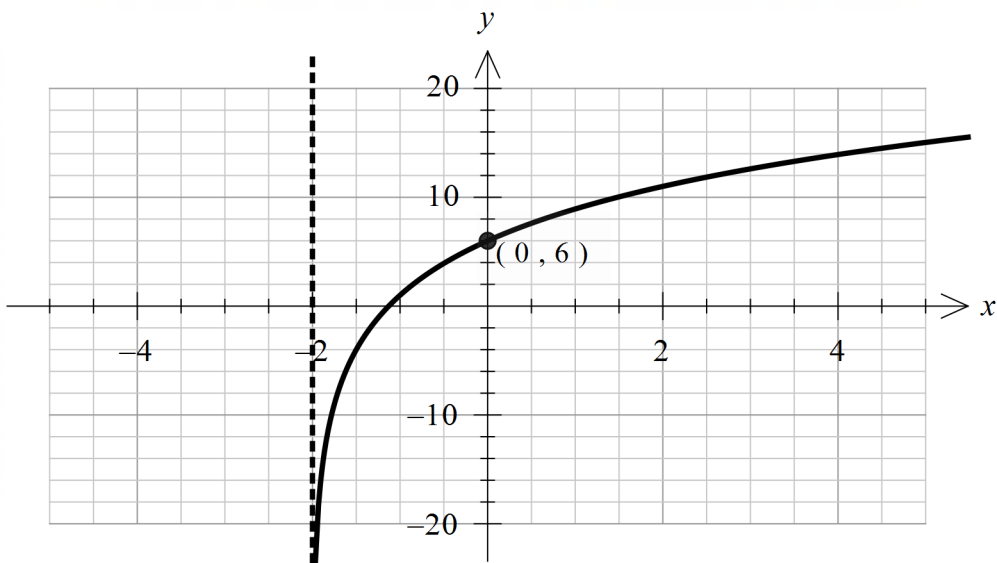
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**Question 4** (4 marks)

The graph of a logarithmic function is shown below.



The graph is of the form  $y = a \log_2(x - b) + c$  where  $a, b, c$  are real constants.

a. Label the asymptote with its equation on the graph above. 1 mark

b. Given that the graph goes through the point  $(0, 6)$  as shown, and the  $x$ -intercept has coordinates  $\left(\frac{1}{2^5} - 2, 0\right)$ , find the values of  $a, b$  and  $c$ . 3 marks

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**Question 5** (5 marks)

Let  $g(x) = \sqrt{x}$  and  $f(x) = \frac{1}{x}$  for their maximal domains.

- a. State, with reasons, why the function  $h(x) = f(g(x))$  does not exist. 1 mark

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Now consider the function  $g_1(x) = \sqrt{x}$ .

- b. State the maximal domain,  $D$ , of  $h_1$  such that  $h_1 : D \rightarrow R$ ,  $h_1(x) = f(g_1(x))$  exists and give the rule for  $h_1$ . 2 marks

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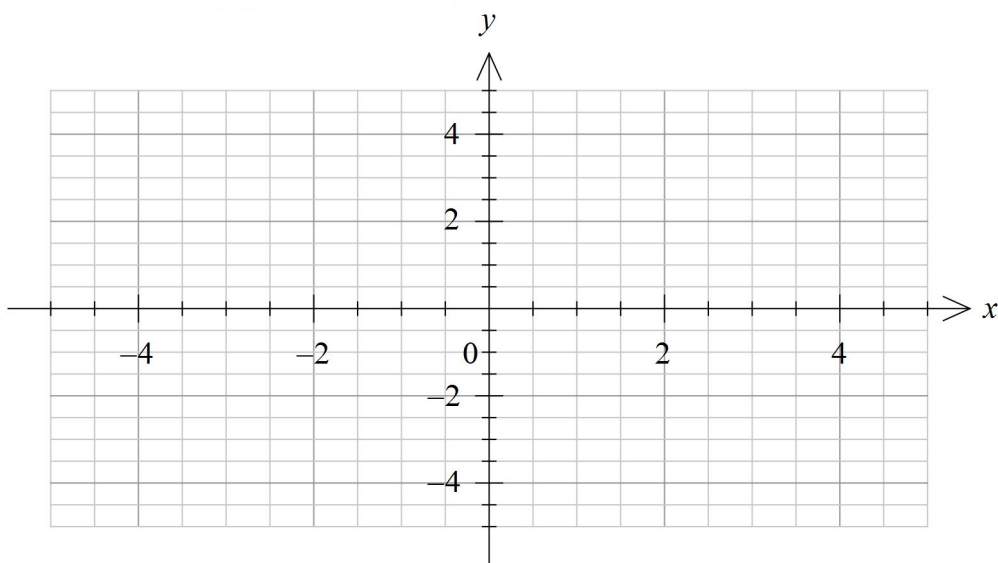


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- c. Hence sketch the graph of  $y = h_1(x)$  on the axes below. Label the asymptotes with their equations. 2 marks



**Question 6** (3 marks)

- a. Find an antiderivative of  $\frac{1}{(3x+1)^3}$ . 2 marks

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- b. Hence if  $f'(x) = \frac{1}{(3x+1)^3}$  and  $f(-1) = 2$  find the rule for  $f(x)$ . 1 mark

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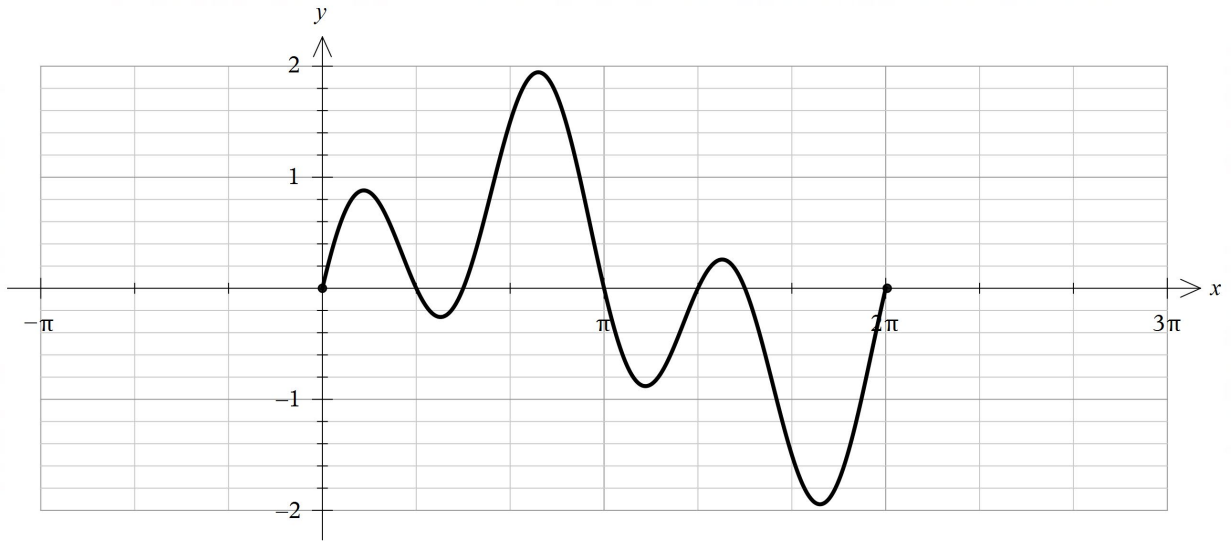
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**Question 7** (7 marks)

The graph of  $f : [0, 2\pi] \rightarrow \mathbb{R}, f(x) = -2 \sin(2x) \sin\left(x - \frac{\pi}{3}\right)$  is shown below.



a. Find the  $x$ -axis intercepts.

2 marks

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The rule for  $f$  can also be expressed as  $f(x) = \sin\left(3x + \frac{\pi}{6}\right) - \cos\left(x + \frac{\pi}{3}\right)$ .

- b.** Find the area bounded by the  $x$ -axis and the curve of  $f$ , between  $x = \frac{\pi}{2}$  and  $x = \pi$ . Express your answer in the form  $\frac{a\sqrt{b} + c}{b}$ , where  $a$ ,  $b$  and  $c$  are integers. 3 marks

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- c.** Sketch the graph of  $h: [-\pi, 0] \rightarrow R$ ,  $h(x) = -2 \sin(2x) \sin\left(x - \frac{\pi}{3}\right)$  on the set of axes on page 8. Label the  $x$ -axis intercepts with their coordinates. Do not label the turning points. 2 marks

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**Question 8** (6 marks)

The Soren family's 4 children, Declan, Faith, Blake and Paris each have an equal chance of 0.25 of going to child care on any one day.

- a. What is the probability that Declan and Faith attend child care, but Blake and Paris do not on a particular day? 1 mark

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- b. What is the probability that more than 1 child in the Soren family attends child care? 2 marks

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A discrete random variable table below records  $\Pr(X = x)$  where the random variable  $X$  describes the number of sultanas in a cupcake at morning tea in the child care centre.

$x$	1	2	3	4	5
$\Pr(X = x)$	0.1	0.25	0.05	0.5	$k$

- c. Given that a randomly chosen cupcake contains less than the mean number of sultanas, what is the probability it contains at most 2 sultanas? 3 marks

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**Question 9** (3 marks)

Let  $g(x) = \frac{x+b}{x+a}$ . For what values of  $a$  and  $b$ , where  $a$  and  $b$  are real constants, will  $g(x) = g^{-1}(x)$  for all  $x \in R$ .

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