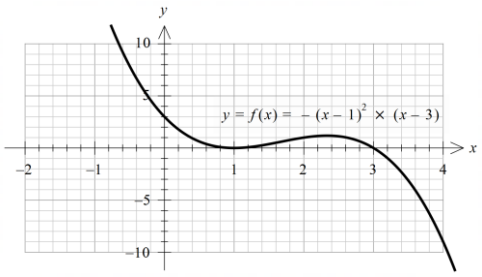
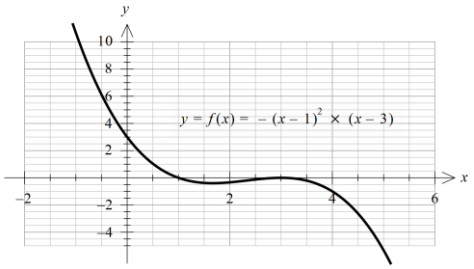
2021 VCE Mathematical Methods 2 (NHT) examination report

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Section A – Multiple-choice questions

Question	Answer	Comment
1	E	
2	A	
3	B	
4	A	$f(x) = x^3 + ax^2 + bx + c$, $c = 9$, $f'(3) = 27 + 6a + b = 0$, $a = \frac{-(b+27)}{6}$, $-5 = \frac{-(3+27)}{6}$, $a = -5,\ b = 3,\ c = 9$
5	C	
6	D	
7	B	
8		
9	C	
10	B	
11	E	$\hat{p} = \frac{0.6299 + 0.6699}{2} = 0.6499$, $1.96\sqrt{\frac{0.6499(1-0.6499)}{n}} = 0.02$, $n = 2185$
12	A	
13	B	
14	A	

Question	Answer	Comment
15	D	
16	C	$X \sim \text{Bi}(720, 0.7)$, $\Pr(X > 720 \times 0.72) = \Pr(X > 518.4) = 0.1187$
17	E	Area of the rectangle = $-b \times f(b)$, Area bounded by the curve *f*, the *x*-axis and the line *x* = *b* is $-\int_a^b (f(x))\,dx$, shaded region = $-b \times f(b) + \int_a^b (f(x))\,dx$
18	D	$\int (\cos(x) - x\sin(x))\,dx = x\cos(x) + c$, $\int (-x\sin(x))\,dx = x\cos(x) - \int (\cos(x))\,dx + c$, $\int (x\sin(x))\,dx = -x\cos(x) + \int (\cos(x))\,dx + c$
19	E	  $g(x) = \sqrt{f(x)}$, $f(x) \geq 0$, so the domain of *g* does not include the interval $(3, \infty)$
20	A	$\frac{4}{5} + \frac{1}{10} - b^3 + \frac{15b^2 - 2}{50} + \frac{9b + 5}{50} = 1$, $b = -\frac{2}{5}, \frac{1}{5}, \frac{1}{2}$ but $0 \leq \Pr(X = x) \leq 1$, $b = -0.4$

Section B

Question 1a.

period = 2π, amplitude = 2

Question 1b.

$c = \frac{\pi}{6}$

Question 1c.

–2

Question 1d.

$p = -\frac{\pi}{3}$ and $q = 1$

Question 1ei.

Derivative of *f* is $-\sqrt{3}$

Derivative of *g* is $-\sqrt{3}$

Question 1eii.

$y = -\sqrt{3}x + \frac{\pi\sqrt{3}+3}{3}$ OR $y = -\sqrt{3}x + \frac{\pi\sqrt{3}}{3} + 1$

Question 1eiii.

$y = \frac{\sqrt{3}x}{3} - \frac{\pi\sqrt{3}-9}{9}$ OR $y = \frac{3\sqrt{3}x - \sqrt{3}\pi + 9}{9}$

Question 1f.

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (g(x) - f(x))dx$

= 4

Question 2a.

28.562 (28.562374…)

Question 2bi.

$$h'(x) = \frac{-(6400\log_e(x-5) - 12800\log_e(2) - 3200)}{(x-5)^3}$$

OR

$$h'(x) = \frac{-3200(2\log_e(x-5) - 4\log_e(2) - 1)}{(x-5)^3}$$

Question 2bii.

36.788 (36.787944 …)

Question 2c.

5.288 (5.28750 …)

Question 2di.

Substituting $x = \frac{x'}{2}$ into $h(x)$ to get the stated answer

Domain: [20,100]

Question 2dii.

Dilation by a factor of 2 from the vertical axis

Question 2ei.

$$\int_{20}^{100} h_1(x)dx$$

≈ 1079.171 (1079.171389…)

Question 2eii.

$$\frac{A}{2}$$

Question 3a.

$$a = -\frac{k}{c^2} \text{ and } b = k$$

Question 3bi.

$$f(x) = -\frac{3}{8}x^2 + 6$$

Question 3bii.

$$\text{Distance} = \sqrt{(x-1.35)^2 + \left(-\frac{3}{8}x^2 + 6 - 3.7\right)^2}$$

x = 2.185 (2.18506 …)

distance = 0.978 (0.9782556 …)

Question 3c.

r = 6

Question 3d.

$$\text{y intercept of the light lines} = \frac{36-6\sqrt{11}}{5} = \frac{30}{6+\sqrt{11}} \approx 3.22 \ldots.$$

Shaded area (using sum of 2 triangles subtract a third)

$$= 2\frac{12\times 5}{2} - \frac{1}{2}\times 12 \times \frac{36-6\sqrt{11}}{5} = \frac{36\sqrt{11}+84}{5} \approx 40.679698 \ldots$$

OR

Shaded area (using trapizum and triangle)

$$= 2\left(\frac{1}{2}\left(\frac{36-6\sqrt{11}}{5}+5\right)\sqrt{11} + \frac{1}{2}(6-\sqrt{11})5\right) = \frac{12\left(3\sqrt{11}+7\right)}{5}$$

$$\approx 40.679698 \ldots.$$

$$\%\text{ of area} = \frac{\frac{\left(36\sqrt{11}+84\right)}{5}}{\frac{36\pi}{2}} \times 100 = \frac{2(3\sqrt{11}+7)}{15\pi} \times 100 \approx 72\%$$

Question 4a.

0.1587 (0.1586552 …)

Question 4b.

Pr($X < a$) = 0.1

$a \approx$ 307 (307.1844843 …)

Question 4c.

$$\int_{310}^{330} x \times b(x)dx$$

= 322

Question 4d.

$\int_{322}^{330} b(x)dx = \frac{328}{625} = 0.5248$

Question 4e.

$\int_{310}^{k} b(x)dx = 0.95$

$k \approx 328$ (328.047770 …)

Question 4f.

$\Pr(A \mid \text{inflation} > 325) = \frac{\Pr(A \cap > 325)}{\Pr(> 325)}$

$= \frac{0.154268766 \ldots}{0.154268766 \ldots + 0.130859 \ldots}$

≈ 0.5411 $(0.5410506 \ldots)$

Question 4g.

D ~ Binomial $n = 25$, $p = 0.08$

$\Pr(D = 4) \approx 0.0899$ (0.0899486 …)

Question 4h.

$\Pr(\hat{P} > 0.15) = \Pr(D > 3.75) = \Pr(D \geq 4)$

≈ 0.1351 (0.1350922 …)

Question 4i.

$\Pr\left(\hat{P}_n < \frac{1}{n}\right) = \Pr(X < 1) = \Pr(X = 0)$

$\binom{n}{0} 0.08^0 \times 0.92^n < 0.15$

$n > 22.75$, i.e. least value = 23

Question 5a.

$\left(\frac{1}{2}, -\frac{2}{5}\right)$ and $\left(\frac{\sqrt{3}+1}{2}, \frac{-2\sqrt{3}-2}{5}\right)$

Question 5b.

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}+1}{2}} (g(x) - f(x))dx$$

$$= \frac{6\sqrt{3} - 9}{10}$$

Question 5c.

$a = 5$

Question 5d.

$$= \frac{1}{\sqrt{a+1}}$$

Question 5e.

$$p'(x) = -\frac{2}{ax^3} = \frac{1-a}{a}$$

$$x = \frac{2^{1/3}}{(a-1)^{1/3}}$$

Question 5fi.

$$\frac{3(a-1)^{2/3}2^{1/3} - 2(a+1)}{2a}$$

Question 5fii.

$$\frac{3(a-1)^{2/3}2^{1/3} - 2(a+1)}{2a} \leq \frac{3 \times \frac{2a}{3} - 2a - 2}{2a} = -\frac{1}{a} < 0$$

Question 5fiii.

Tangent is always parallel to and less than *q*. *q* goes through the origin therefore must cross *p* twice creating a bounded area.

Question 5g.

$$\frac{1}{2}$$