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YEAR 12 *Trial Exam Paper*

2022

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = x\left(2 - x\right)^3$.

Find $\dfrac{dy}{dx}$.

2 marks

b. Let $f(x) = 2e^{-\frac{1}{2}(x-1)}$.

Evaluate $f'(1)$.

2 marks

Question 2 (4 marks)

a. Let $y = \dfrac{1}{\sin(x) + 1}$.

Show that $\dfrac{dy}{dx} = \dfrac{-\cos(x)}{(\sin(x)+1)^2}$.

1 mark

b. Let $f'(x) = \dfrac{\cos(x)}{(\sin(x)+1)^2}$.

Find $f(x)$ given that $f\left(\dfrac{\pi}{2}\right) = 0$.

3 marks

Question 3 (4 marks)

In a board game, a player takes a turn by rolling two fair dice. One die has four sides, numbered 1 to 4, and the other has six sides, numbered 1 to 6. A turn is called a success if at least one die rolls a four or higher.

a. Find the probability that a player's turn is a success.

1 mark

b. If both dice roll a four or more, the turn is called an exceptional success.

Find the probability that a turn that is a success is also an exceptional success.

2 marks

c. Let $\hat{P}$ be the proportion of a player's first two turns in the game that are a success.

Find $\Pr\left(\hat{P} > 0\right)$.

1 mark

Question 4 (3 marks)

a. Solve $e^{2x} - 5e^{x} + 4 = 0$ for $x \in R$.

2 marks

b. Let $f:[k,\infty) \to R,\ f(x) = e^{2x} - 5e^{x} + 4$ and $g:[0,\infty) \to R,\ g(x) = \sqrt{2x} + 1$.

Find the lowest value of k for which $g\left(f\left(x\right)\right)$ is defined.

1 mark

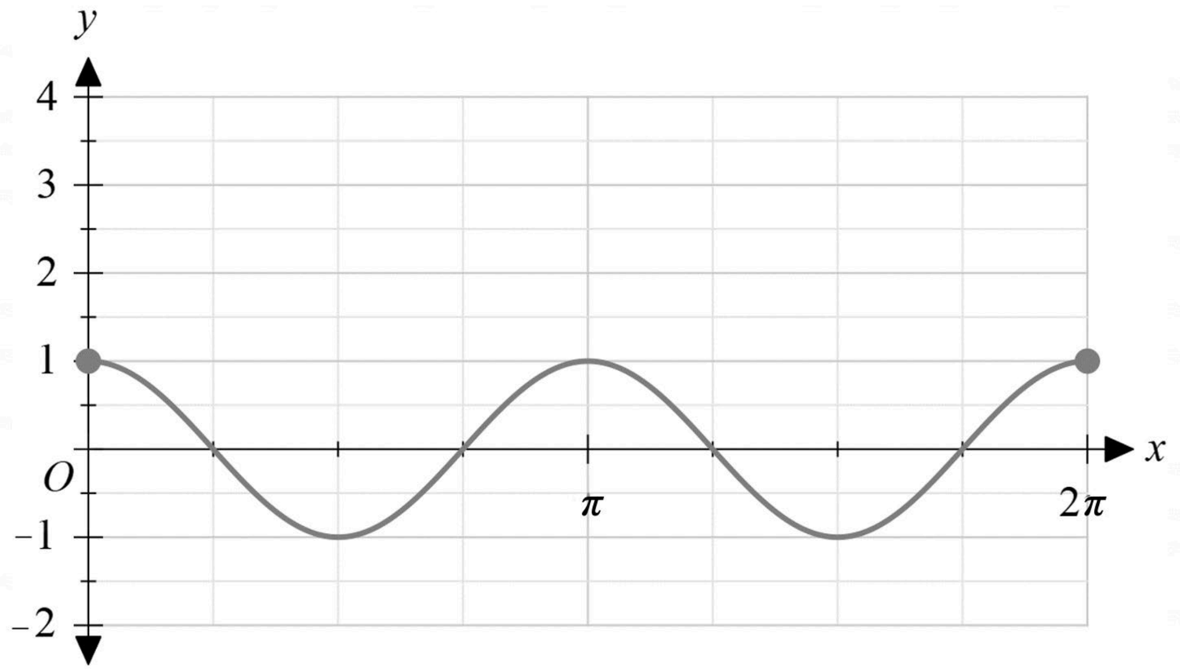
Question 5 (5 marks)

Let $f: R \to R,\ f(x) = \cos(2x)$ and $g: R \to R,\ g(x) = 2\sin(x) + 1$.

a. The graph of $y = f(x)$ for $x \in [0,\ 2\pi]$ is shown on the axes below.

Sketch the graph of $y = g(x)$ for $x \in [0, 2\pi]$ on the axes below and label the coordinates of all points of intersection with the graph of $y = f(x)$.

2 marks



b. The transformation $T: R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$ maps the graph of $y = g(x)$ onto the graph of $y = f(x)$, where $a, b, d \in R$ and $c \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Find the values of a, b, c and d.

3 marks

Question 6 (3 marks)

Let X be a normally distributed random variable with a mean of 10 and a standard deviation of 5.

a. Find $\Pr(X < 10)$.

1 mark

b. Let Y be a normally distributed random variable with a mean of 15 and a standard deviation of σ.

Find the value of σ if $\Pr\left(X < 12\right) = \Pr(Y > 12)$.

2 marks

Question 7 (3 marks)

Let $f : [0,\ a] \to R,\ f(x) = \dfrac{k}{x+1}$.

If f is a probability density function, find k in terms of a.

Question 8 (8 marks)

In a game, the results of three independent and identical Bernoulli trials with a probability of success p are used to determine the result for a player.

- If all three trials are failures, then the player gains no points.
- If only one trial is a success, then the player loses two points.
- If only two trials are a success, then the player gains one point.
- If all three trials are a success, then the player gains three points.

a. Show that the expected number of points that would be gained by a player is given by the expression $-6p^3+15p^2-6p$.

2 marks

b. Find the values of p for which a player would expect to gain points in the game.

3 marks

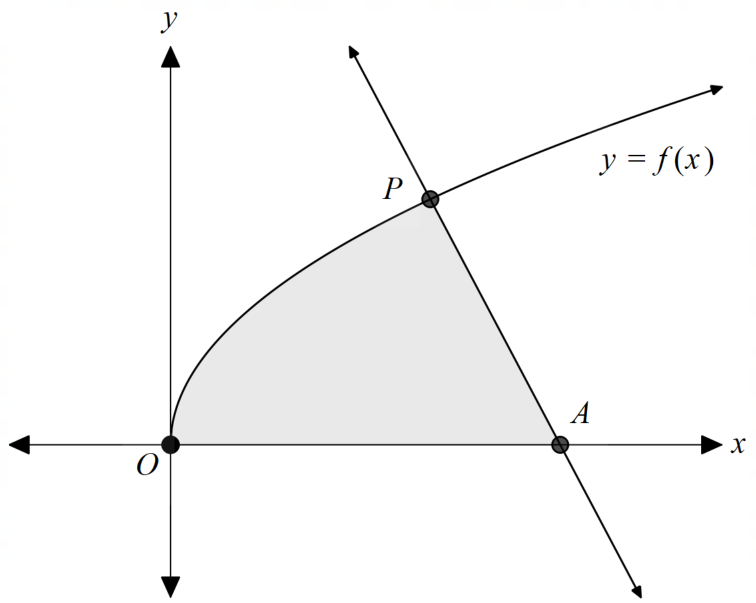
c. Find the value of p for which a player would expect to lose the most points.

3 marks

Question 9 (6 marks)

Let $f:[0,\infty)\to R,\ f(x)=\sqrt{x}$. Let P be the point $\left(p,\sqrt{p}\right)$ on f with $p>0$. Let A be the x-intercept of the line perpendicular to f at P.

The shaded region in the diagram is enclosed by the horizontal axis, the graph of $y=f(x)$ and the graph of the line perpendicular to f at P.



a. Show that the x-coordinate of A is $p+\frac{1}{2}$.

2 marks

b. Find an expression for the area of the shaded region in terms of p.

2 marks

c. Show that the area of the shaded region strictly increases as p increases.

2 marks

END OF QUESTION AND ANSWER BOOK