

 **MATHS METHODS 3 & 4**

 **TRIAL** **EXAMINATION 2**

 **SOLUTIONS**

 **2022**

# SECTION A – Multiple-choice answers

1. **E 9. A 17. B**
2. **B 10. C 18. A**
3. **D 11. A 19. A**
4. **A 12. B 20. D**
5. **E 13. B**
6. **C 14. C**
7. **D 15. D**
8. **C 16. D**

# SECTION A – Multiple-choice solutions

# Question 1



The answer is E.

**Question 2**





The answer is B.

**Question 3**



The answer is D.

**Question 4**



The number of left-handed people in the sample is .

Using CAS, the 95% confidence interval for the proportion of left-handed people in the population is ( correct to four decimal places.

The answer is A.

**Question 5**



Draw a graph and remember that the domain is restricted to .



From the graph, .

The answer is E.

**Question 6**



The graph of *f* is symmetrical about the *y*-axis so .

Option A is true.

Since .

Option B is true.



So option C is **not** true.

 so option D is true.

.

So option E is true.

The answer is C.

**Question 7**

The graph of *h* has a gradient that is negative and decreasing as .

We can eliminate option A because whilst its gradient is negative, it is increasing as .

The same is true for option B and option C.

Option D is correct because its gradient is negative and decreasing as .

Option E has part of its gradient which is positive.

The answer is D.

**Question 8**

Define *h* on your CAS.



The answer is C.

**Question 9**

The graph of  has been dilated by a factor of 3 from the *y*-axis to produce the graph of  then reflected in the *x*-axis to become the graph of

 

then translated 1 unit down to become

 

The sequence of transformations is defined by

 

The answer is A.

**Question 10**

Draw a diagram. Note that .





 (conditional probability formula – formula sheet)

 

The answer is C.

**Question 11**

The four rectangles used in the approximation are shown below.



Note that the width of each rectangle is 2 units since we are using four rectangles of equal width between .



Note that we are not being asked for the area but rather an approximation to the value of the integral, hence the negative values are kept negative.

The answer is A.

# Question 12

A possible area represented by  is shaded in the diagram below and is 12 units2.



The area represented by  is shaded in the diagram below.

It occurs because  has been dilated by a factor of  (i.e. compressed) to become

.

Note that *f* is compressed and so too are the terminals of integration and so too is the area i.e. .



The area  is shaded in the diagram below.

It occurs because  has been translated 1 unit left to become .

Note that the terminals are translated also. The area is unaffected by the translation so 



The answer is B.

**Question 13**

Solve  for *k*.

 

The answer is B.

**Question 14**

Let *X* be the number of green balls that are selected in the 3 selections.





(binomPdf*)*

(binomCdf)

The closest answer is 0.2308.

The answer is C.

**Question 15**

Note that *x* and *y* are both second quadrant angles so



 will be positive and 

will be negative.









The answer is D.

**Question 16**

Let *X* be the number of games that Emma wins when she plays *n* games.

 





So .

The answer is D.

**Question 17**





This is what we are given.

|  |  |  |  |
| --- | --- | --- | --- |
|  | *A* |  |  |
| *B* |  | ? |  |
|  |  |  |  |
|  | *a* |  | 1 |



The answer is B.

**Question 18**



So, an antiderivative of  is given by .

The answer is A.

**Question 19**

Two possible graphs of *f* are shown below.



The possible graphs for  are shown as dotted lines below.



For  to be defined,  to be defined, .

So  is defined for .

Only the former option is offered.

The answer is A.

**Question 20**

A function is differentiable for  if it is smoothly continuous over that interval.

Method 1 - algebraically Method 2 - graphically







So the graph is continuous for .

 

 For option A the graph is continuous

So the graph is not smooth at . at  but not smooth.

Reject option A. Reject option A.







So the graph is not continuous for .

Reject option B. For option B the graph is

 discontinuous at .

 Reject option B.







So the graph is continuous for .

  For option C the graph is continuous

 at  but not smooth.

So the graph is not smooth at . Reject option C.

Reject option C.







So the graph is continuous for .

 



So the graph is smoothly continuous and therefore For option D the graph is smoothly

differentiable for . continuous for .







So the graph is continuous for .

  For option E the graph is continuous

  at  but not smooth.

So the graph is not smooth at . Reject option E.

Reject option E.

The answer is D. The answer is D.

**SECTION B**

**Question 1 (12 marks)**

1. Define the function *h* on your CAS.

 

The initial height is 2 metres.

 **(1 mark)**

1. The wheel rotates at the rate of 0.1 revolutions per second, which is the same as 1 revolution per 10 seconds, so the period is 10 seconds.

 **(1 mark)**

1. Solve . **(1 mark)**



During each revolution, the point *P* is less than 10 m above the water for



(Alternatively, using symmetry )

This represents a percentage of each revolution of  to the nearest whole percentage.

**(1 mark)**

1. Define the function on your CAS.



**(1 mark)**

1. The water pipe runs between point 



**(1 mark)**

Note that *c* is the *y*-coordinate of point *B* when .

Since the water pipe is a tangent to the water wheel at point *B*, the gradient at point *B* will be the same i.e.



Solve this for *b*. **(1 mark)**



*B* is the point (64.32, 25.20), where the coordinates are expressed correct to two decimal places.

 **(1 mark)**

1. **i.** The maximum distance between points *P* and *B* is 24 metres since the radius of the water wheel is 12 metres.

**(1 mark)**

1. When *P* and *B* are 24 m apart, the line passing through these points is perpendicular to the waterpipe.

Using CAS, the normal to the path of *P* when  where the gradient and *y*-intercept are expressed correct to two decimal places.

**(1 mark)**

1. Let  be the angle between the line found in part **ii** and a horizontal line running through the centre of the wheel.





**(1 mark)**

The initial position of *P* is at .

*P* moves through an angle of

**

*P* moves at the rate of 0.1 revolutions per second i.e. at  per second i.e. at 36° per second.



**(1 mark)**

**Question 2** (9 marks)

1. Define *f* on your CAS.



 

The maximum value of *f* is  and this occurs when .

**(1 mark)** – maximum value **(1 mark)** – value of *x*

1. **i.** Since  and *g* has an inverse and a maximal domain, then  must be a 1:1 function so .

If  then the graph of the inverse function (shown as the dotted line below) has an *x*-intercept at  so reject this option.



If , then the graph of the inverse function (shown as the dotted line below) has no *x*-intercepts as required.



So . **(1 mark)**

1. 

 

Let 

Swap *x* and *y* for inverse.

 

Solve for *y* using CAS.



From part **i.** we know that the graph of  passes through . This point satisfies the first equation but not the second.

So .

**(1 mark)**

1. The value of *k* gives the average value of the function *f* between .

 **(1 mark)**

 

  **(1 mark)**

1. Draw a diagram.



 

**(1 mark)**

 **(1 mark)**

So the maximum area is 8.17 square units and this occurs when  (both values correct to two decimal places).

**(1 mark)**

**Question 3** (9 marks)

1.  (from the graph) **(1 mark)**

To double-check using your CAS, define *f* on your CAS.

 (using domain **)**

The graph of *f* has a maximum turning point at .

So .

**(1 mark)**

1. *f* is strictly increasing for .

 **(1 mark)**

1. The graph of *f* is
* translated 0.5 units to the right then
* reflected in the *x*-axis (or vice versa).

 **(1 mark)**

1. Method 1 - graphically

The graph of *f* has a stationary point at .

After a translation of 0.5 units to the right this point becomes .

After a reflection in the *x*-axis this point becomes .

So the graph of *g* has a stationary point at  as required.

 **(1 mark)**

Method 2 – algebraically



So *g* has a stationary point at  as required.

**(1 mark)**

1. For ****, the graph of *f* has a horizontal asymptote with equation  which *f* approaches from above and the graph of *g* has a horizontal asymptote with equation  which *g* approaches from below.



**(1 mark)**

1. Sketch the graph of *g*.



The two areas are shaded.



**(1 mark)**



**(1 mark)**

**(1 mark)**

**Question 4** (16 marks)

1. *T* is the flight time, in hours, of a flight from Los Angeles to London.





 

**(1 mark)**

1. Let *Y* be the number of flights that exceed 11 hours out of the sample of 100.





  **(1 mark)**

Note that we use binomCdf(100, 0.0562, 0, 9) i.e. *Y* has to be less than 10 not equal to it and *Y* is an integer, that is, a whole number because it represents the number of flights.

**c.** 



**(1 mark)**

 **(1 mark)**

1. Let ** from part **b.**



**(1 mark)**

Note that we use binomCdf(100, 0.0562, 9, 100) to find this result. **(1 mark)**

1. Let George’s flight time be *g*.





from part **a.**

Using invNorm on your CAS, 

The minimum duration that George’s flight could have been was 11.47 hours (correct to 2 decimal places).

**(1 mark)**

1. 



**(1 mark)**

(from part **e.**)

(from part **b.**)

 **(1 mark)**

1. Sketch the graph of .



The value of a probability density function cannot be negative.

Since 

we see from the graph that the next *x*-intercept that occurs for  is  i.e. the maximum possible value of *a* is .

**(1 mark)**

1. We know that



**(1 mark)**

**(1 mark)**

1.  **(1 mark)**

Note, given that  from part **h.**

 

**(1 mark)**

1. The median flight time of the original airline (the airline that George flew on) is 10 hours. This is because the distribution of flight times for the original airline is normal and the normal distribution is symmetrical and therefore the mean is equal to the median.

Let the median flight time of the rival airline be *m* where

 **(1 mark)**

Solve for *m*, remembering that



So the difference in median flight times is  (correct to 2 decimal places).

**(1 mark)**

**Question 5** (14 marks)

1. **i.** Define *f* on your CAS.



The tangent passes through the point .

The gradient of the tangent is . **(1 mark)**

Equation of tangent is 



 **(1 mark)**

1. Point *P* is the *y*-intercept of *g* which from part **a. i.**  is .



**(1 mark)**

1. Method 1 – finding *k*  in terms of *a*.



Using part **a. i.** .

Sketch the graph.

.

 **(1 mark)** – left endpoint

**(1 mark)** – right endpoint

Method 2 – testing different values of *a*

The graph of *f* has a negative gradient (i.e. slopes up to the left) for .

At the point  there is a stationary point of inflection so the tangent at this point will be a horizontal line, i.e. .

So the minimum value of *k* (the *y*-intercept of *g*) will be 1.

For .

The value of *k* increases over the interval  with a maximum value occurring when .

When , the *y*-intercept of *g* (i.e. *k*) is



So the maximum value of *k* is 6.

The possible values of *k* are .

**(1 mark)** – left endpoint

**(1 mark)** – right endpoint

1. **i.**  from part **a. i**.



 

Now .

 **(1 mark)**

* 1. Looking at the graph, as *a* increases, so does the shaded area.

The maximum possible value of *a* is 2 (from part **c.i.**), so the maximum area of the shaded region occurs when . **(1 mark)**





Since the point of tangency occurs at (2, 0) then the maximum area is given by

 **(1 mark)**

1. From part **c. i.**, when .

From part **c. ii.**, as *a* increases in value so does the shaded area.

So the minimum area occurs when  but since this value of *a* is undefined when , the area is always greater than this but never equal to it.

When  and the *x*-intercept of *g* is .



**(1 mark)**

**(1 mark)**

Hence .

So .

**(1 mark)**

1. Method 1

By recognition that the graph of *f* has a stationary point of inflection at the point , then this would imply that 

So .  **(1 mark)**

Method 2 – expand and equate coefficients





**(1 mark)**

1. 

 Let 

Swap *x* and *y* for inverse.



For the tangents to the functions *f* and  to have no points of intersection, their gradients must be equal.





**(1 mark)**

So 



**(1 mark)**

(Check that these values of *a* satisfy . They do.)