

Trial Examination 2021

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (3 marks)

a.
$$y = \frac{3}{2} \cos\left(\frac{3x}{2}\right)$$
$$\frac{dy}{dx} = -\frac{3}{2} \times \frac{3}{2} \sin\left(\frac{3x}{2}\right)$$
$$= -\frac{9}{4} \sin\left(\frac{3x}{2}\right)$$
A1
b.
$$f'(x) = -e^{-x} \times \log_e(-x) + e^{-x} \times -\frac{1}{-x}$$
M1

Note: Product or quotient rule should be used.

A1

$$= -e^{-x} \log_e(-x) + \frac{1}{xe^x}$$

$$f'(-1) = -e^{-(-1)} \log_e(-(-1)) + \frac{1}{(-1)e^{(-1)}}$$

$$= -e$$

Question 2 (3 marks)

$$\int_{1}^{5} \frac{1}{1-2x} dx = -\int_{1}^{5} \frac{1}{2x-1} dx$$

$$= -\frac{1}{2} [\log_{e}(2x-1)]_{1}^{5} \qquad M1$$

$$= -\frac{1}{2} (\log_{e}(9) - \log_{e}(1))$$

$$= -\log_{e}(3) \qquad M1$$

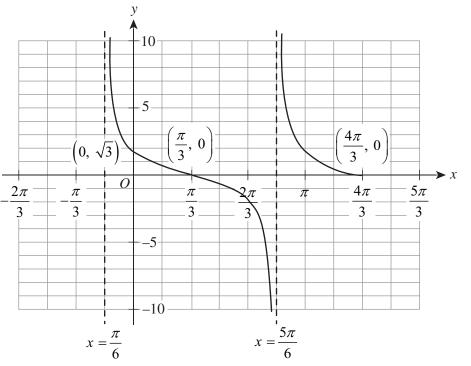
$$= \log_{e} \left(\frac{1}{3}\right)$$

$$\therefore b = \frac{1}{3}$$
A1

Question 3 (7 marks)

a.
$$\tan\left(\frac{\pi}{3} - x\right) = 0$$
$$\tan\left(x - \frac{\pi}{3}\right) = 0$$
M1
$$x - \frac{\pi}{3} = 0 + n\pi, \ n \in \mathbb{Z}$$
$$x = \frac{\pi}{3} \text{ and } x = \frac{4\pi}{3}$$
A1





correct intercepts A1 correct asymptotes A1

correct shape A1

M1

c.
$$f(x) = \tan\left(\frac{\pi}{3} - x\right)$$
$$f'(x) = -\sec^2\left(\frac{\pi}{3} - x\right)$$
$$f'(0) = -\sec^2\left(\frac{\pi}{3}\right)$$
$$= -\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right)^2$$
$$= -4$$
$$m_T = -4 \text{ and point } \left(0, \sqrt{3}\right)$$
$$y - \sqrt{3} = -4x$$
$$y = -4x + \sqrt{3}$$

Question 4 (3 marks) $log_{2}(2x + 4) - 2log_{2}(x + 2) - 1 = 0$ $log_{2}(2x + 4) - log_{2}(x + 2)^{2} = 1$ $log_{2}\left(\frac{2x + 4}{(x + 2)^{2}}\right) = log_{2}(2)$ $\frac{2x + 4}{(x + 2)^{2}} = 2$ M1 $2x + 4 = 2(x + 2)^{2}$ $x + 2 = (x + 2)^{2}$ $(x + 2)^{2} - (x + 2) = 0$ (x + 2)(x + 2 - 1) = 0 (x + 2)(x + 1) = 0 x = -2 or x = -1

But x > -2 and therefore x = -1 only.

Question 5 (3 marks)

$$X \sim Bi(4, p)$$

 $Pr(X = 1) = {}^{4}C_{1}p(1-p)^{3}$
 $= 4p(1-p)^{3}$
 $Pr(X = 3) = {}^{4}C_{3}p^{3}(1-p)$ M1
 $= 4p^{3}(1-p)$
 $4Pr(X = 1) = Pr(X = 3)$
 $4 \times 4p(1-p)^{3} = 4p^{3}(1-p)$
 $4p(1-p)^{3} = p^{3}(1-p)$
 $4p(1-p)^{3} - p^{3}(1-p) = 0$
 $p(1-p)(4(1-2p+p^{2})-p^{2}) = 0$
 $p(1-p)(4(1-2p+p^{2})-p^{2}) = 0$
 $p(1-p)(2p-2)(2p-2) = 0$
 $p(1-p)(2p-2)(2p-2) = 0$
 $p(1-p)(p-2)(2p-2) = 0$
 $p(1-p)(2p-2)(2p-2) = 0$

Question 6 (2 marks)

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

Let $\sqrt{\frac{\frac{3}{5} \times \frac{2}{5}}{n}} = \frac{1}{10}$.
 $\sqrt{\frac{\frac{3}{5} \times \frac{2}{5}}{n}} = \frac{1}{10}$
 $\sqrt{\frac{6}{25n}} = \frac{1}{10}$
 $\frac{6}{25n} = \frac{1}{100}$
 $25n = 600$
 $n = 24$

Question 7 (5 marks)

a. Let
$$f(x) = 0$$
.
 $x \cos(x^2) = 0$
 $x = 0 \text{ or } \cos(x^2) = 0$
 $x^2 = \frac{\pi}{2}$ gives first positive solution.
 $a = \sqrt{\frac{\pi}{2}}$ as $a > 0$
 $= \frac{\sqrt{\pi}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
 $= \frac{\sqrt{2\pi}}{2}$

M1

A1

b.
$$\frac{d(\sin(x^2))}{dx} = 2x \times \cos(x^2)$$
 A1

$$\int 2x \cos(x^2) dx = \sin(x^2)$$
 M1
Area
$$= \int_0^{\sqrt{2\pi}} x \cos(x^2) dx$$
 M1

$$= \left[\frac{1}{2} \sin(x^2)\right]_0^{\sqrt{2\pi}}$$
 =
$$\left[\frac{1}{2} \sin\left(\frac{\sqrt{2\pi}}{2}\right)^2\right] - \left[\frac{1}{2} \sin((0)^2)\right]$$
 =
$$\left[\frac{1}{2} \sin\left(\frac{\pi}{2}\right)\right] = \left[\frac{1}{2} \sin\left(\frac{\pi}{2}\right)\right]$$
 A1

Question 8 (6 marks)

a.
$$\int_{0}^{p} a\sqrt{p-x} \, dx = 1$$

$$\left[-\frac{2a}{3}(p-x)^{\frac{3}{2}} \right]_{0}^{p} = 1$$

(0) $-\left(-\frac{2a}{3}p^{\frac{3}{2}} \right) = 1$

$$\frac{2a}{3}p^{\frac{3}{2}} = 1$$

$$a = \frac{3}{2p^{\frac{3}{2}}}$$

b. $f(x) = \frac{3}{2p^{\frac{3}{2}}} \sqrt{p-x}$
 $q = f(0)$

$$= \frac{3}{2p^{\frac{3}{2}}} \times p^{\frac{1}{2}}$$

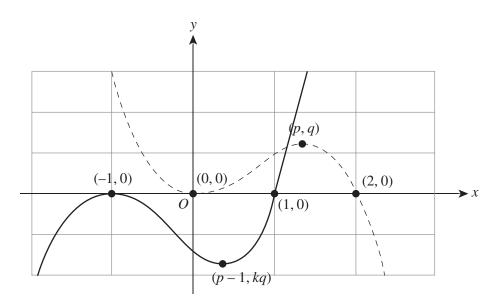
 $= \frac{3}{2p}$
A1

7

c. Let
$$g(p) = p + q$$
.
 $g(p) = p + \frac{3}{2p}$
 $g'(p) = 1 - \frac{3}{2p^2}$
Let $g'(p) = 0$.
 $1 - \frac{3}{2p^2} = 0$
 $p^2 = \frac{3}{2}$
 $p = \sqrt{\frac{3}{2}} \text{ as } p > 0$
 $p + q = \sqrt{\frac{3}{2}} + \frac{3}{2(\sqrt{\frac{3}{2}})}$
 $= \frac{\sqrt{3}}{\sqrt{2}} + \frac{3\sqrt{2}}{2\sqrt{3}}$
 $= \frac{6+6}{2\sqrt{6}}$
 $= \sqrt{6}$
As $(p+q) = \sqrt{m}, \sqrt{m} = \sqrt{6}$
 $\therefore m = 6$

Question 9 (8 marks)





correct x-intercepts A1

M1

M1

A1

correct turning point A1

correct shape and scale A1

b.
$$f(x) = ax^2(x-2)$$
 where $a < 0$
 $f(x) = a(x^3 - 2x^2)$
 $f'(x) = a(3x^2 - 4x)$
Let $f'(x) = 0$.
 $3x^2 - 4x = 0$
 $x(3x - 4) = 0$
 $x = 0$ or $x = \frac{4}{3}$
 $\therefore p = \frac{4}{3}$

A1

M1

c. The average value of g(x) over the interval [-1,1] is equal to -1. Find q in terms of k.

$$\frac{1}{1--1} \int_{-1}^{1} g(x) dx = -1$$

$$\frac{1}{2} \int_{-1}^{1} kf(x+1) dx = -1$$

$$\frac{k}{2} \int_{-1}^{1} f(x+1) dx = -1$$

$$\frac{k}{2} \int_{0}^{2} dx^{2}(x-2) dx = -1$$

$$\frac{ka}{2} \int_{0}^{2} x^{2}(x-2) dx = -1$$

$$\frac{ka}{2} \int_{0}^{2} x^{3} - 2x^{2} dx = -1$$

$$\int_{0}^{2} x^{3} - 2x^{2} dx = -\frac{2}{ka}$$

$$\left[\frac{2^{4}}{4} - \frac{2(2)^{3}}{3}\right] = \frac{-2}{ka}$$

$$\frac{-4}{3} = \frac{-2}{ka}$$

$$a = \frac{3}{2k}$$

$$f(x) = \frac{3}{2k} (x^{3} - 2x^{2})$$

$$q = f\left(\frac{4}{3}\right)$$

$$= \frac{-16}{9k}$$

$$q = -\frac{16}{9k}$$

M1

M1