

Student Name……………………………………

### MATHEMATICAL METHODS UNITS 3 & 4

### TRIAL EXAMINATION 2

**2022**

#### Reading Time: 15 minutes

Writing time: 2 hours

###### Instructions to students

This exam consists of Section A and Section B.

Section A consists of 20 multiple-choice questions, which should be answered on the detachable answer sheet which can be found on page 27 of this exam.

Section B consists of 5 extended-answer questions.

Section A begins on page 2 of this exam and is worth 20 marks.

Section B begins on page 10 of this exam and is worth 60 marks.

There is a total of 80 marks available.

All questions in Section A and Section B should be answered.

In Section B, where more than one mark is allocated to a question, appropriate working must be shown.

Where a numerical answer is required, an exact value must be given unless otherwise directed.

Diagrams in this exam are not to scale except where otherwise stated.

Students may bring one bound reference into the exam.

Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.

A formula sheet can be found on pages 25 and 26 of this exam.

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**SECTION A – Multiple-choice questions**

**Question 1**

Let ****.

The period and amplitude of *g* are respectively

1.  and 3
2. 
3.  and 4
4.  and 3
5.  and 4

**Question 2**

Let .

The range of *f* is

1. 
2. 
3. 
4. 
5. 

**Question 3**

The expression  is equivalent to

1. 
2. 
3. 
4. 
5. 

**Question 4**

A random sample of 100 people is taken from the population of a large city. It is found that the proportion of left-handed people in the sample is 0.13. Based on this, a 95% confidence interval for the proportion of left-handed people in the population is

1. 
2. 
3. 
4. 
5. 

**Question 5**

Let .

The composite function *f* is given by .

The range of *f* is

1. 
2. 
3. 
4. 
5. 

**Question 6**

Let .

Which one of the statements below is **not** true for *f* ?

1. 
2. 
3. 
4. 
5. 

**Question 7**

Part of the graph of  is shown below.



The graph corresponding to the graph of the derivative function is



**Question 8**

Let .

The average rate of change between  is

1. 
2. 
3. 
4. 
5. 

**Question 9**

A transformation  maps the graph of  onto the graph of .

The transformation *T* could be defined by

1. 
2. 
3. 
4. 
5. 

**Question 10**

A continuous random variable *X* is normally distributed with a mean of 14 and a variance of 4.

The continuous random variable *Z* has the standard normal distribution.

Let 

 is equal to

1. 
2. 
3. 
4. 
5. 

**Question 11**

Part of the graph of the function *g* is shown below.



An approximation to  is made using four rectangles of equal width and the right endpoint of each rectangle.

The value of the approximation is

1. – 4
2. – 2
3. 10
4. 14
5. 16

**Question 12**

If is equal to

1. 3
2. 4
3. 5
4. 6
5. 9

**Question 13**

The average value of  over the interval  is equal to the average value of  over the same interval.

The value of *k* is

1. 
2. 
3. 
4. 2
5. 4

**Question 14**

A barrel contains ten balls, six of them are green and four of them are blue.

A ball is randomly selected from the barrel, the colour is noted and then it is returned to the barrel.

This is done three times.

The outcome of each selection is independent of the outcome of any other selection.

The probability that all the balls selected are green, given that at least one of the balls selected is green, is closest to

1. 0.075
2. 0.206
3. 0.2308
4. 0.3784
5. 0.936

**Question 15**

Given that  then the expression  is equal to

1. 
2. 
3. 
4. 
5. 

**Question 16**

Emma and Thomas regularly play tic-tac-toe, a two-player game in which either Emma wins, or Thomas wins or it is a draw.

Emma has calculated that the probability of her winning any given game is , and the result of any one game is independent of the result in any other game.

The number of games that Emma needs to play so that the probability of her winning at least one game, is more than 75%, is

1. 2
2. 3
3. 4
4. 5
5. 6

**Question 17**

A sample space has two independent events *A* and *B*.

If  is equal to

1. 
2. 
3. 
4. 
5. 

**Question 18**

Given that  then an antiderivative of  is

1. 
2. 
3. 
4. 
5. 

**Question 19**

A cubic polynomial function .

The graph of *f* has a positive *y*-intercept.

Consider the function *h* where .

A possible domain of *h* is

1. 
2. 
3. 
4. 
5. 

**Question 20**

Which one of the functions below is differentiable for ?

1. 
2. 
3. 
4. 
5. 

**SECTION B**

Answer all questions in this section.

**Question 1** (12 marks)

A circular water wheel rotates in a clockwise direction at the rate of 0.1 revolutions per second.

A logo, showing the builder of the water wheel, is attached to its outer rim at point *P*, as shown in the diagram below.

The height, *h*, in metres, of point *P* above the water, can be modelled by the function , where *t* is the time, in seconds, after the water wheel starts rotating.



1. Find the initial height, in metres, of point *P* above the water. 1 mark
2. Show that . 1 mark
3. Find the proportion of each revolution, for which point *P* is less than ten metres above the water. Give your answer correct to the nearest whole percent. 2 marks

The centre of the water wheel is positioned 60 metres horizontally from a vertical wall. The water pipe that pours water into the water wheel, is attached to this wall at point *A*, which is 50 metres above the water.

The water pipe makes a smooth and continuous join with the water wheel at point *B* as shown below.



Point *B* lies on that part of the path of *P* that is given by  where, and *x* and *y* are expressed in metres.

1. Find . 1 mark
2. Find the gradient of the water pipe in terms of *b* and hence find the coordinates of point *B*, correct to two decimal places. 3 marks
3. **i.** State the maximum possible distance, in metres, between points *P* and *B*. 1 mark
   1. Find the equation of the straight line that passes through the points *P* and *B* when their distance apart is a maximum. Give the gradient and *y*-intercept correct to two decimal places. 1 mark
   2. Hence or otherwise find the time, in seconds, that it takes point *P* to first reach the water pipe from its initial position.

Give your answer correct to one decimal place. 2 marks

**Question 2** (9 marks)

Let .

Part of the graph of *f* is shown below.



1. Find the maximum value of *f* and the value of *x* at which this occurs. 2 marks

The function *g* has a maximal domain and an inverse function .

1. **i.** Find the range of , given that  has no *x*-intercepts. 1 mark
2. Find the rule for . 1 mark
3. Consider the horizontal line with equation .

The area enclosed by this line, the *x*-axis, the *y*-axis and the line with equation , is equal to the area enclosed by *f* and the *x*-axis.

Find the value of *k*. 2 marks

1. Let *R* be the point .

Right-angled triangles can be formed with the vertices  where  and *Q* lies on the graph of *f*.

Find the maximum area, in square units, of triangle *PQR* and the value of *p* for which this occurs. Give your answers correct to two decimal places. 3 marks

**Question 3** (9 marks)

Let .

Part of the graph of *f* is shown below.



1. State the maximal domain and range of *f*. 2 marks
   1. Find the values of *x* for which *f* is strictly increasing. 1 mark

The function *f* undergoes a transformation to become the function *g*, where .

* 1. Write a sequence of two transformations that map the graph of *f* onto the graph of *g*. 1 mark

1. Show that the graph of *g* has a stationary point at the point . 1 mark
2. Let *D* be the vertical distance between the graphs of *f* and *g*.

Explain why . 1 mark

The area enclosed by the graphs *f* and *g* and the *y*-axis is equal to the area enclosed by the graphs of *f* and *g* and the line .

1. Find the *x*-coordinate of the point of intersection between the graphs of *f* and *g* and hence find the value of *a*, both correct to three decimal places. 3 marks

**Question 4** (16 marks)

An airline operates flights from Los Angeles to London. The flight time, in hours, of each of these flights is a normally distributed random variable *T* with a mean of 10 hours and a standard deviation of 0.63 hours. Assume that the flight times are independent of one another.

1. What proportion of flights have a flight time greater than 10.5 hours? Give your answer correct to four decimal places. 1 mark

The airline has a policy to issue a 50% refund to passengers whose flight time exceeds 11 hours.

The proportion of flights that have passengers receiving this 50% refund is 0.0562

A random sample of 100 flights is taken.

1. Find the probability that passengers will receive a 50% refund on less than ten of these

flights. Give your answer correct to four decimal places. 1 mark

Let  be the random variable representing the sample proportion of flights for which passengers receive a 50% refund in random samples of 100 flights.

1. Find the mean and standard deviation of , correct to three decimal places. 2 marks
2. Use the binomial distribution to find , correct to three decimal places. 2 marks

The airline issues a new policy that replaces the old refund policy. Under this new policy, passengers on the slowest 1% of flights from Los Angeles to London receive a full refund.

1. George flew from Los Angeles to London on this airline and received a full refund.

What was the minimum duration, in hours, that George’s flight could have been, correct to two decimal places? 1 mark

1. Given that a passenger was eligible for a 50% refund under the old refund policy, what is the probability that this passenger is eligible for a full refund under the new policy? Give your answer correct to four decimal places. 2 marks

A rival airline starts operating flights from Los Angeles to London.

The probability density function that describes the flight time, *X*, in hours, for these flights is given by



where *k* is a positive real number.

1. Explain why the maximum possible value of *a* is . 1 mark
2. If . 2 marks
3. Given that , find the mean flight time, in hours, for the rival airline’s flights from Los Angeles to London, correct to two decimal places. 2 marks
4. Given that , find the difference in the median flight times of the Los Angeles to London flights of the airline that George flew on and the rival airline. Give your answer, in hours, correct to two decimal places. 2 marks

**Question 5** (14 marks)

Let .

Let , be the function which represents the tangent to *f* at .

The point , where *k* is a constant, lies on the graph of *g* as shown below.



1. **i.** Show that . 2 marks
2. Find the values of *a* for which the graphs of *f* and *g* intersect at point *P*. 1 mark
3. Find all the possible values of *k*. 2 marks

*Question 5 continues on the next page.*

Consider the region bounded by the graphs of *f* and *g,* and the *x*-axis where .

This region is shaded in the diagram below.



Let  represent the total area of the shaded region in terms of *a.*

1. **i.** Find all the values of *a* such that . 1 mark
2. Hence, find the maximum area of the shaded region and the value of *a* for which this occurs. 2 marks

**d.** State the largest value of *b* such that . 3 marks

Consider the tangents to the graphs of *f* and .

**e.** Express in the form , where *m* and *n* are

integers. 1 mark

1. Hence find the values of *a* for which the tangents to the graphs of *f* and ** at  have no points of intersection. Give your answers correct to two decimal places. 2 marks

**END OF EXAMINATION**

**Mathematical Methods formulas**

## Mensuration

|  |  |  |  |
| --- | --- | --- | --- |
| area of a trapezium |  | volume of a pyramid |  |
| curved surface area of a cylinder |  | volume of a sphere |  |
| volume of a cylinder |  | area of a triangle |  |
| volume of a cone |  |  | |

## Calculus

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
| product rule |  | quotient rule |  |
| chain rule |  |  | |

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**Probability**

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
| mean |  | variance |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **Probability distribution** | | **Mean** | **Variance** |
| discrete | = |  |  |
| continuous |  |  |  |

## Sample proportions

|  |  |  |  |
| --- | --- | --- | --- |
|  | | mean |  |
| standard  deviation |  | approximate  confidence  interval |  |

