The Mathematical Association of Victoria

Trial Examination 2021

MATHEMATICAL METHODS

Written Examination 2

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of examination

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 24 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet.
  Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A- Muliple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple – choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the graph of $y=-2\cos\left(\frac{x}{3}\right)-\frac{1}{2}$, the amplitude and period are respectively

A. -2 and $-\frac{1}{2}$

B. -2 and $\frac{2\pi}{3}$

C. -2 and 6π

D. 2 and 6π

E. 2 and $\frac{\pi}{6}$

Question 2

The function $g: D \to R, g(x)=\frac{1}{1+3x}$ has range $[-0.5,\ -0.2)$.

The domain D is

A. $[-2,\ -1)$

B. $(-2,\ -1]$

C. $[-2,\ 2.5)$

D. $(-2,\ 2.5]$

E. $(2,\ 1]$

SECTION A - continued
TURN OVER

Question 3

Let $f(x) = x^5 + mx^3 - nx^2 - 1$, where $m, n \in R$. If $x+2$ is a factor of *f* and when *f* is divided by $x-1$ the remainder is 5, the values of *m* and *n* are respectively

A. $-\frac{13}{12}$ and $-\frac{73}{12}$

B. $-\frac{59}{12}$ and $-\frac{25}{12}$

C. $-\frac{5}{4}$ and $-\frac{23}{4}$

D. $\frac{13}{12}$ and $-\frac{47}{12}$

E. $-\frac{59}{12}$ and $-\frac{143}{12}$

Question 4

The following set of simultaneous equations will have no solutions for

$$nx - 2y = m$$
$$n^2x + 6y = m + 1$$

A. $n \in R \setminus \{-3\}$, $m \in R \setminus \left\{-\frac{1}{4}\right\}$

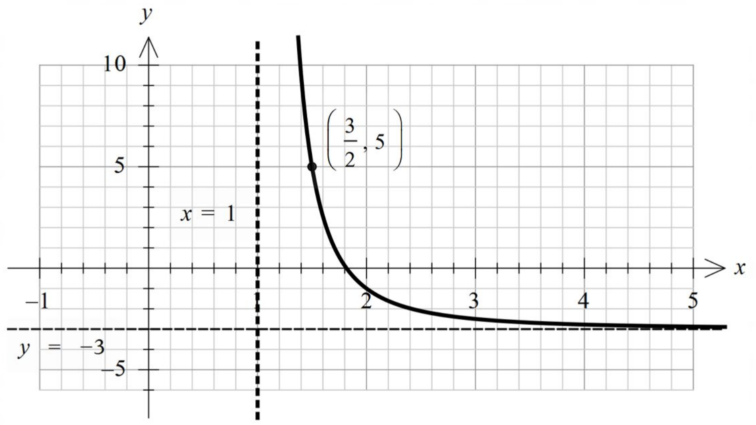
B. $n = -3$, $m = -\frac{1}{4}$ only

C. $n = -3$, $m \in R \setminus \left\{-\frac{1}{4}\right\}$ only

D. $n = 0$, $m = -\frac{1}{4}$ or $n = -3$, $m = -\frac{1}{4}$

E. $n = 0$, $m \in R \setminus \left\{-\frac{1}{4}\right\}$ or $n = -3$, $m \in R \setminus \left\{-\frac{1}{4}\right\}$

Question 5



An equation for the inverse function of the above graph, $y = f(x)$ could be

A. $f^{-1}(x) = \dfrac{1}{(x-1)^2} - 3$

B. $f^{-1}(x) = \dfrac{2}{(x-1)^2} - 3$

C. $f^{-1}(x) = 1 + \sqrt{\dfrac{1}{x+3}}$

D. $f^{-1}(x) = 1 - \sqrt{\dfrac{2}{x+3}}$

E. $f^{-1}(x) = 1 + \sqrt{\dfrac{2}{x+3}}$

Question 6

Let $f: \left[-\frac{\pi}{3}, \pi\right] \to R, f(x) = -2\sin(3x) + \sqrt{3}$. The maximum rate of change of *f* with respect to *x* occurs when

A. $x = 0, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

B. $x = \dfrac{-\pi}{3}, \dfrac{\pi}{3}, \pi$

C. $x = \dfrac{\pi}{3}$ only

D. $x = \dfrac{-\pi}{6}, \dfrac{\pi}{2}$

E. $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

SECTION A - continued
TURN OVER

Question 7

Consider the functions

$f:\left[0,\frac{\pi}{2}\right) \to R, f(x)=\tan(x)$ and $g(x)=\sqrt{2x+1}$ over its maximal domain.

The rule and domain, respectively, for the function $h(x)=g(f(x))$ are

A. $h(x)=\sqrt{2\tan(x)+1}$ and $x \in \left[0,\frac{\pi}{2}\right)$

B. $h(x)=\sqrt{2\tan(x)}+1$ and $x \in \left[0,\frac{\pi}{2}\right)$

C. $h(x)=\sqrt{2\tan(x)+1}$ and $x \in \left[-\frac{1}{2},\infty\right)$

D. $h(x)=\sqrt{2\tan(x)+1}$ and $x \in [0,\infty)$

E. $h(x)=\tan\left(\sqrt{2x+1}\right)$ and $x \in \left[-\frac{1}{2},\infty\right)$

Question 8

A sequence of transformations applied to create the image rule $y=-\sqrt{3x+1}-\frac{1}{4}$ from the original function $y=-\sqrt{x}$, in an appropriate order, could be

A a reflection in the x-axis, then a translation 1 unit to the right, and then a dilation by a factor of 3 from the x-axis and finally a translation 4 units down.

B. a translation of 1 unit to the left, then a dilation by a factor of 3 from the y-axis and finally a translation of 4 units down.

C. a translation of 1 unit left, then a dilation by a factor of $\frac{1}{3}$ from the y-axis and finally a translation of $\frac{1}{4}$ of a unit down.

D. a reflection in the x-axis, a translation of 1 unit left, then a dilation by a factor of $\frac{1}{3}$ from the y-axis and finally a translation of $\frac{1}{4}$ of a unit down.

E. a translation of $\frac{1}{3}$ of a unit left, then a dilation by a factor of $\frac{1}{3}$ from the y-axis and finally a translation of $\frac{1}{4}$ of a unit down.

Question 9

When the transformation described by $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is applied to the function $y = -x^4$ its image rule is

A. $y = -3(1-2x)^4$

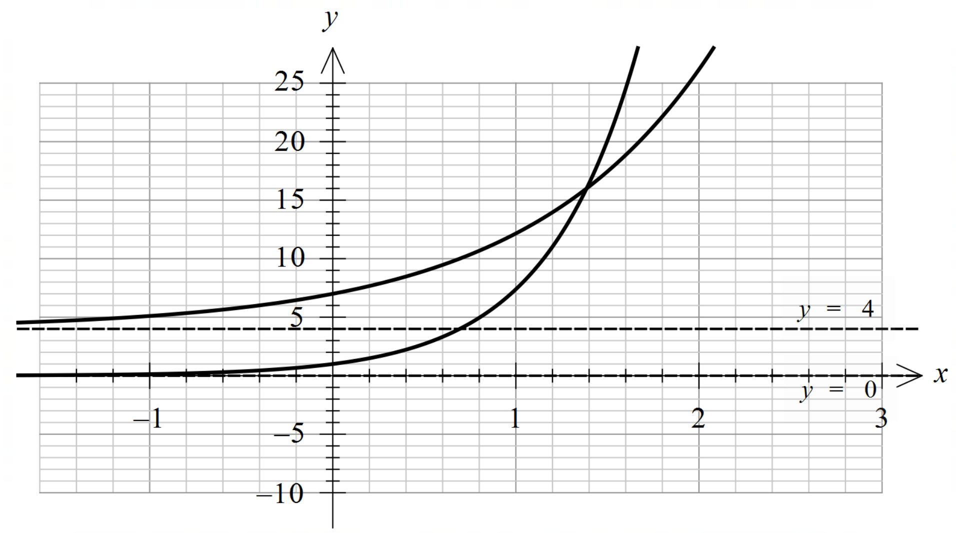
B. $y = -\dfrac{(3x+1)^4}{2}$

C. $y = \dfrac{(3x+1)^4}{2}$

D. $y = \dfrac{2(x-1)^4}{81}$

E. $y = -\dfrac{2(x-1)^4}{81}$

Question 10



The area enclosed by the two graphs shown above with equations $y = f(x) = e^{2x}$ and $y = g(x) = 3e^x + 4$ and the line $x = 0$ can be found by evaluating

A. $\displaystyle\int_{\log_e(4)}^{0} \left(f(x) - g(x)\right) dx$

B. $\displaystyle\int_{0}^{2\log_e(2)} \left(f(x) - g(x)\right) dx$

C. $\displaystyle\int_{-\infty}^{2\log_e(2)} \left(g(x) - f(x)\right) dx$

D. $\displaystyle\int_{0}^{\log_e(2)} \left(g(x) - f(x)\right) dx$

E. $\displaystyle\int_{-1}^{2\log_e(2)} \left(f(x) - g(x)\right) dx$

SECTION A - continued
TURN OVER

Question 11

If $g(x)=\frac{1}{2^x}$ then $g(1-x)$ equals

A. $g(-x)+\frac{1}{2}$

B. $\frac{1}{2}g(x)$

C. $2g(-x)$

D. $\frac{1}{2}g(-x)$

E. $2g(x)$

Question 12

The solution to the equation $\frac{1}{2}e^{x+2}\times e^{2x-1}=1$ is

A. $x=\frac{1}{3}\log_e(2)-1$

B. $x=\log_e(2)-\frac{1}{3}$

C. $x=\frac{\log_4(2)}{3\log_2(e)}-\frac{1}{3}$

D. $x=\frac{1}{3\log_2(e)}-1$

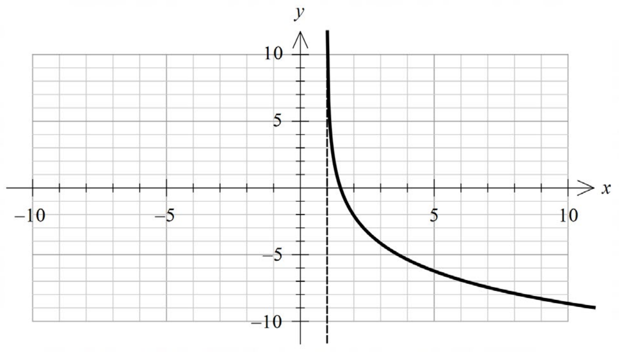
E. $x=\frac{1}{3\log_2(e)}-\frac{1}{3}$

SECTION A - continued

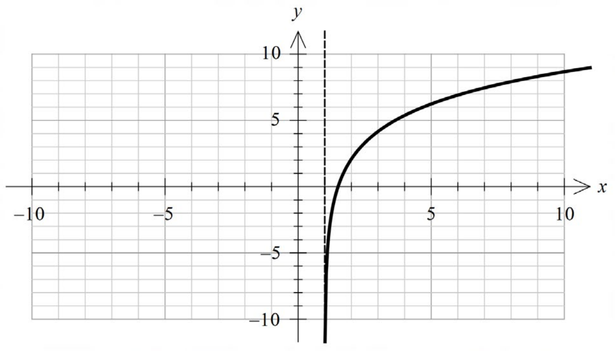
Question 13

The graph of the inverse f^{-1} where $f(x) = -3\log_e(2x-2)$ is

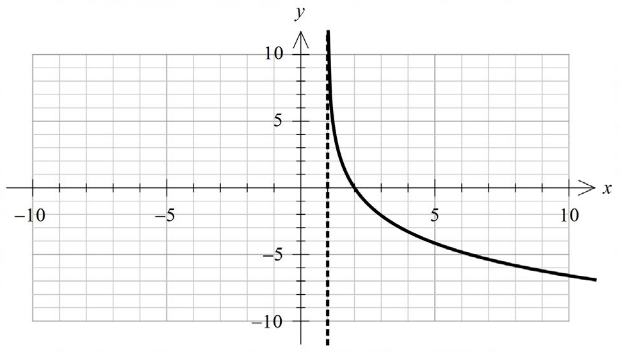
A.



B.



C.



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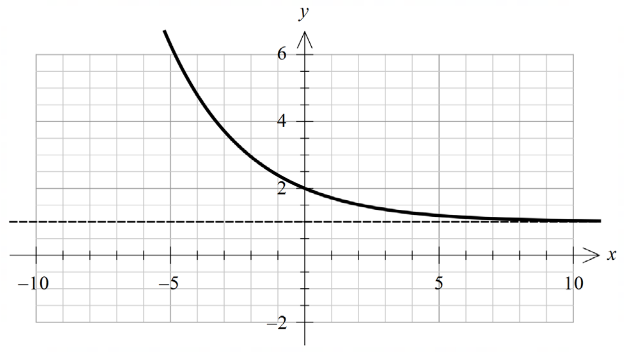
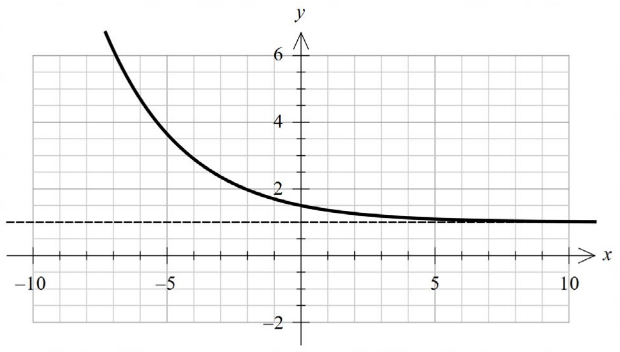


E.



SECTION A - continued
TURN OVER

Question 14

A 20 cm piece of wire is cut into two pieces. One of the pieces is made into a square with side length x cm. The other is made into a rectangle where the length is double the width.
The minimum total area, in cm^2, that can be formed is

A. $\dfrac{40}{17}$

B. $\dfrac{200}{17}$

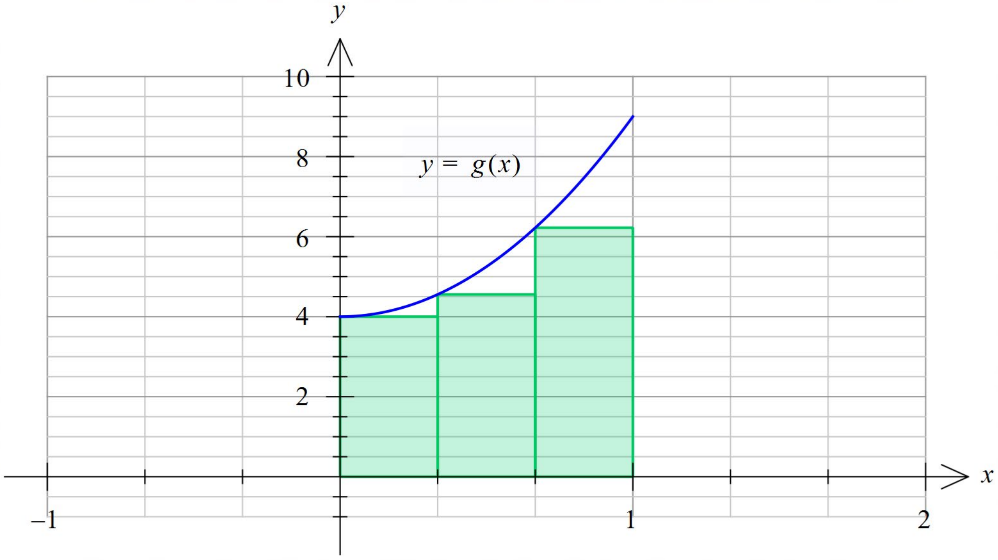
C. $\dfrac{1600}{289}$

D. 5

E. 25

Question 15

The area bounded by the graph of $y = g(x) = 5x^2 + 4$, the x-axis and the lines $x = 0$ and $x = 1$ is estimated by taking the average of the areas of the left-endpoint and right-endpoint rectangles with width $\frac{1}{3}$ of a unit. The left-endpoint rectangles are shown in the diagram below.



The approximation, using the average of the areas, written as a percentage, correct to one decimal place, of the exact area is

A. 69.9
B. 86.9
C. 101.6
D. 116.3
E. 304.9

Question 16

Consider the function h with rule

$$h(x)=\begin{cases}\log_e(1-x), & -3\le x\le 0\\ \log_e(x+1), & 0<x\le 3\end{cases}.$$

The average value of the function can be found by evaluating

A. $\dfrac{h(3)-h(-3)}{6}$

B. $\dfrac{1}{3}\int_{-3}^{0}\left(h(x)\right)dx$

C. $\int_{-3}^{3}\left(xh(x)\right)dx$

D. $\dfrac{1}{6}\int_{-3}^{3}\left(xh(x)\right)dx$

E. $\dfrac{1}{3}\int_{-3}^{0}\left(\log_e\left(1-x\right)\right)dx+\dfrac{1}{3}\int_{0}^{3}\left(\log_e\left(x+1\right)\right)dx$

Question 17

A probability distribution function for a discrete random variable X is defined as $\Pr\left(X=x\right)=\dfrac{20}{859}x^2-\dfrac{20}{859x}$.

A possible set of values for X is

A. $\{1,4,5\}$

B. $\{-1,4,5\}$

C. $\{-1,4,-5\}$

D. $\{1,-4,5\}$

E. $\{1,4,-5\}$

Question 18

A random sample of 300 Mathematical Methods students were selected and it was found that 80% of them did more than one hour of mathematics every day. The 90% confidence interval for the proportion of Mathematical Methods students who do less than one hour of mathematics per day, correct to three decimal places, is

A. (0.155, 0.245)
B. (0.162, 0.238)
C. (0.762, 0.838)
D. (0.755, 0.845)
E. (0.741, 0.859)

Question 19

The birth weight of new born babies is normally distributed.
0.575% of new borns have a weight less than 2 kg and 0.518% greater than 4.8 kg.
The rule for the distribution is closest to

A. $f(x)=\dfrac{1}{0.55\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-3.39}{0.55}\right)^2}$

B. $f(x)=\dfrac{1}{3.39\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-0.55}{3.39}\right)^2}$

C. $f(x)=\dfrac{1}{0.55\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-3.38}{0.55}\right)^2}$

D. $f(x)=\dfrac{1}{3.38\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-0.55}{3.38}\right)^2}$

E. $f(x)=\dfrac{1}{0.87\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-3.38}{0.87}\right)^2}$

Question 20

Let p be the probability function for the number of times, x, a prime number is rolled on a 10-sided fair die in n trials. The sides of the die are numbered from one to ten. One is not a prime number. Let f be the probability function for the number of times, m, a number greater than four is rolled in n trials.
$f(m)$ is given by

A. $p\left(1-\dfrac{m}{n}\right)$

B. $1-p(x)$

C. $p(n-m)$

D. $1-p(m)$

E. $p(m-n)$

END OF SECTION A

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SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

a. If $\frac{2x+1}{2x-3} = a + \frac{b}{2x-3}$ show that $a = 1$ and $b = 4$. 1 mark

Let $f(x) = \frac{2x+1}{2x-3}$.

b. i. State the equations of any asymptotes of the graph of *f*. 1 mark

ii. State the maximal domain and range of f. 2 marks

Let $f_1 : \left[-\frac{1}{2}, \frac{1}{6}\right] \to R,\ f_1(x) = \frac{2x+1}{2x-3}$.

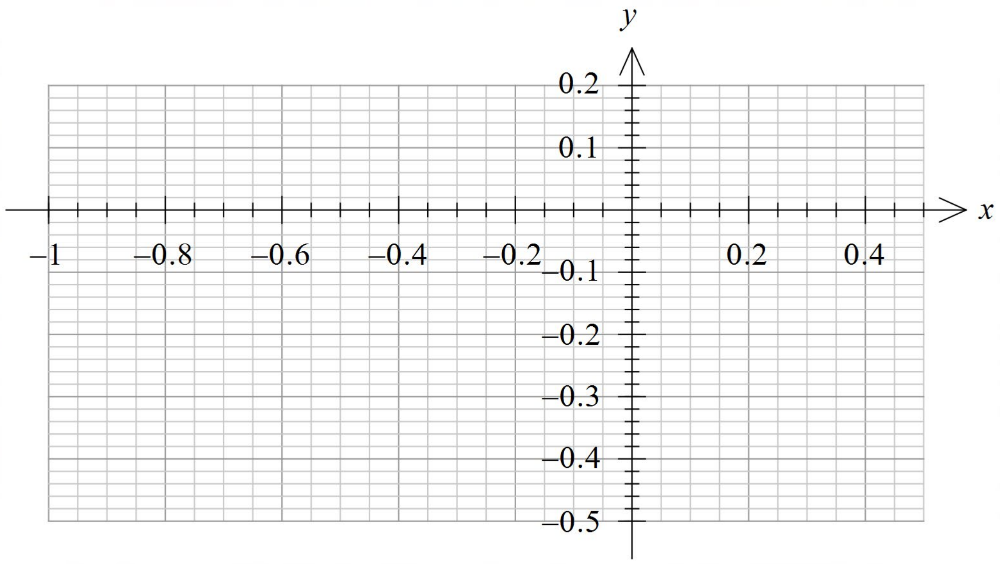
c. Explain why $f_1\left(f_1(x)\right)$ exists. 1 mark

SECTION B - Question 1 - continued

Let $g(x) = f_1(f_1(x)) = \dfrac{1-6x}{2x-11}$.

d. i. State the coordinates of the axial intercepts of the graph of g. 2 marks

ii. Sketch the graph of $y = g(x)$, labelling axial intercepts and endpoints with their coordinates. 2 marks



e. i. Write down a suitable integral statement that would find the area bounded by the graph of g and the lines $y = x$ and $x = \frac{1}{6}$. 2 marks

ii. Hence find this area, described in **part e.i.**, giving your answer correct to three decimal places. 1 mark

iii. State the area enclosed between the graph of g^{-1} and the lines $y = x$ and $y = \frac{1}{6}$, correct to three decimal places. 1 mark

SECTION B - continued
TURN OVER

Question 2 (15 marks)

The graph of $d(t) = e^{-kt}\sin(kt)$, can be used to describe drug absorption in the bloodstream, where d is in mg/litre and time t is in minutes, $t \geq 0$. k is a positive real constant.

a. Find $d'(t)$. 1 mark

b. **i.** Hence show that the general solution for the t-ordinates of the stationary points in the graph of $d(t)$ are found by solving $\tan(kt) = 1$. 2 marks

ii. State the general solution for the equation $d'(t) = 0$. Give your answer in terms of k. 1 mark

iii. Hence find the t-ordinate, in terms of k, of the maximum stationary point of the graph of

$$d: \left[0, \frac{2}{k}\right] \to R, d(t) = e^{-kt}\sin(kt).$$

1 mark

SECTION B - Question 2 - continued

Juniper is in hospital and needs a particular drug for her pain.

Let $d_j : [0,10] \to R, d_j(t) = 10e^{-0.2t} \sin(0.2t)$ where the particular drug in Juniper's bloodstream is measured in mg/litre at time, t minutes.

c. After how many minutes will Juniper's bloodstream contain the maximum amount of the drug? State the maximum amount of the drug. 2 marks

d. Find the average rate of change of the amount of the drug in Juniper's blood stream, in mg/L/min, over the interval $t \in [0,10]$. Give your answer correct to three decimal places. 2 marks

e. Show that the graphs of
$d_j : [0,10] \to R, d_j(t) = 10e^{-0.2t} \sin(0.2t)$
and
$p : [0,10] \to R, p(t) = 10e^{-0.2t}$
intersect at only one point and that it is not at the stationary point found in **part c**. 2 marks

SECTION B - Question 2 - continued
TURN OVER

f. Find the difference in time between the maximum stationary point on the graph of $d:\left[0,\frac{2}{k}\right]\to R, d(t)=e^{-kt}\sin(kt)$ and the point where the graph of $y=e^{-kt}$ is tangential to d for **(i)** $k=0.2$ and **(ii)** $k=0.02$. 2 marks

g. Find the difference in time between the maximum stationary point on the graph of $d:\left[0,\frac{2}{k}\right]\to R, d(t)=e^{-kt}\sin(kt)$ and the point where the graph of $y=e^{-kt}$ is tangential to d. Give your answer in terms of k. 2 marks

SECTION B - continued

Question 3 (17 marks)

Carlin, the manager of Office Supplies, is always looking for ways to improve customer service. He found that the waiting time, T minutes, for customers to be served has a probability density function w defined by

$$w(t) = \begin{cases} \dfrac{3}{64}t(4-t)^2 & 0 \le t \le 4 \\ 0 & \text{elsewhere} \end{cases}.$$

a. i. Find the expected value of T. 1 mark

ii. Find the median value of T. Give your answer in minutes correct to two decimal places. 1 mark

iii. Find the standard deviation of T. 2 marks

Carlin introduces a customer self-service option and finds that the mean waiting time is halved. The waiting time, T_1 minutes, for customers to be served now has a probability density function w_1 defined by

$$w_1(t) = \begin{cases} at(b-t)^2 & 0 \le t \le b \\ 0 & \text{elsewhere} \end{cases}$$ where a and b are real constants.

b. i. Find the values of a and b. 2 marks

ii. By what percentage was the standard deviation reduced? 1 mark

SECTION B - Question 3 - continued
TURN OVER

Carlin noticed that their 9 kg Ecostore laundry powder was not selling. A customer survey revealed that the most common complaint was that less than 9 kg of laundry powder was found in the container. Carlin rang the factory that supplies the laundry powder and they claimed that at least 80% of their 9 kg Ecostore laundry powder containers weighed more than 9 kg.

Carlin organized quality control inspectors to visit the factory. They decided to take a random sample of 1000 containers and they found that 867 of them weighed at least 9 kg.

c. **i.** Find a 95% confidence interval for the proportion of containers that weigh at least 9 kg. Give your answer correct to three decimal places. 1 mark

ii. According to the 95% confidence interval, is the factory misleading Office Supplies? Explain. What should the factory be claiming? 1 mark

The life time of the ink cartridges in Office Supplies is normally distributed with a mean of 3005.5 pages and a standard deviation of 50 pages when using plain text. Carlin would like to claim that 95% of the cartridges will print at least k pages.

d. **i.** Find the value of k correct to one decimal place. 1 mark

ii. A company buys five cartridges. What is the probability that more than 3 of them will print at least 3030 pages? Give your answer correct to four decimal places. 2 marks

Carlin keeps two boxes of chocolates in the staffroom for the staff. The blue box contains 20 white Ferrero chocolates and 15 dark Ferrero chocolates. The green box contains p white Ferrero chocolates and p^2+1 dark Ferrero chocolates where $p \in Z^+$. The probability Carlin selects the blue box is $\frac{1}{3}$. Carlin selects a box and then randomly selects a chocolate for his morning tea. It is a white chocolate.

e. **i.** What is the probability it was from the green box? Give your answer in terms of p. 2 marks

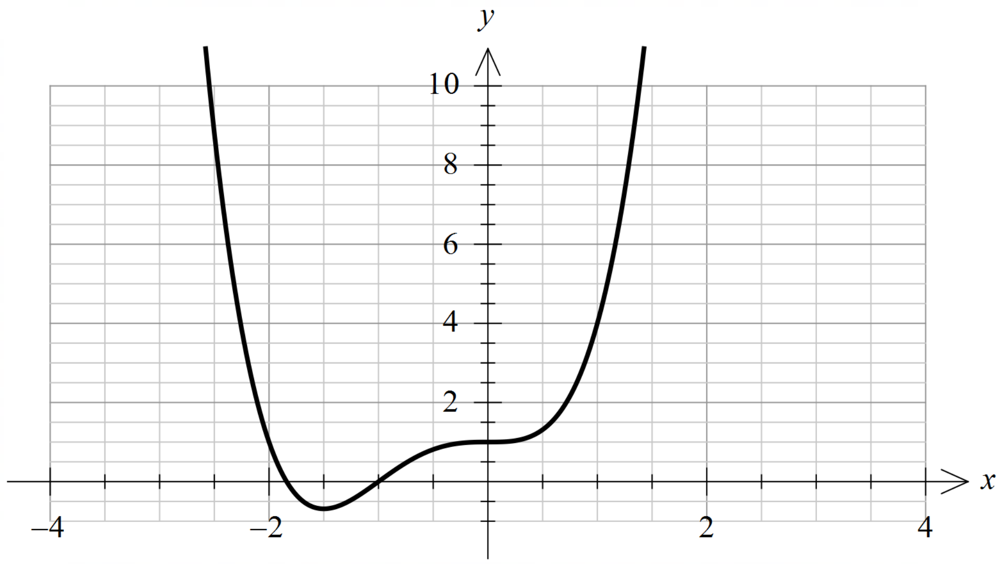
ii. What is the maximum value the probability in **part e.i.** can have? 1 mark

Whether a chocolate is classified as white or dark depends on the amount of cocoa it contains. The amount of cocoa in the white Ferrero chocolates is normally distributed with a mean of 12 g and a standard deviation of 1 g. The amount of cocoa in the dark Ferrero chocolates is normally distributed with a mean of 15 g and a standard deviation of 2 g. A chocolate is classified as white if it has less than b g of cocoa and dark otherwise. The probability that a white chocolate is misclassified is the same as a dark chocolate being misclassified.

f. If the white chocolate Carlin is eating has been misclassified, what is the smallest amount of cocoa it could contain? 2 marks

Question 4 (15 marks)

The graph of $y = f(x) = (x+1)\left(x^3 + x^2 - x + 1\right)$ is shown below.



a. Label the stationary points with their coordinates on the graph above. 1 mark

Now consider the graphs of $y = f(x) = (x+1)\left(x^3 + mx^2 - x + 1\right)$ where m is a real constant.

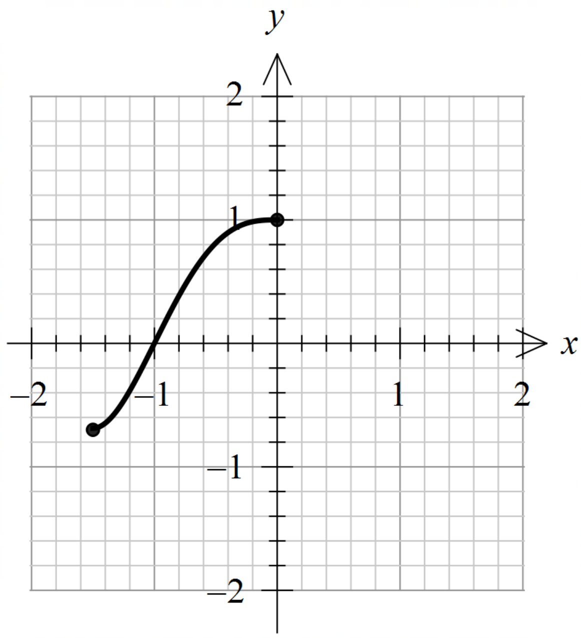
b. Find the x coordinate of each of the stationary points, giving answers in terms of m where appropriate. 2 marks

c. Hence, explain why there will always be three stationary points for $m \in R \setminus \{1\}$ and only two when $m = 1$. 2 marks

d. For what values of m will there be two negative x coordinates of the turning points? 1 mark

e. State the nature of the stationary point at (0, 1) for the different values of m. 2 marks

The graph of $g:\left[-\frac{3}{2},0\right]\to R, g(x)=(x+1)\left(x^3+x^2-x+1\right)$ is shown below.



f. Sketch the graph of the inverse function, $y=g^{-1}(x)$ on the set of axes above, labelling the endpoints with their coordinates. 1 mark

The graph of g^{-1} is translated left 1 unit and up 1 unit.

g. Find the area bounded by the graph of g and the translated graph of g^{-1}. 2 marks

h. Find the equation of the tangent line to the graph of g where the gradient is a maximum. 2 marks

i. State a sequence of transformations that will transform the tangent line found in **part h.** to the line that passes through the endpoints of g. 2 marks

END OF QUESTION AND ANSWER BOOK