Neap

Trial Examination 2021

VCE Mathematical Methods Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (4 marks)

a.

	People younger than 40	**People older than 40**	**Total**
Yes	**7**	**22**	**29**
No	**8**	**13**	**21**
Total	**15**	**35**	50

any 4 correct values A1
all 8 correct values A2

b. For independence, $\Pr(Y \cap F) = \Pr(Y) \times \Pr(F)$. M1

Note: For independence $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$, *is also acceptable, but the response must also specify that* $\Pr(A \cap B) = \Pr(Y \cap F)$, $\Pr(A) = \Pr(Y)$, $\Pr(B) = \Pr(F)$.

$\Pr(Y \cap F) = \frac{7}{50}$ $\quad \Pr(Y) = \frac{29}{50}$ $\quad \Pr(F) = \frac{15}{50} = \frac{3}{10}$

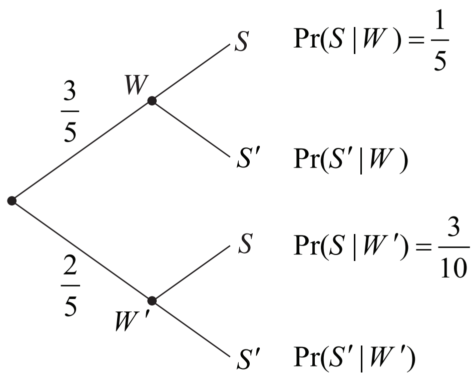
$$\begin{aligned}\Pr(Y) \times \Pr(F) &= \frac{29}{50} \times \frac{3}{10} \\ &= \frac{87}{500}\end{aligned}$$

Since $\Pr(Y \cap F) \neq \Pr(Y) \times \Pr(F)$, the relationship is not independent. A1

Note: Consequential on answer to ***Question 1a.***

Question 2 (3 marks)

a.



correct branches A1
correct outcomes and possibilities A1

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b. Let $b = \Pr(S)$ when W' has occurred.

$$\Pr(S \mid W') = \Pr(W') \times \Pr(S)$$

$$\frac{3}{10} = \frac{2}{5} \times b$$

$$b = \frac{3}{10} \times \frac{5}{2}$$

$$= \frac{3}{4}$$

$$\Pr(S') \text{ when } W' \text{ has occurred} = 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

$$\Pr(W' \cap S') = \frac{2}{5} \times \frac{1}{4}$$

$$= \frac{1}{10}$$ A1

Note: Consequential on answer to ***Question 2a.***

Question 3 (2 marks)

$$\log_3\left(\frac{\frac{x}{3}}{y}\right) = \log_3\left(\frac{1}{9^{-1}}\right)$$

$$\log_3\left(\frac{x}{3y}\right) = \log_3(9)$$

$$\log_3\left(\frac{x}{3y}\right) = 2$$

$$3^2 = \frac{x}{3y}$$ M1

$$9 = \frac{x}{3y}$$

$$3y = \frac{x}{9}$$

$$y = \frac{x}{27}$$ A1

Question 4 (3 marks)

a. $\text{LHS} = \dfrac{x^{\frac{1}{2}} - y^{\frac{1}{2}}}{x^{-\frac{1}{2}} - y^{-\frac{1}{2}}}$

$= \dfrac{\left(x^{\frac{1}{2}} - y^{\frac{1}{2}}\right)}{\left(x^{-\frac{1}{2}} - y^{-\frac{1}{2}}\right)}$

$= \dfrac{\left(x^{\frac{1}{2}} - y^{\frac{1}{2}}\right)}{\left(\dfrac{1}{x^{\frac{1}{2}}} - \dfrac{1}{y^{\frac{1}{2}}}\right)}$

$= \dfrac{\left(x^{\frac{1}{2}} - y^{\frac{1}{2}}\right)}{\left(\dfrac{y^{\frac{1}{2}} - x^{\frac{1}{2}}}{x^{\frac{1}{2}}y^{\frac{1}{2}}}\right)}$ M1

$= \left(x^{\frac{1}{2}} - y^{\frac{1}{2}}\right) \times \left(\dfrac{x^{\frac{1}{2}}y^{\frac{1}{2}}}{y^{\frac{1}{2}} - x^{\frac{1}{2}}}\right)$

$= \left(x^{\frac{1}{2}} - y^{\frac{1}{2}}\right) \times -\left(\dfrac{x^{\frac{1}{2}}y^{\frac{1}{2}}}{x^{\frac{1}{2}} - y^{\frac{1}{2}}}\right)$

$= -x^{\frac{1}{2}}y^{\frac{1}{2}}$ A1

b. $\dfrac{x^{\frac{1}{2}} - y^{\frac{1}{2}}}{x^{-\frac{1}{2}} - y^{-\frac{1}{2}}} > 2$

$-x^{\frac{1}{2}}y^{\frac{1}{2}} > 2$

$-x^{\frac{1}{2}} \times 1 > 2$

$-x^{\frac{1}{2}} > 2$

$x^{\frac{1}{2}} < -2$

$x < 4$

Therefore, $x > 0$.

Hence, $0 < x < 4$. A1

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Question 5 (7 marks)

a. Amplitude can be directly read from the equation of type $f(x) = a\sin(nx + h) + k$, where the magnitude of a represents the amplitude.

The amplitude is 3. A1

b. $P = \frac{2\pi}{n}$, where $n = \frac{\pi}{4}$.

$$\begin{aligned} P &= \frac{2\pi}{\frac{\pi}{4}} \\ &= 2\pi \times \frac{4}{\pi} \\ &= 8 \end{aligned}$$

A1

c. **i.**

$$\begin{aligned} f(4) &= -3\sin\left(\frac{\pi x}{4}\right) + 1 \\ &= -3\sin\left(\frac{\pi \times 4}{4}\right) + 1 \\ &= -3\sin(\pi) + 1 \\ &= -3 \times 0 + 1 \\ &= 1 \end{aligned}$$

A1

ii.

$$\begin{aligned} f\left(\frac{4}{3}\right) &= -3\sin\left(\frac{\frac{4\pi}{3}}{4}\right) + 1 \\ &= -3\sin\left(\frac{4\pi}{3} \times \frac{1}{4}\right) + 1 \\ &= -3\sin\left(\frac{\pi}{3}\right) + 1 \end{aligned}$$

M1

$$\begin{aligned} &= -3 \times \frac{\sqrt{3}}{2} + 1 \\ &= 1 - \frac{3\sqrt{3}}{2} \quad \text{or} \quad \frac{2 - 3\sqrt{3}}{2} \end{aligned}$$

A1

iii. $-3\sin\left(\frac{\pi x}{4}\right)+1=-\frac{1}{2}$

$-3\sin\left(\frac{\pi x}{4}\right)=-\frac{3}{2}$

$\sin\left(\frac{\pi x}{4}\right)=\frac{1}{2}$

$\frac{\pi x}{4}=\frac{\pi}{6},\ \frac{5\pi}{6}$ M1

Note: Both values must be given.

$\frac{x}{4}=\frac{1}{6},\ \frac{5}{6}$

$x=\frac{4}{6},\ \frac{20}{6}$

$=\frac{2}{3},\ \frac{10}{3}$ A1

Note: Both values must be given.

Question 6 (11 marks)

a. For the x-intercept, solve $g(x) = 0$.

$0=-(x-1)(x+2)^2$

$x-1=0 \qquad x+2=0$

$x=1 \qquad x=-2$

Therefore, the coordinates of the x-intercepts are (1, 0) and (–2, 0). A1

b. Find the y-intercept by substituting $x = 0$.

$g(0)=-(0-1)(0+2)^2$

$=-(-1)(2)^2$

$=4$

The coordinates of the y-intercept are (0, 4). A1

c. $g(x)=-(x-1)(x+2)^2$

$=-(x-1)\left(x^2+4x+4\right)$

$=-\left(x^3+4x^2+4x-x^2-4x-4\right)$ M1

$=-\left(x^3+3x^2-4\right)$

$=-x^3-3x^2+4$

$g'(x)=-3x^2-6x$ A1

d. $g'(x) = -3x^2 - 6x$

$-3x - 6x = 0$

$-3x(x+2) = 0$

$x = 0$ or -2 M1

From **part b.**, $g(0) = 4$:

$g(-2) = -(-2)^3 - 3(-2)^2 + 4$

$= 8 - 12 + 4$

$= 0$ M1

*Note: The answer for g(–2) can also be drawn from **Question 6b**.*

stationary points = (0, 4) and (–2, 0) A1

*Note: Consequential on answer to **Question 6c.***

e. Since $g(x) = -(x-1)(x+2)^2$ is an inverted cubic function with x-intercepts of (1, 0) and (–2, 0), given $(x+2)^2$, this indicates a stationary point on the x-axis. From **part d.**, (0, 4) is also a stationary point.

Following the coordinates from (–2, 0) to (0, 4) to (1, 0):

(0, 4) is a local maximum and (–2, 0) is a local minimum. A1

f. Endpoints are found at:

$g(-3) = -(-3-1)(-3+2)^2$

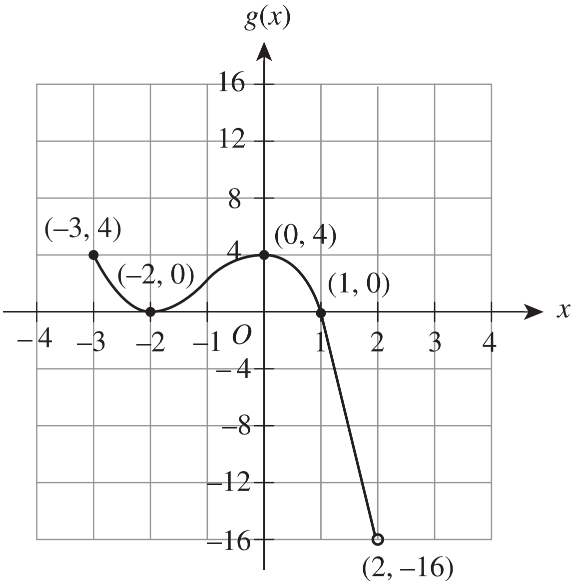
$= 4$

$(-3, 4)$

$g(2) = -(2-1)(2+2)^2$

$= -16$

$(2, -16)$



(–2, 0), (0, 4), (1, 0) all correctly labelled A1

correct shape A1

(–3, 4) labelled with a closed circle and (2, –16) labelled with an open circle A1

*Note: Consequential on answer to **Questions 6c., d. and e.***

Question 7 (10 marks)

a. $\lim\limits_{x \to 1} f(x) = 2 - 3 + 2 - 1 + 1$

$= 1$ A1

b. $f'(x) = 8x^3 - 9x^2 + 4x - 1$ A1

c. $g(3x) = 2(3x)^2 + 3x - 5$

$= 18x^2 + 3x - 5$ A1

d. **i.** $h(x) = f(x) - g(x)$

$= 2x^4 - 3x^3 + 2x^2 - x + 1 - \left(2x^2 + x - 5\right)$

$= 2x^4 - 3x^3 + 2x^2 - x + 1 - 2x^2 - x + 5$

$= 2x^4 - 3x^3 - 2x + 6$ A1

ii. $\int_1^2 h(x)dx = \int_1^2 \left(2x^4 - 3x^3 - 2x + 6\right) \cdot dx$

$= \left[\frac{2x^5}{5} - \frac{3x^4}{4} - x^2 + 6x\right]_1^2$ M1

$= \left(\frac{64}{5} - \frac{3 \times 16}{4} - 4 + 12\right) - \left(\frac{2}{5} - \frac{3}{4} - 1 + 6\right)$ M1

$= \frac{64}{5} - 4 - \frac{2}{5} + \frac{3}{4} - 5$

$= \frac{62}{5} + \frac{3}{4} - \frac{45}{5}$

$= \frac{17}{5} + \frac{3}{4}$

$= \frac{68}{20} + \frac{15}{20}$

$= \frac{83}{20}$ A1

Note: Consequential on answer to ***Question 7d.i.***

e. **Method 1:**

$$2x^4 - 3x^3 + 2x^2 - x + 1 = 2x^3(x-2) + x^2(x-2) + 4x(x-2) + 7(x-2) + 15 \quad \text{M1}$$

$$= 2x^4 - 4x^3 + x^3 - 2x^2 + 4x^2 - 8x + 7x - 14 + 15$$

$$= 2x^4 - 3x^3 + 2x^2 - x + 1 \quad \text{A1}$$

$$\frac{f(x)}{x-2} = \frac{2x^4 - 3x^3 + 2x^2 - x + 1}{(x-2)}$$

$$= 2x^3 + x^2 + 4x + 7 + \frac{15}{x-2} \quad \text{A1}$$

Method 2:

$$\begin{array}{r|l} & 2x^3 + x^2 + 4x + 7 \\ x-2 & 2x^4 - 3x^3 + 2x^2 - x + 1 \end{array} \quad \text{A1}$$

$$-\underline{(2x^4 - 4x^3)}$$

$$x^3 + 2x^2 - x + 1$$

$$-\underline{(x^3 - 2x^2)}$$

$$4x^2 - x + 1$$

$$-\underline{(4x^2 - 8x)}$$

$$7x + 1$$

$$-\underline{(7x - 14)} \quad \text{M1}$$

$$15$$

$$\frac{f(x)}{x-2} = \frac{2x^4 - 3x^3 + 2x^2 - x + 1}{(x-2)}$$

$$= 2x^3 + x^2 + 4x + 7 + \frac{15}{x-2} \quad \text{A1}$$