The Mathematical Association of Victoria

Trial Examination 2022

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

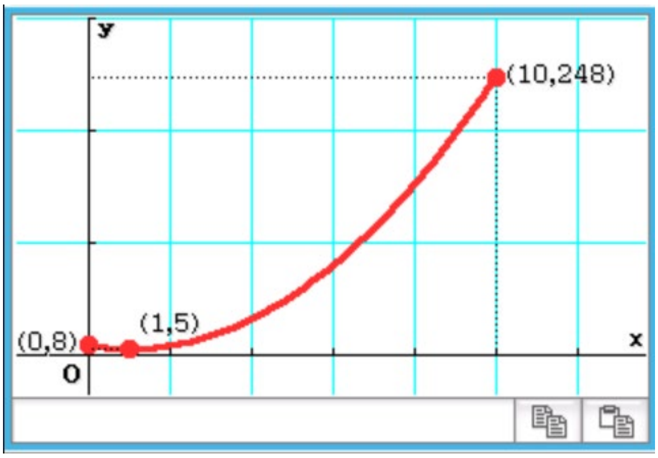
SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	B	**11**	C
2	A	**12**	D
3	E	**13**	C
4	E	**14**	D
5	B	**15**	E
6	C	**16**	A
7	A	**17**	A
8	D	**18**	C
9	B	**19**	E
10	D	**20**	B

Question 1 **Answer B**

$f:[0,10]\to R, f(x)=3(x-1)^2+5$ has restricted domain $[0,10]$.

Minimum turning point is at $(1,5)$ and the y-intercept is at (0, 8)



range $[5,248]$

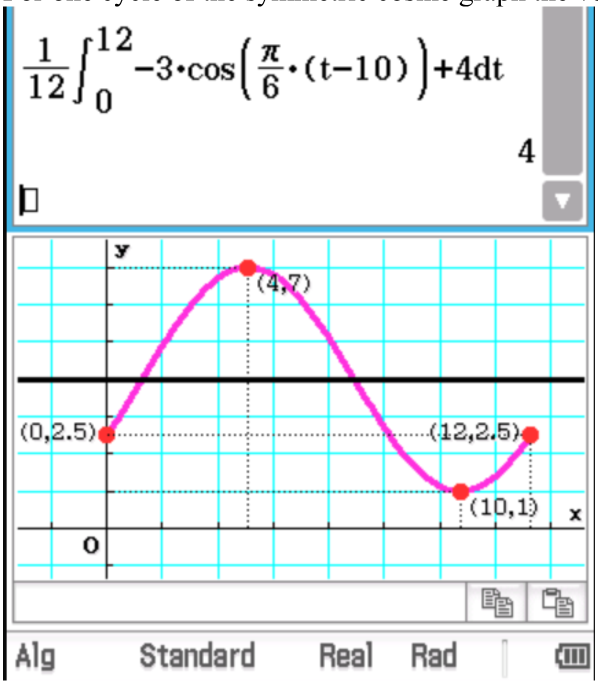
Question 2 **Answer A**

$h(t) = -3\cos\left(\frac{\pi}{6}(t-10)\right)+4$

The average height of water from 9 am $(t=0)$ to 9 pm $(t=12)$ is found by

$\frac{1}{12-0}\int_0^{12} h(t)dt = 4$

Alternatively
For one cycle of the symmetric cosine graph the vertical translation of 4 becomes the average value.



Question 3 **Answer E**

For $y = -\pi\sin(2\pi x)-\pi$, the amplitude is π

Question 4 **Answer E**

$f(x) = 2^{x-1}$

$f(x)\times f(y)$

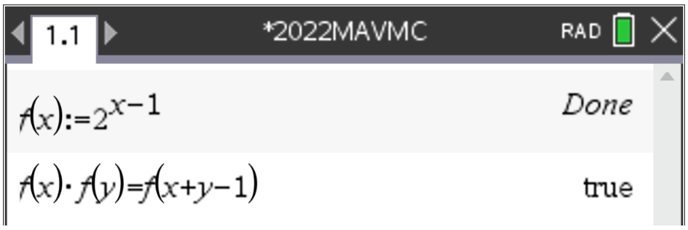
$= 2^{x-1}\times 2^{y-1}$

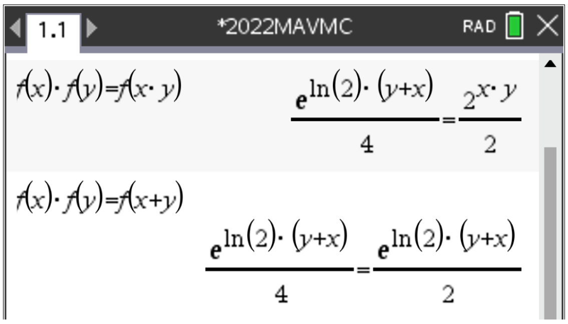
$= 2^{x+y-2}$

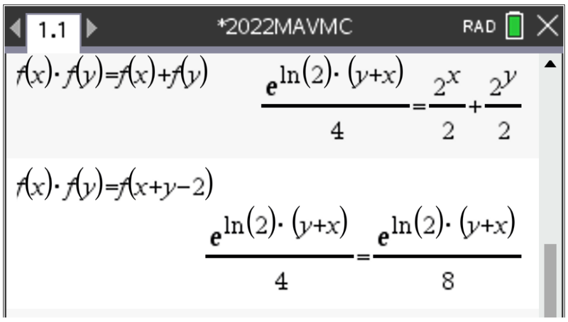
$= 2^{x+y-1-1}$

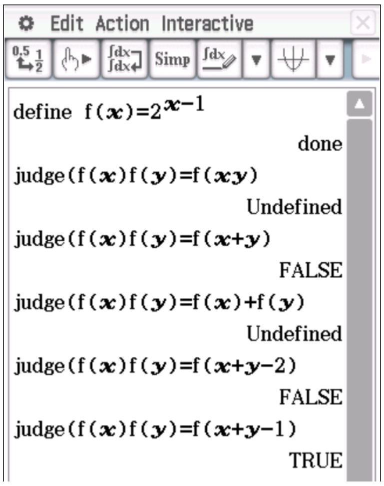
$= f(x+y-1)$

Question 4 (continued)









Question 5 **Answer B**

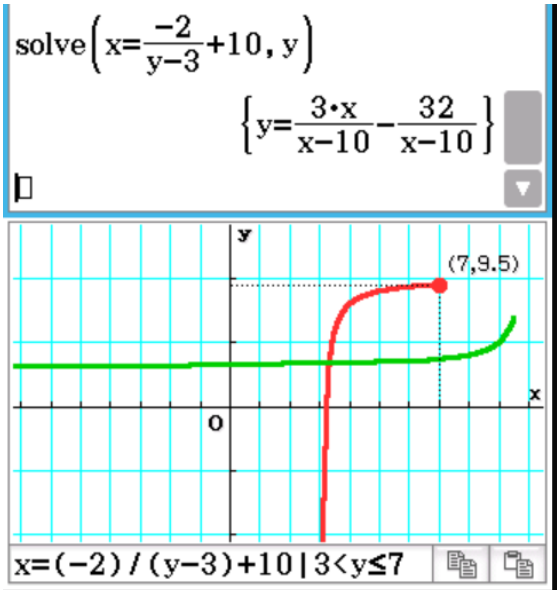
$g:(3,7]\to R,\ g(x)=\dfrac{-2}{x-3}+10$ with domain $(3,7]$ and range $(-\infty,9.5]$

Using CAS to swap x and y to get the rule for inverse

$g^{-1}(x)=\dfrac{3x}{x-10}-\dfrac{32}{x-10}$

Dom $g^{-1}(x)=$ Ran $g(x)=(-\infty,9.5]$

Question 5 (continued)



Question 6 **Answer C**

$T : R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ has been applied to get the image $y = e^{2x}$.

We need to find the original exponential rule.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ 3y \end{bmatrix}$$

Giving $x' = -x$ and $y' = 3y$

Substitute into image equation $y' = e^{2x'}$

$3y = e^{-2x}$

$y = \frac{1}{3}e^{-2x}$

Question 7 **Answer A**

The gradient of both equations is 1

$2x - 2y = m$ and $6x - 6y = m$

$y = x - \frac{m}{2}$ and $y = x - \frac{m}{6}$

Hence parallel lines

Question 8 **Answer D**

$y = a\log_e(bx + c)$

$(3, -6\log_e(2))$ and $\left(\frac{2}{3}, 0\right)$

From the graph the vertical asymptote is at $x = \frac{1}{3}$ and the graph is dilated by $\frac{1}{3}$ from the y-axis to get the x-intercept of $\left(\frac{2}{3}, 0\right)$

So the rule is $y = a\log_e(3x - 1)$

Question 8 (continued)

Substitute $\left(3, -6\log_e(2)\right)$

$-6\log_e(2) = a\log_e(8)$

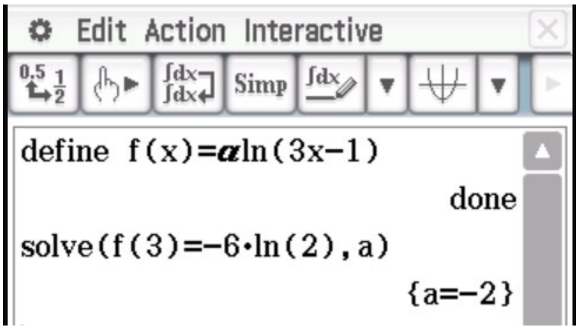
$$a = \frac{-6\log_e(2)}{\log_e(8)}$$

$$a = \frac{-6\log_e(2)}{\log_e(2)^3}$$

$$a = \frac{-6\log_e(2)}{3\log_e(2)}$$

Giving $a = -2$

Equation of graph is $y = -2\log_e(3x-1)$



Question 9 **Answer B**

A and *B* are two independent events.

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$= \frac{\Pr(A) \times \Pr(B)}{\Pr(B)} \quad \text{for independent events}$$

$$= \Pr(A) = \frac{1}{2}$$

$$\Pr(A') = \frac{1}{2}$$

$$\Pr(A' \cap B') = \Pr(A') \times \Pr(B') = \frac{1}{5}$$

$$\Pr(B') = \frac{2}{5}$$

$$\Pr(A \cap B') = \Pr(A) \times \Pr(B') = \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$$

OR

$$\Pr(A' \cap B') = \frac{1}{5} \text{ given}$$

$$\Pr(A' \cap B) = 1 - \left(\frac{1}{2} + \frac{1}{5}\right) = \frac{3}{10}$$

Question 9 (continued)

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B) = \frac{1}{2} \times \left(\frac{3}{10} + \Pr(A \cap B)\right)$$

$$\Pr(A \cap B) = \frac{3}{10}$$

$$\Pr(A \cap B') = \frac{1}{2} - \frac{3}{10} = \frac{1}{5}$$

	A	A'	
B	$\frac{3}{10}$	$\frac{3}{10}$	$\frac{3}{5}$
B'	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

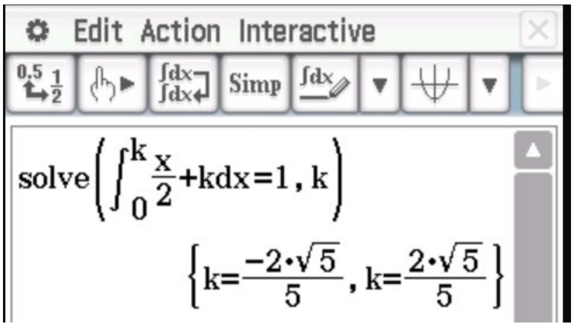
Question 10 **Answer D**

$$f(x) = \begin{cases} \frac{x}{2} + k, & 0 \le x \le k \\ 0, & \text{otherwise} \end{cases} \quad \text{where } k \in R^+.$$

Firstly, find constant k

For PDF let $\int_0^k \left(\frac{x}{2} + k\right) dx = 1$

For $k \in R^+$, $k = \frac{2\sqrt{5}}{5}$

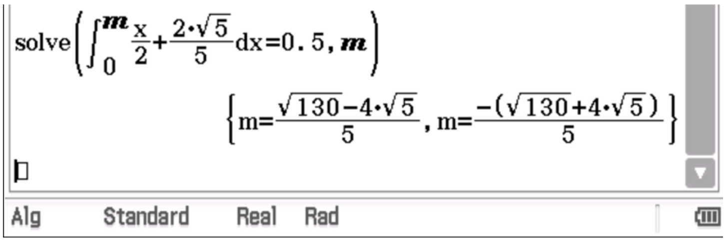


To find the median of $f(x)$ solve

$$\int_0^m \left(\frac{x}{2} + \frac{2\sqrt{5}}{5}\right) dx = 0.5$$

$$m = \frac{\sqrt{130} - 4\sqrt{5}}{5}$$

Question 10 (continued)



Median is $\frac{\sqrt{130}-4\sqrt{5}}{5}$

Question 11 **Answer C**

To find the mean of the sample, find the midpoint of the confidence interval $(0.3346, 0.4654)$

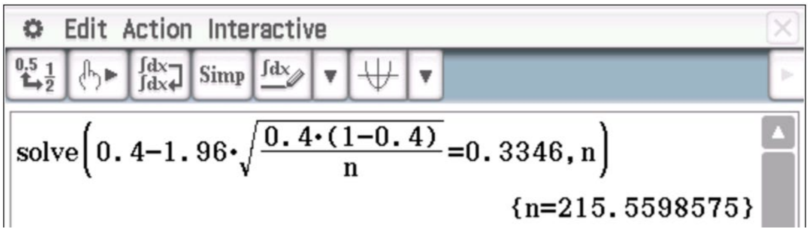
$$\frac{0.3346+0.4654}{2}=0.4$$

Using the left edge of the CI solve for n where

$$0.4-1.96\sqrt{\frac{0.4(1-0.4)}{n}}=0.3346$$

Giving $n=215.5598...$

Number of students in the random sample is closest to 216



Question 12 **Answer D**

To find k solve $0.2+\frac{1+k^2}{10}+\frac{4-k}{10}+\frac{k}{20}=1$

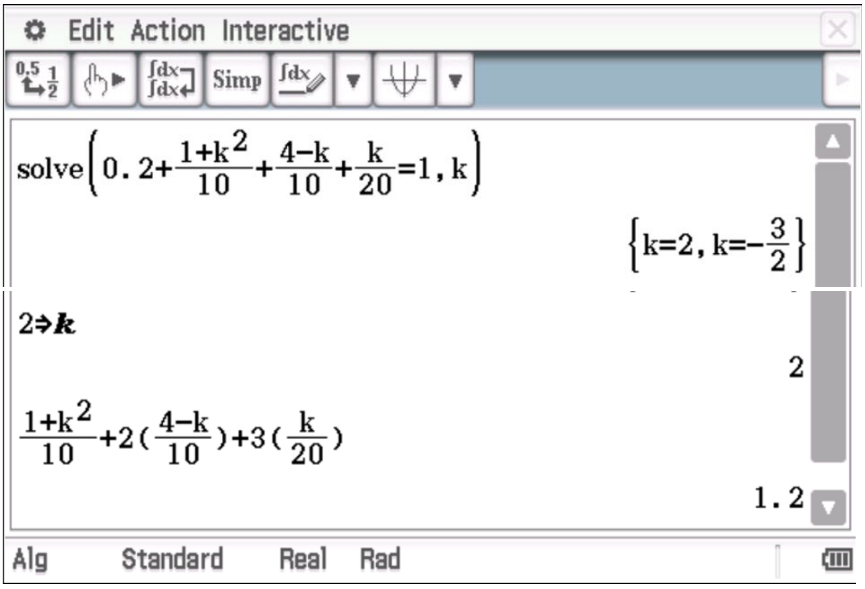
gives $k=2, k=-\frac{3}{2}$

As $k\in R^+$, $k=2$

To find the mean evaluate $0\times0.2+1\times\frac{1+k^2}{10}+2\times\frac{4-k}{10}+3\times\frac{k}{20}$ where $k=2$

Mean = 1.2

Question 12 (continued)



Question 13 **Answer C**

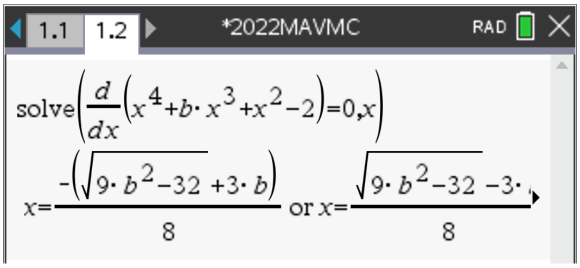
$f(x) = x^4 + bx^3 + x^2 - 2$

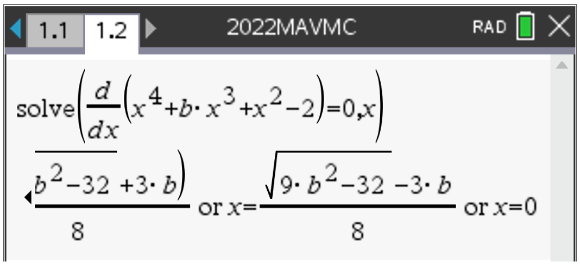
Solve $\frac{d}{dx}\left(x^4 + bx^3 + x^2 - 2\right) = 0$ for x

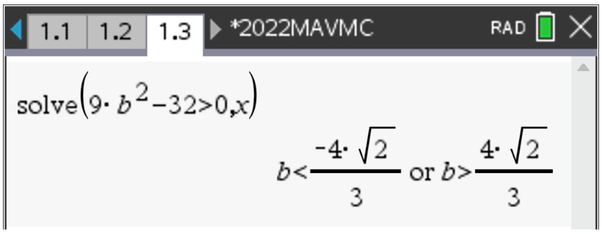
$x = \frac{\pm\sqrt{9b^2 - 32} + 3b}{8}$ or $x = 0$

There will be three solutions when $9b^2 - 32 > 0$.

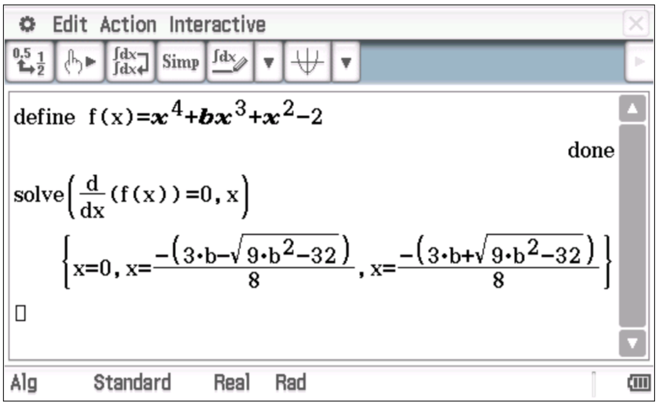
$b < -\frac{4\sqrt{2}}{3}$ or $b > \frac{4\sqrt{2}}{3}$







Question 13 (continued)



Question 14 **Answer D**

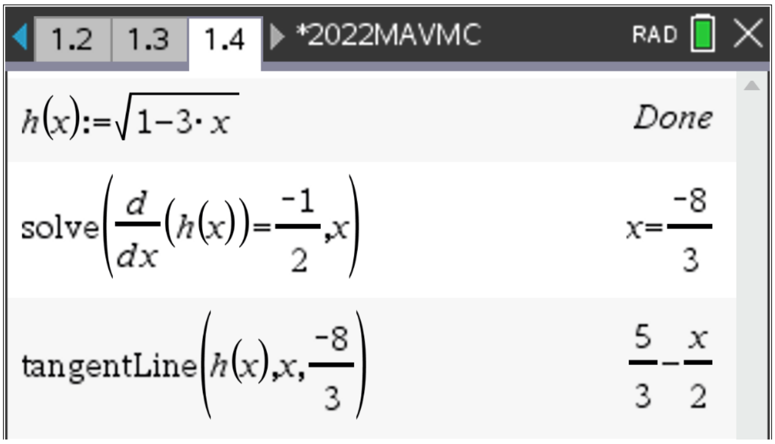
$h(x)=\sqrt{1-3x}$

The gradient of the line perpendicular to the line $y=2x$ is $-\frac{1}{2}$.

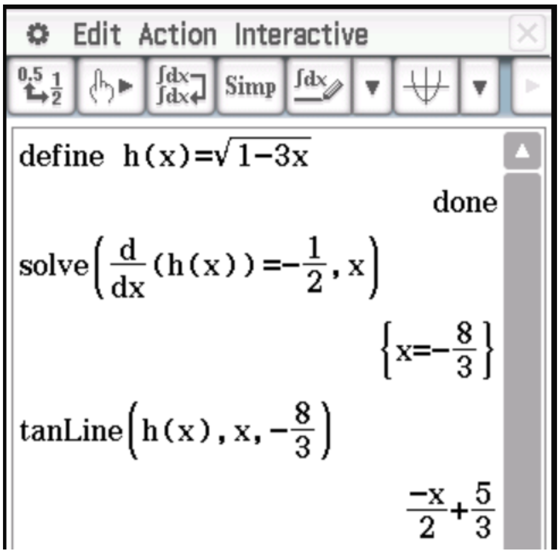
Solve $h'(x)=-\frac{1}{2}$ for x

$x=-\frac{8}{3}$

The tangent is $y=-\frac{x}{2}+\frac{5}{3}$.



Question 14



Question 15 **Answer E**

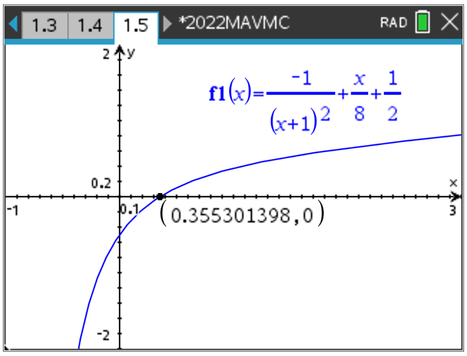
$v=\frac{-1}{(t+1)^2}+\frac{t}{8}+\frac{1}{2}$

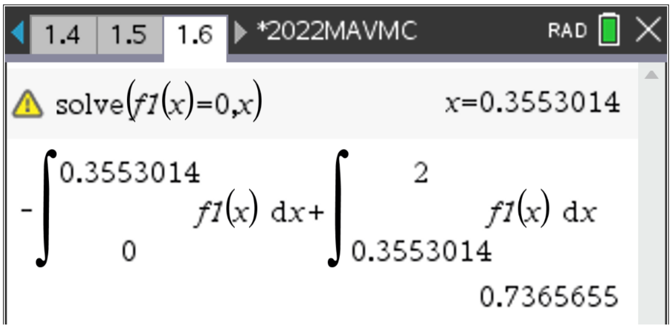
Solve $v(t)=0$ for t

$t=0.3553\ldots$

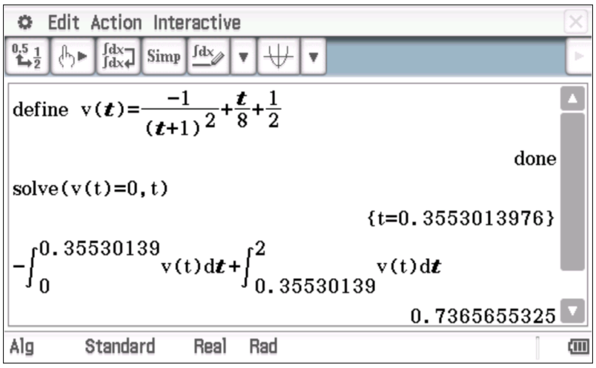
$$-\int_{0}^{0.3553\ldots}(v(t))\,dt+\int_{0.3553\ldots}^{2}(v(t))\,dt$$

$=0.737$ correct to three decimal places





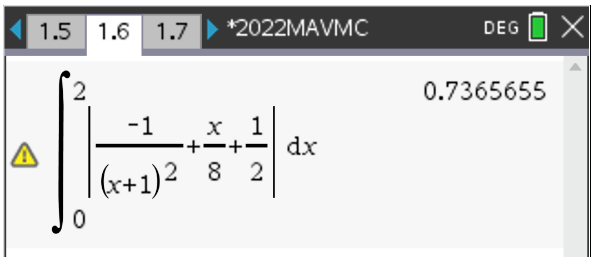
Question 15 (continued)

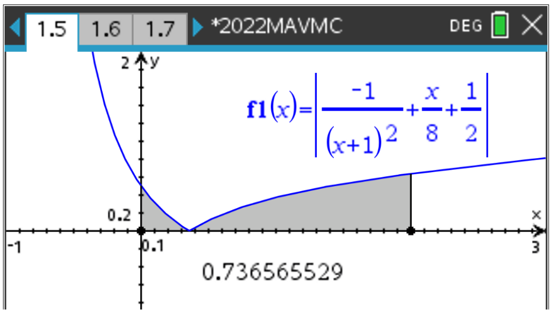


OR

Please note that absolute values are not on the course but can be used for these types of questions.

$$\int_0^2 \left(\left|v(t)\right|\right) dt = 0.737$$



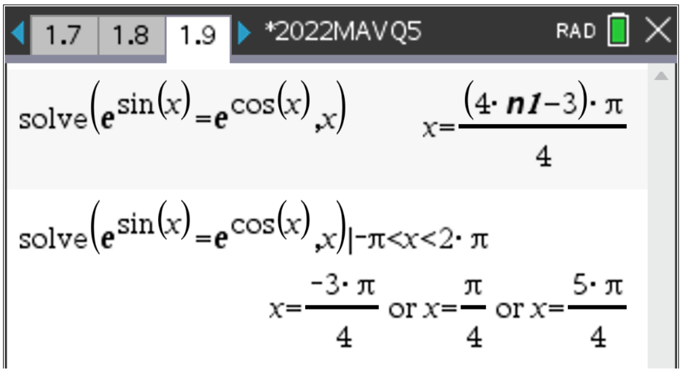


Question 16 **Answer A**

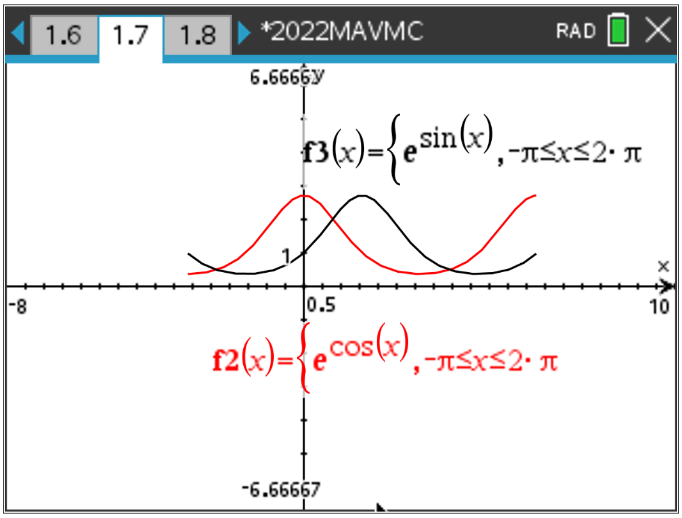
$f(x) = e^{\sin(x)}$ and $g(x) = e^{\cos(x)}$

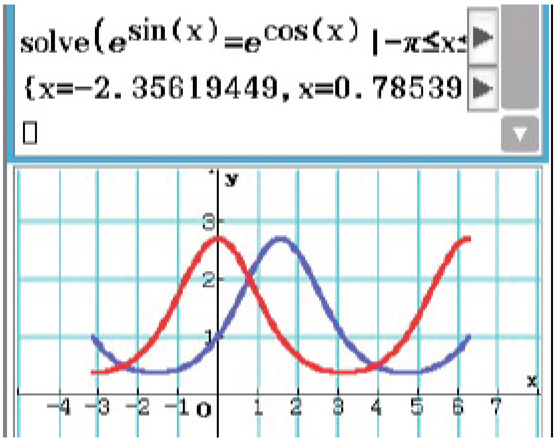
$$\text{Area } = \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \left(g(x) - f(x)\right) dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \left(f(x) - g(x)\right) dx$$

$$= 2\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \left(g(x) - f(x)\right) dx$$



Question 16





Question 17 **Answer A**

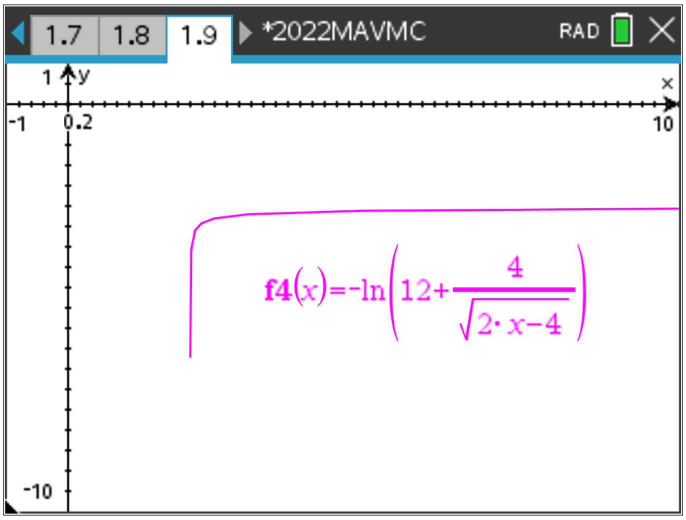
$f(x) = -\log_e(12 - 4x)$ and $g(x) = -\dfrac{1}{\sqrt{2x-4}}$

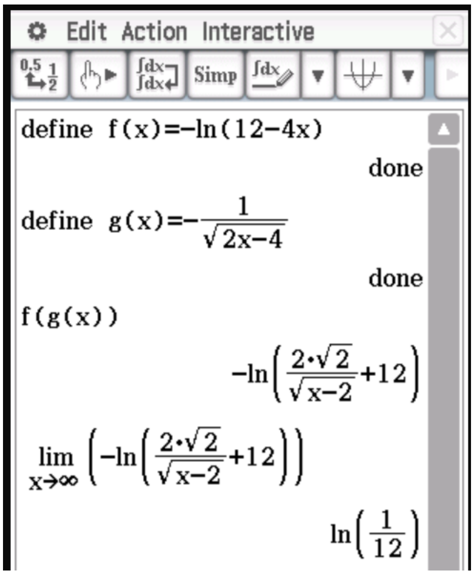
$f(g(x)) = -\log_e\left(12 + \dfrac{4}{\sqrt{2x-4}}\right)$

Domain of $f(g(x))$ is $x > 2$

Range is $(-\infty, -\log_e(12))$

Question 17 (continued)





Question 18 **Answer C**

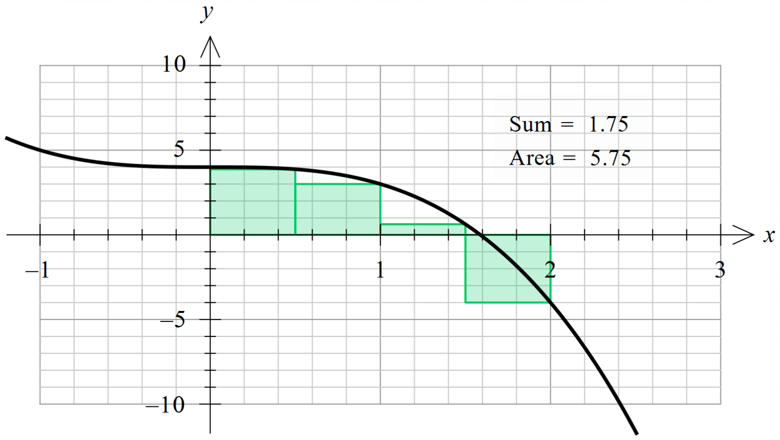
$y = f(x) = -x^3 + 4$

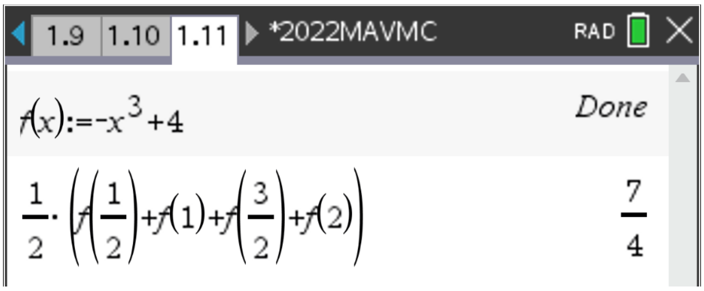
$\int_0^2 \left(-x^3 + 4\right) dx$

$\approx \frac{1}{2}\left(f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) \right)$

$= \frac{7}{4}$

Question 18 (continued)





Question 19 **Answer E**

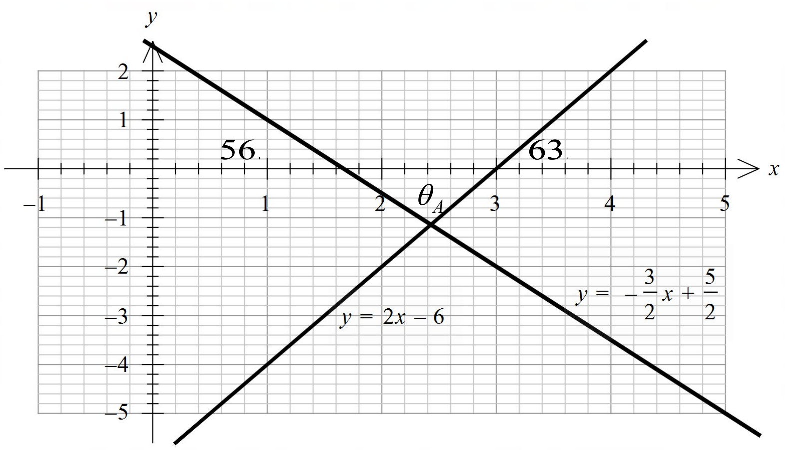
$2x - y = 6$ and $3x + 2y = 5$

$y = 2x - 6$ and $y = -\frac{3}{2}x + \frac{5}{2}$

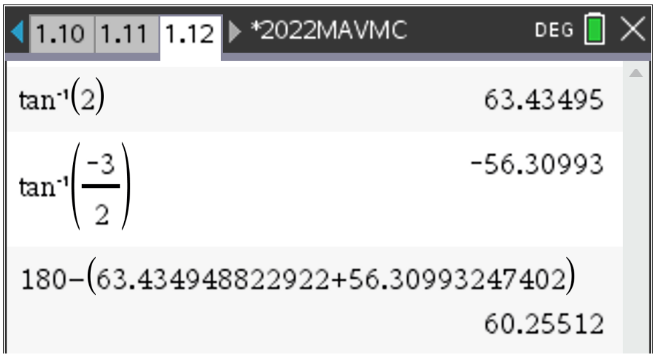
$\tan(\theta_1) = 2$ and $\tan(\theta_2) = -\frac{3}{2}$

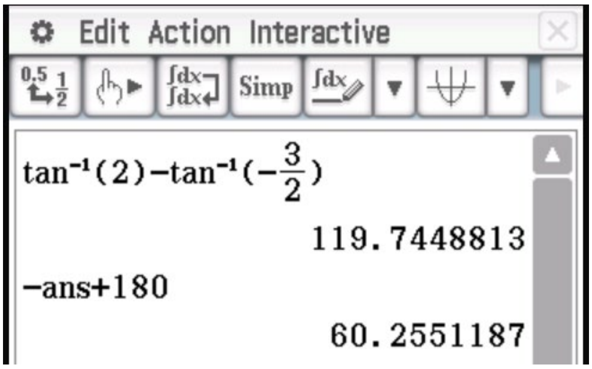
$\theta_1 = \tan^{-1}(2) = 63.43\ldots,\ \theta_2 = \tan^{-1}\left(-\frac{3}{2}\right) = -56.30\ldots$

Acute angle $= \theta_A = 180 - (63.43\ldots + 56.30\ldots) = 60.26$ correct to two decimal places



Question 19 (continued)



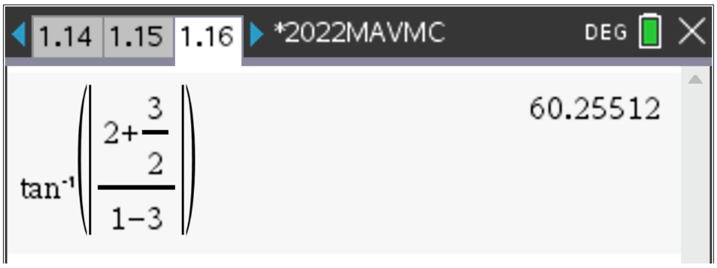


OR

Please note absolute values are not on the course but can be used for these types of questions.

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{2 + \frac{3}{2}}{1 - 3}\right|$$

Acute angle $= 60.26$ correct to two decimal places



Question 20 **Answer B**

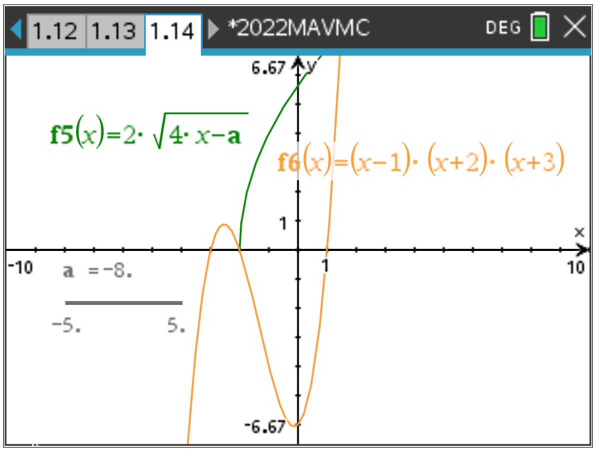
$f(x) = 2\sqrt{4x - a}$ and $g(x) = (x-1)(x+2)(x+3)$

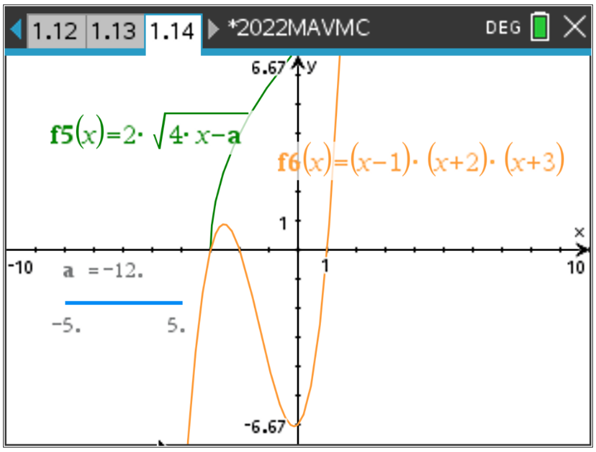
The maximum number of points of intersection is two.

When $x = -2$, $2\sqrt{4 \times -2 - a} = 0$, $a = -8$

When $x = -3$, $2\sqrt{4 \times -3 - a} = 0$, $a = -12$

So two points of intersection occur when $-12 \le a \le -8$.





END OF SECTION A SOLUTIONS

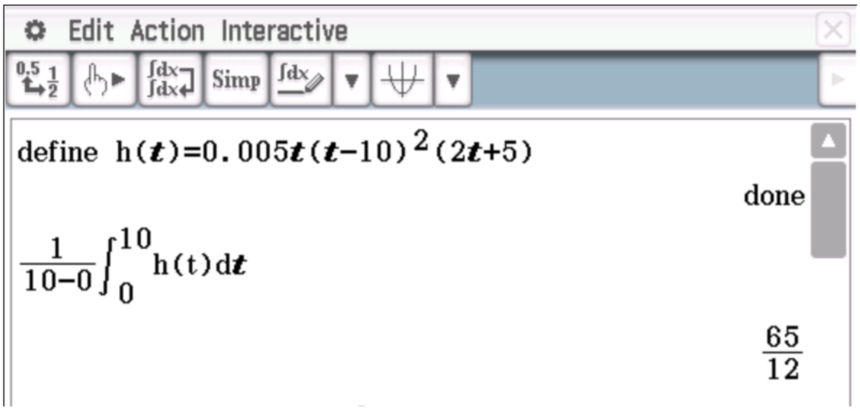
SECTION B

Question 1

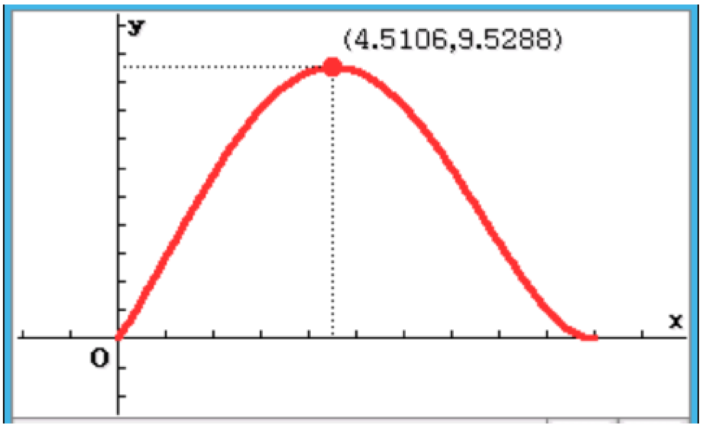
$h:[0,10] \to R, h(t) = 0.005t(t-10)^2(2t+5)$

a. Average height $= \dfrac{1}{10-0}\int_0^{10} h(t)dt$

$= \dfrac{65}{12}$ cm **1A**



b. Maximum height $= 9.5$ cm correct to one decimal place **1A**

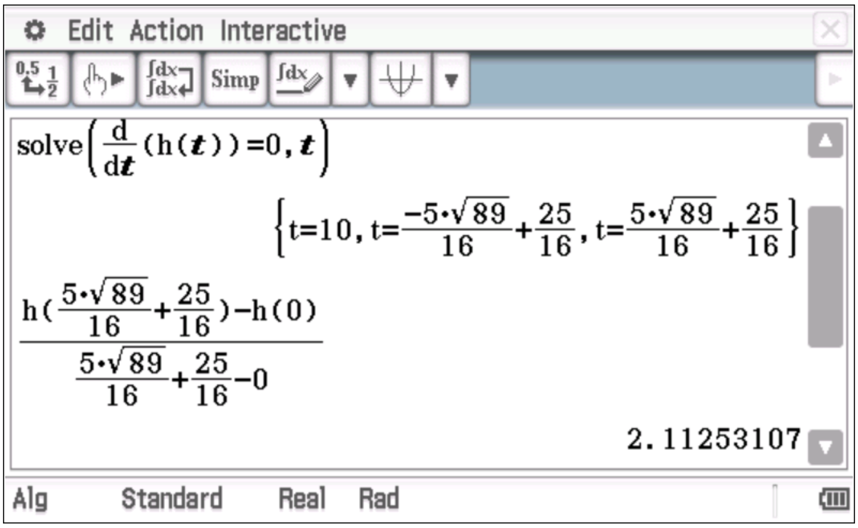


c. Average rate of change of h for $t \in [0, 4.5106...]$ or $t \in \left[0, \dfrac{25+5\sqrt{89}}{16}\right]$

$= \dfrac{h\left(\dfrac{25+5\sqrt{89}}{16}\right) - h(0)}{\dfrac{25+5\sqrt{89}}{16} - 0}$ **1M**

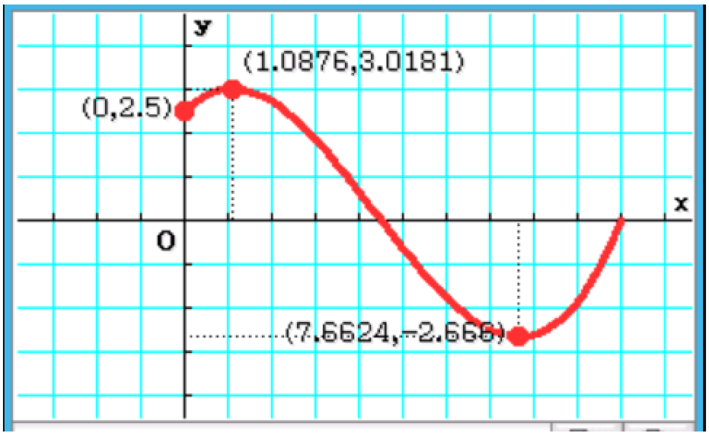
$= 2.11$ cm/min **1A**

Question 1 (continued)



d. From the gradient graph the height of liquid is increasing at its maximum rate at

$t = 1.09$ min correct to two decimal places. **1A**



e. Volume of cone $= \frac{1}{3}\pi r^2 h$ where $r = \frac{1}{4}h$ **1M**

$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h = \frac{1}{3}\pi\frac{h^3}{16}$

$V = \frac{\pi}{48}h^3$ as required **1M** *Show that*

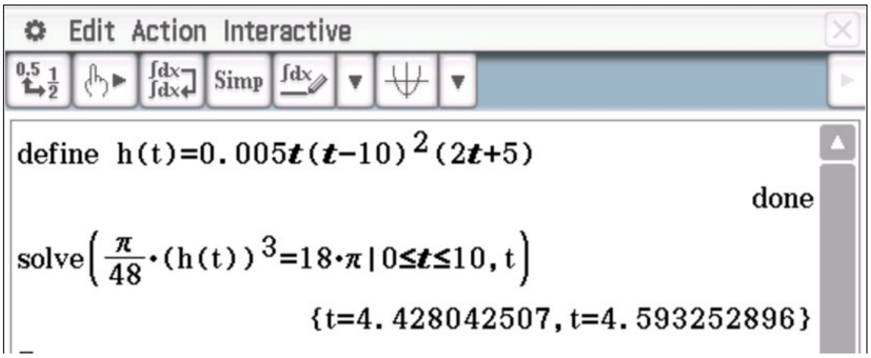
f. Let glass have half volume.

When full, volume of glass $= \frac{1}{3}\pi \times 3^2 \times 12 = 36\pi \text{ cm}^3$

Solve for t, $\frac{\pi}{48}(h(t))^3 = 18\pi$ **1M**

Glass is half full at $t = 4.43, 4.59$ mins correct to two decimal places **1A**

Question 1 (continued)

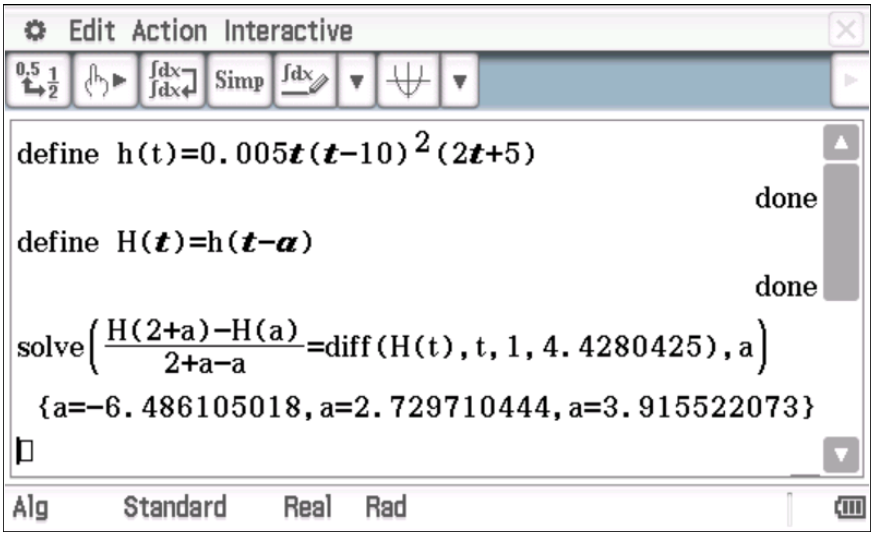


g. $h_1:[a, 10+a] \to R, h_1(t) = 0.005(t-a)(t-a-10)^2(2(t-a)+5)$

Recognise $h_1(t) = h(t-a)$ for $t \in [a, 10+a]$

Equate $\dfrac{h_1(2+a) - h_1(a)}{2+a-a} = h_1'(4.428...)$ **1M**

$a = 2.73$ or $a = 3.92$ correct to 2 decimal places **1A**



Question 2

$w:[-4,4] \to R, w(x) = \sqrt{r^2 - x^2} + c$ where $r > 0$ and $c \in R^+ \cup \{0\}$.

a. Radius = 4 dm **1A**

b. $w(x) = \sqrt{16 - x^2} + 1$ **1A**

c. Area of the rectangle is $A_r = 2x_1(y_1 - 1)$.

$A_r = 2x_1\left(\sqrt{16 - x^2} + 1 - 1\right)$

$A_r = 2x_1\sqrt{16 - x_1^2}$ as required **1M** *Show that*

d. $A_r = 2x_1\sqrt{16 - x_1^2}$

Question 2 (continued)

Solve $\frac{d}{dx_1}\left(A_r\right)=0$

Gives $x_1=\pm 2\sqrt{2}$

for $x_1>0, x_1=2\sqrt{2}$

maximum area of rectangle = 16 dm^2. **1A**

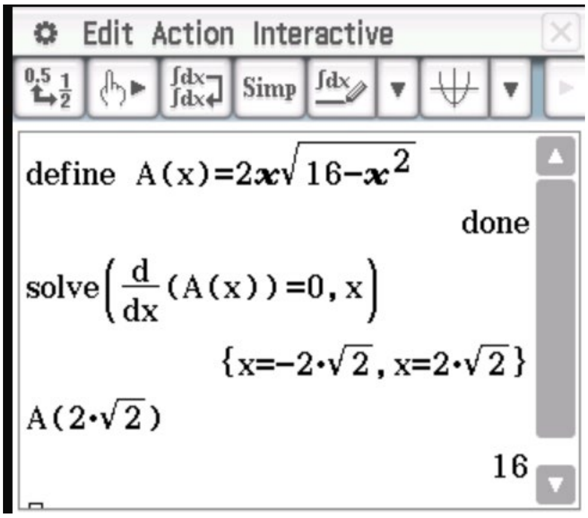
Justification

x	1	$2\sqrt{2}$	3
$A_r'(x)$	$\frac{28\sqrt{15}}{15}$	0	$\frac{-4\sqrt{7}}{7}$
	/	—	\

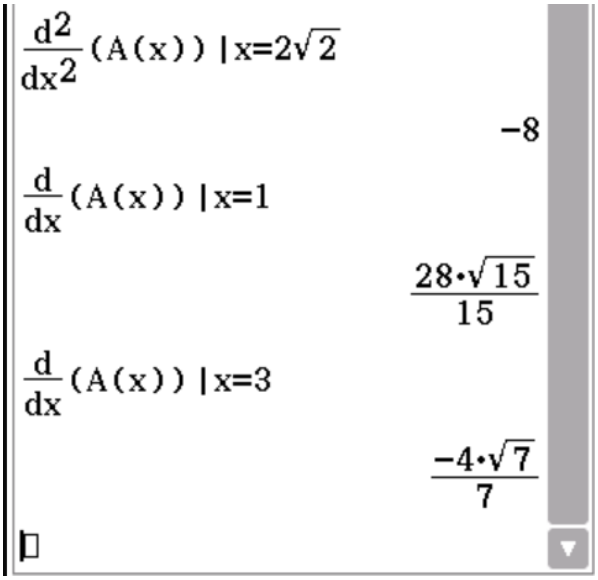
1M

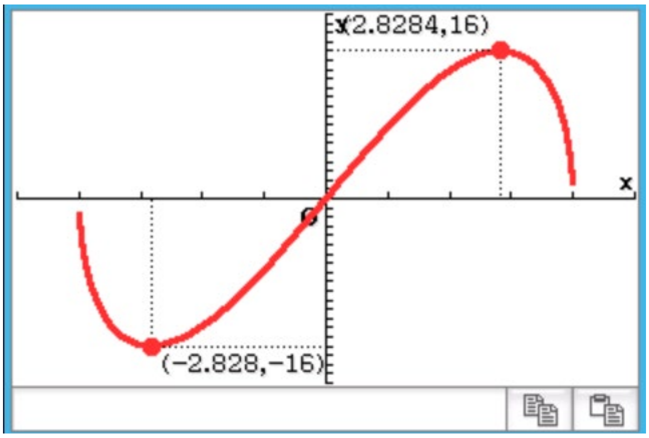
OR

$A_r''\left(2\sqrt{2}\right)=-8$, $A_r''<0$, hence a maximum **1M**



Question 2 (continued)





e. Given radius = 4 dm

Area of semicircle $=\frac{1}{2}\pi 4^2=8\pi$

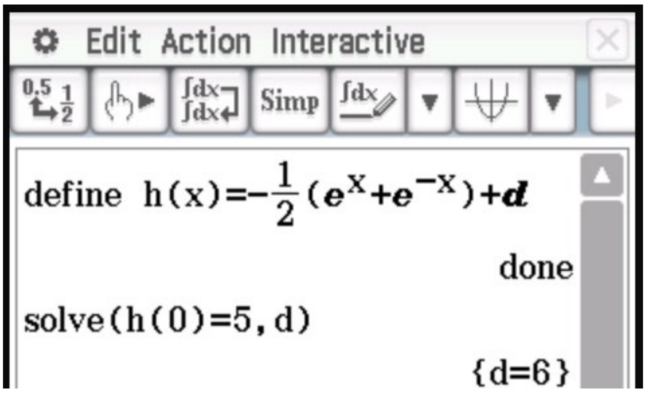
Proportion stained glass $=\frac{16}{8\pi}=\frac{2}{\pi}$ **1A**

f. $h(x)=-\frac{1}{2}\left(e^x+e^{-x}\right)+d$ where $d\in R^+\cup\{0\}$.

Substitute $(0,5)$ into $h(x)=-\frac{1}{2}\left(e^x+e^{-x}\right)+d$

$d=6$ **1A**

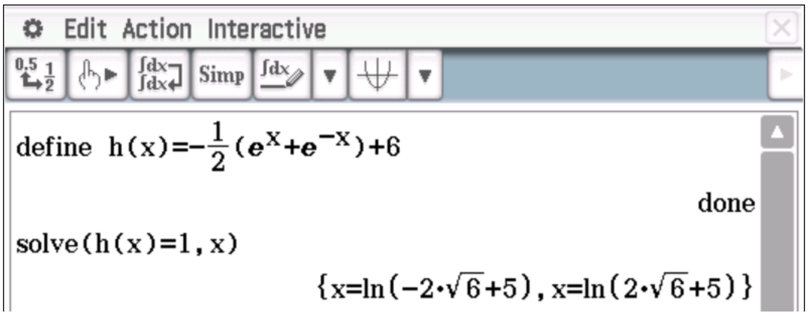
Question 2 (continued)



g. Solve $-\frac{1}{2}\left(e^{x}+e^{-x}\right)+6=1$

Intersects with line $y=1$ at

$\left(\log_e\left(5-2\sqrt{6}\right),1\right)$ and $\left(\log_e\left(5+2\sqrt{6}\right),1\right)$ **1A**



h. For the rectangle:

solve $16=2\times 2\sqrt{2}\times \text{width}$

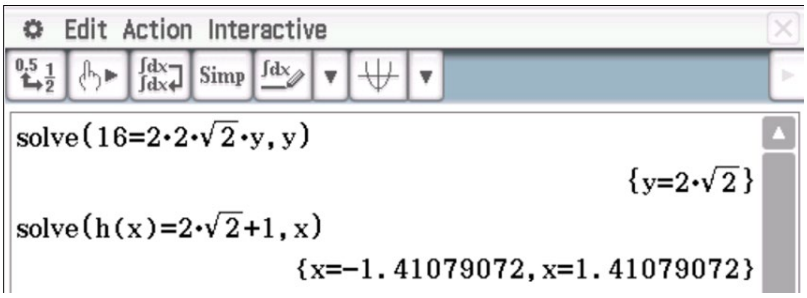
$\text{width}=2\sqrt{2}$

Solve $h(x)=2\sqrt{2}+1$ for x

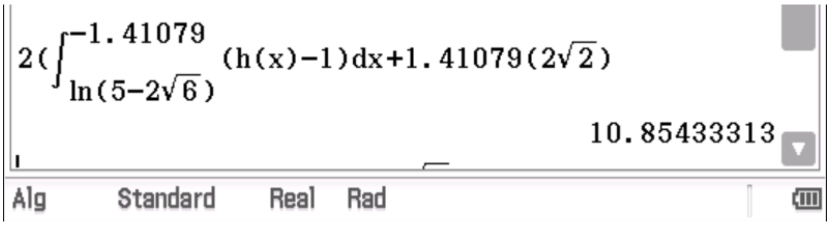
$x=-1.410...$ **1M**

$$2\left(\int_{\log_e(5-2\sqrt{6})}^{-1.410...}\left(h(x)-1\right)dx+1.410...\times 2\sqrt{2}\right)$$ **1M**

$=10.9\text{ dm}^2$ **1A**



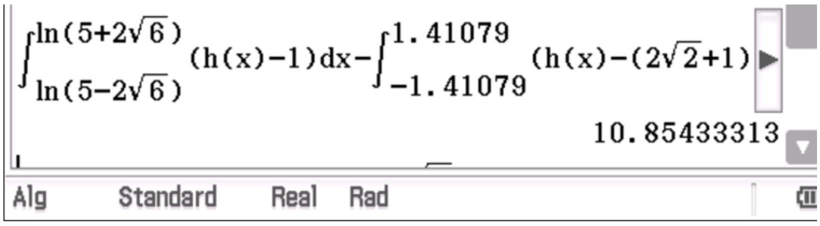
Question 2 (continued)



OR

$$\int_{\log_e(5-2\sqrt{6})}^{\log_e(5+2\sqrt{6})} \left(h(x)-1\right)dx - \int_{-1.410...}^{1.410...} \left(h(x)-\left(2\sqrt{2}+1\right)\right)dx$$ **2M**

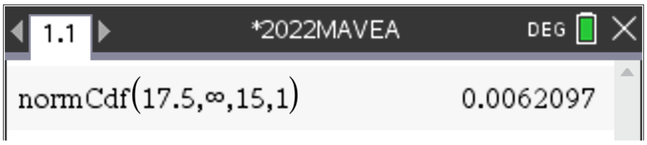
$=10.9\ \text{dm}^2$ **1A**



Question 3

a.i. $X \sim \text{N}(15,1)$

$\Pr(X > 17.5) = 0.0062$ **1A**

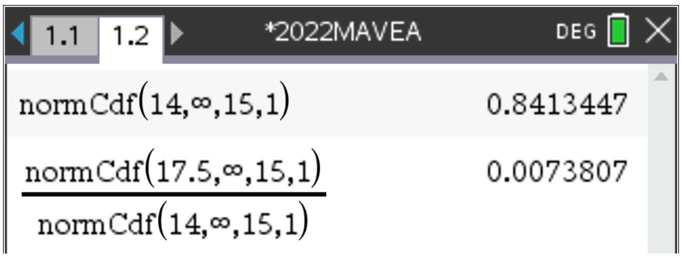


a.ii. $\Pr(X > 17.5 \mid X \geq 14)$

$= \dfrac{\Pr(X > 17.5 \cap X \geq 14)}{\Pr(X \geq 14)}$

$= \dfrac{\Pr(X > 17.5)}{\Pr(X \geq 14)}$ **1M**

$= 0.0074$ correct to four decimal places **1A**



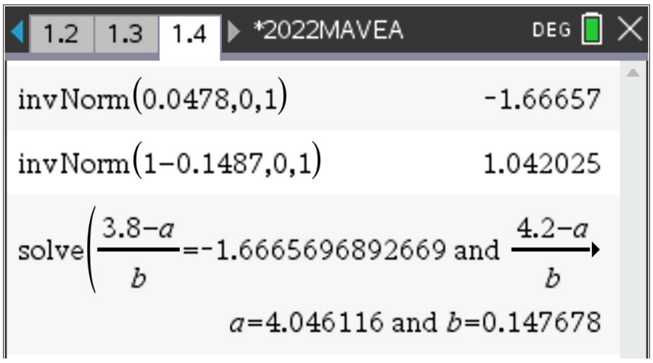
Question 3 (continued)

b. $M \sim \text{N}(\mu, \sigma)$

$\dfrac{3.8-\mu}{\sigma} = -1.666....$

$\dfrac{4.2-\mu}{\sigma} = 1.042...$ **1M**

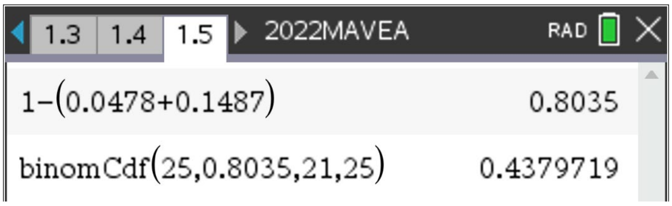
$\mu = 4.05$ kg, $\sigma = 0.15$ kg **1A**



c. $X \sim \text{Bi}(25, 0.8035)$

$p = 1-(0.0478+0.1487) = 0.8035$ **1M**

$\Pr(X > 20) = \Pr(21 \le X \le 25) = 0.4380$ correct to four decimal places **1A**



d. $X_1 \sim \text{Bi}(n, 0.8035)$

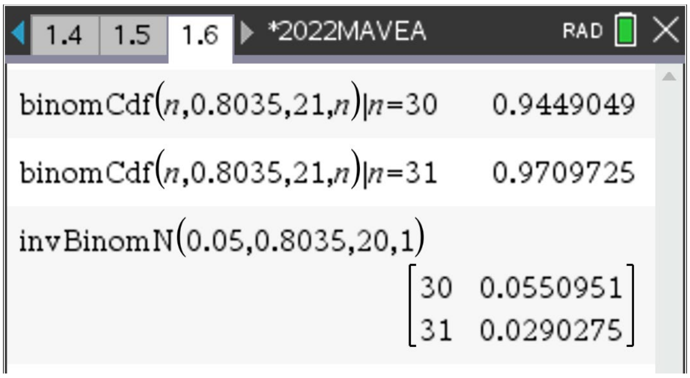
$\Pr(21 \le X_1 \le n) > 0.95$ **1M**

Trial and error

$n = 30$ $\Pr(21 \le X_1 \le 30) = 0.944....$

$n = 31$ $\Pr(21 \le X_1 \le 31) = 0.970....$

$n = 31$ **1A**



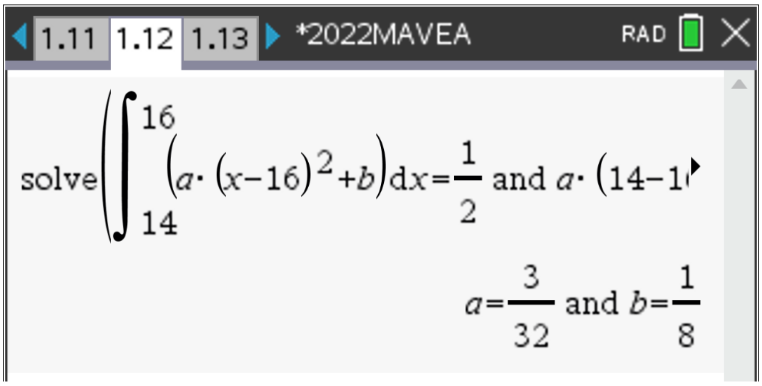
Question 3 (continued)

e. $s(x)=\begin{cases}\frac{x}{4}-3 & 12\le x\le 14\\ a(x-16)^2+b & 14<x\le 16\\ 0 & \text{elsewhere}\end{cases}$

$\int_{12}^{14}\left(\frac{x}{4}-3\right)dx=\frac{1}{2}$

Solve $\int_{14}^{16}\left(a(x-16)^2+b\right)dx=\frac{1}{2}$ and $a(14-16)^2+b=\frac{14}{4}-3$ for a and b **1M**

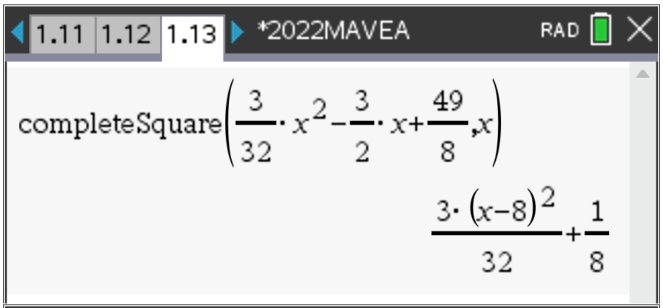
$a=\frac{3}{32}$, $b=\frac{1}{8}$ **1A**

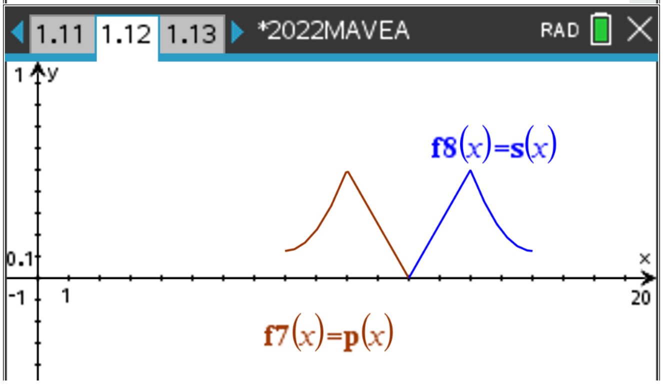


f.i. $p(x)=\begin{cases}\frac{3x^2}{32}-\frac{3x}{2}+\frac{49}{8} & 8\le x\le 10\\ -\frac{x}{4}+3 & 10<x\le 12\\ 0 & \text{elsewhere}\end{cases}$

Reflection in the line $x=12$ **1A**
OR
Reflection in the y-axis
Translation of 24 units to the right **1A**

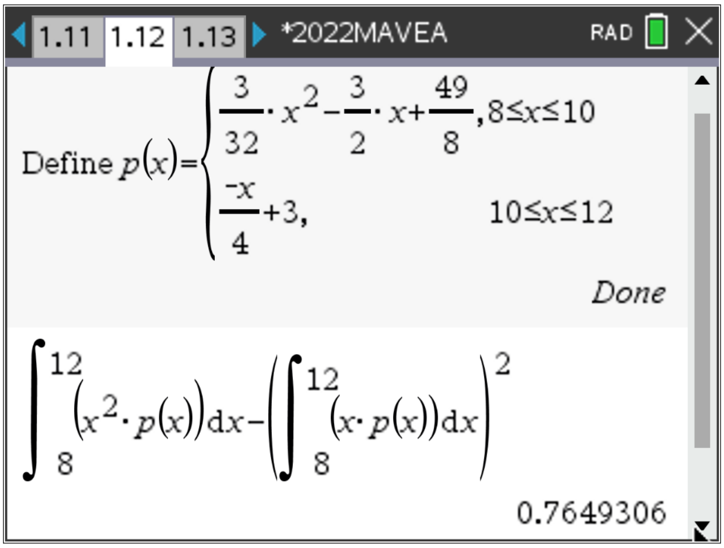
Question 3 (continued)





f.ii. $\int_8^{12}\left(x^2p(x)\right)dx-\left(\int_8^{12}\left(xp(x)\right)dx\right)^2$ **1M**

$=0.765$ correct to three decimal places **1A**



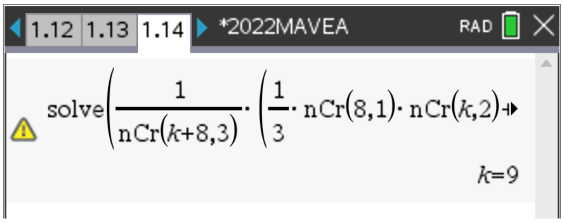
g. Solve $0\times\frac{\binom{8}{0}\binom{k}{3}}{\binom{8+k}{3}}+\frac{1}{3}\times\frac{\binom{8}{1}\binom{k}{2}}{\binom{8+k}{3}}+\frac{2}{3}\times\frac{\binom{8}{2}\binom{k}{1}}{\binom{8+k}{3}}+1\times\frac{\binom{8}{3}\binom{k}{0}}{\binom{8+k}{3}}=\frac{8}{17}$ for k **1M**

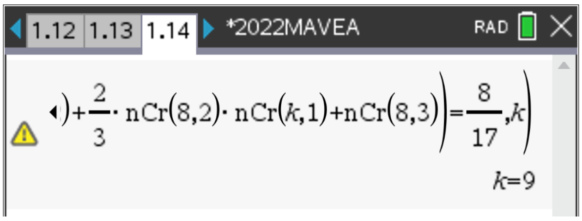
OR

Solve $\frac{3}{3}\times\frac{8}{8+k}=\frac{8}{17}$ for k **1M**

$k=9$ **1A**

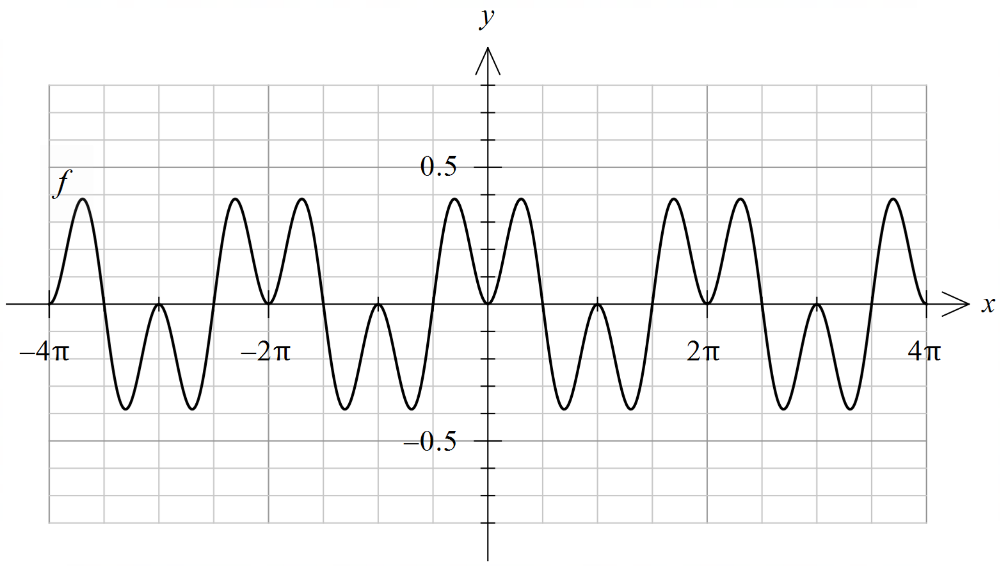
Question 3 (continued)





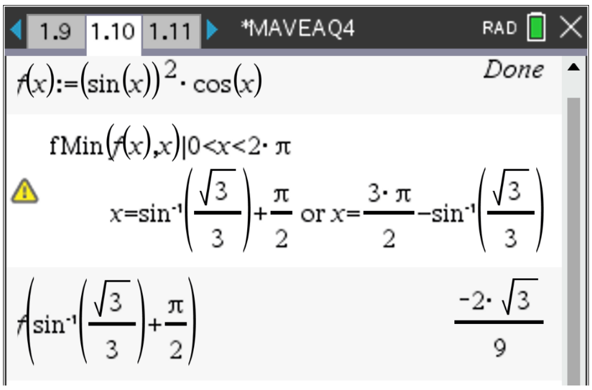
Question 4

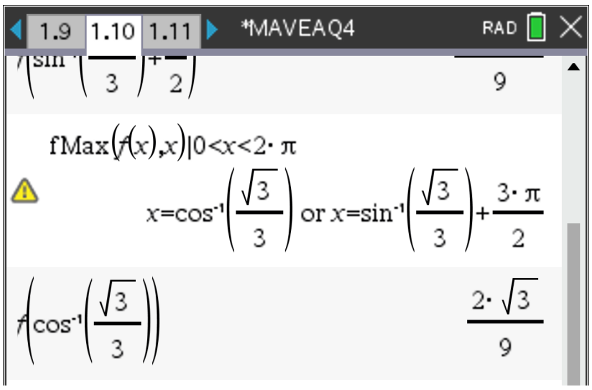
$f: R \to R, f(x) = \sin^2(x)\cos(x)$



a. 2π **1A**

b. $\left[-\frac{2\sqrt{3}}{9}, \frac{2\sqrt{3}}{9}\right]$ **1A**



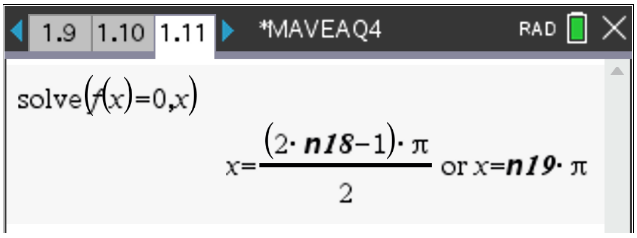


c. $x = \frac{\pi k}{2},\ k \in Z$ **1A**

OR

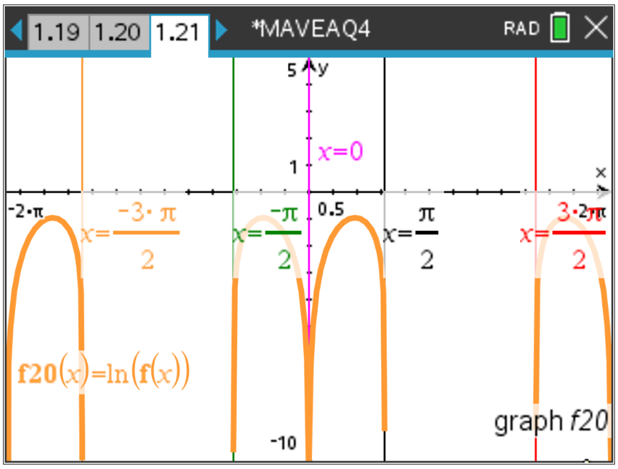
$x = \frac{(2k-1)\pi}{2},\ x = k\pi,\ k \in Z$ **1A**

Question 4 (continued)



d. $2\pi k < x < \frac{\pi}{2} + 2\pi k, k \in Z$ or **1A**

$-\frac{\pi}{2} + 2\pi k < x < 2\pi k, k \in Z$ **1A**

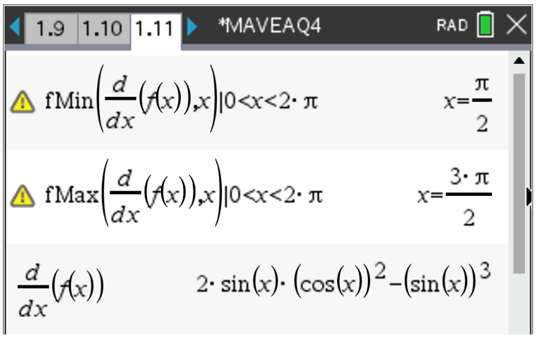


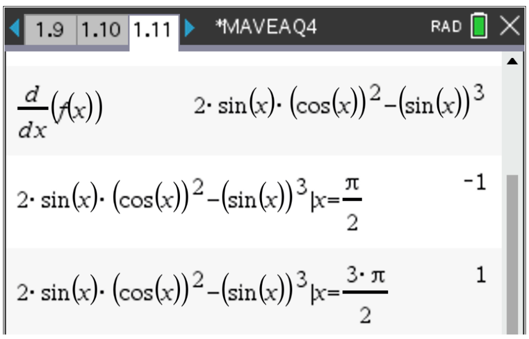
e. $A\left(\frac{\pi}{2},-1\right), B\left(\frac{3\pi}{2},1\right)$ **1A**

$m = \frac{2}{\pi}$

$y + 1 = \frac{2}{\pi}\left(x - \frac{\pi}{2}\right)$

$y = \frac{2}{\pi}x - 2$ **1A**

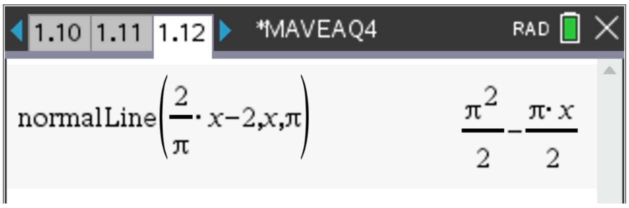




f.i. Midpoint $(\pi,0)$, $m = -\frac{\pi}{2}$

Question 4 (continued)

$y=-\frac{\pi}{2}(x-\pi)=-\frac{\pi}{2}x+\frac{\pi^2}{2}$ **1A**



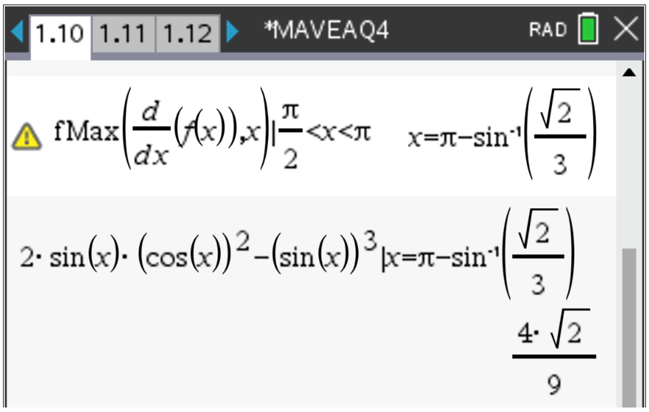
f.ii. $C\left(\pi-\sin^{-1}\left(\frac{\sqrt{2}}{3}\right),\frac{4\sqrt{2}}{9}\right)$ **1M**

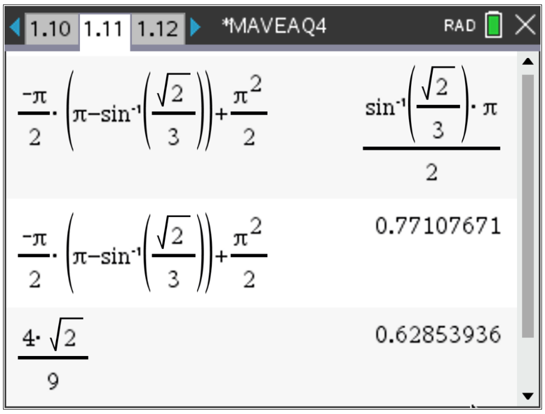
$$y=-\frac{\pi}{2}\times\left(\pi-\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)\right)+\frac{\pi^2}{2}$$

$$=\frac{\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)\pi}{2}$$

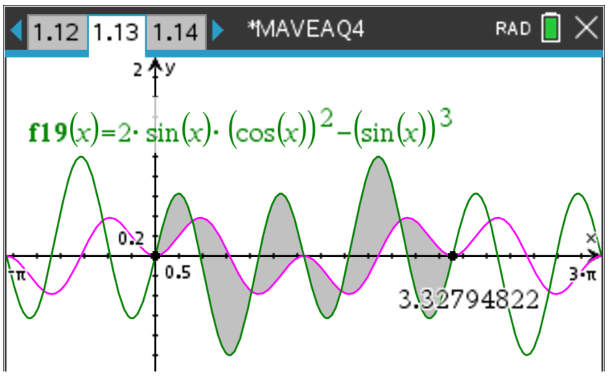
$$\neq\frac{4\sqrt{2}}{9}$$

1M *Show that*





g. Area = 3.33 correct to two decimal places **1A**



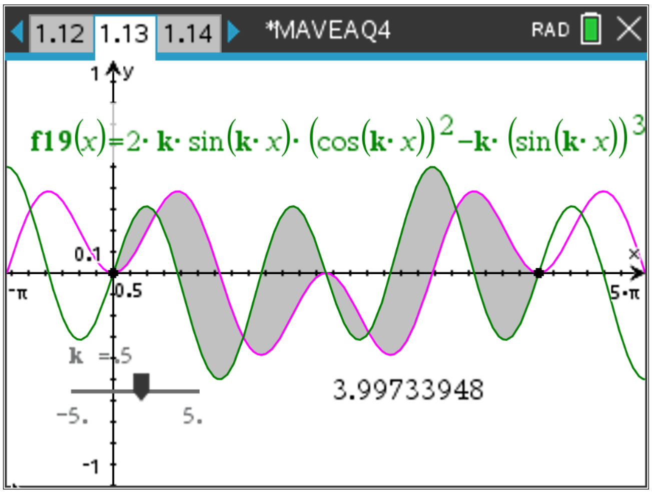
Question 4 (continued)

h. $\dfrac{2\pi}{k} = 4\pi$

$k = \dfrac{1}{2}$

$g(x) = \sin^2(kx)\cos(kx)$, $g'(x) = 2k\sin(kx)\cos^2(kx) - k\sin^3(kx)$ **1M**

Area = 4.00 **1A**



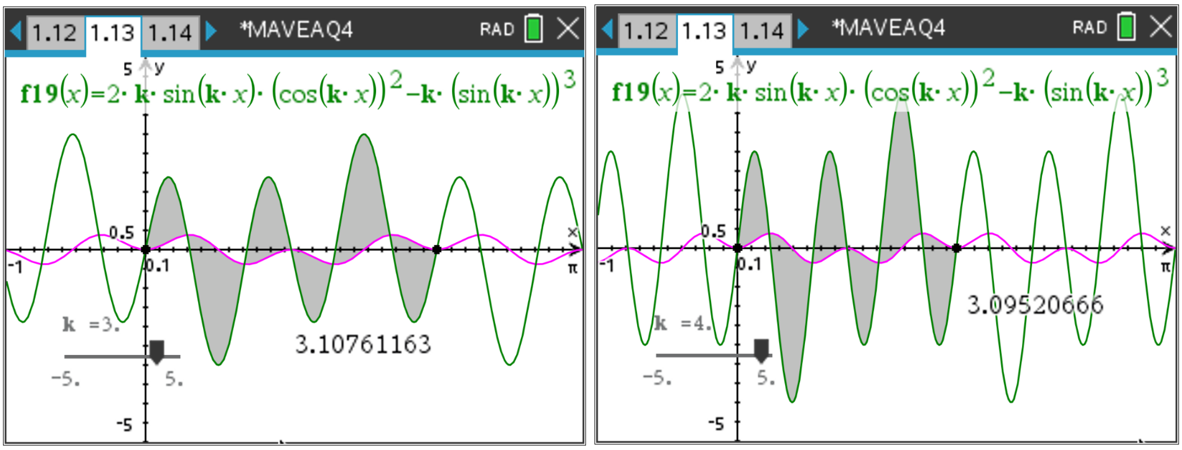
i. Find the bounded area over the interval $\left[0, \dfrac{2\pi}{k}\right]$.

Use trial and error

When $k = 3$, the bounded area is 3.107…

When $k = 4$, the bounded area is 3.095…

$k = 4$ **1A**



Question 5

$p: R \to R, p(x) = -(x-1)^3$

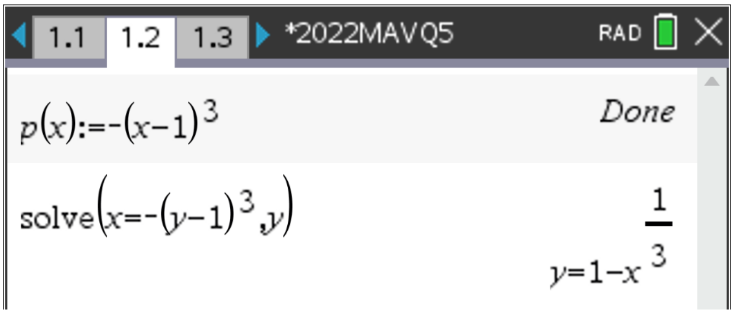
a. Let $y = -(x-1)^3$

Inverse swap x and y

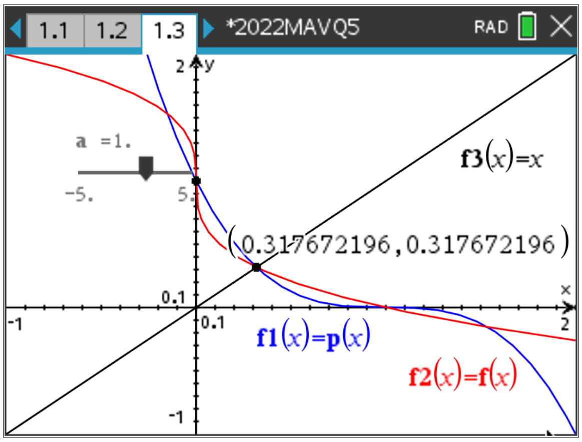
$x = -(y-1)^3$

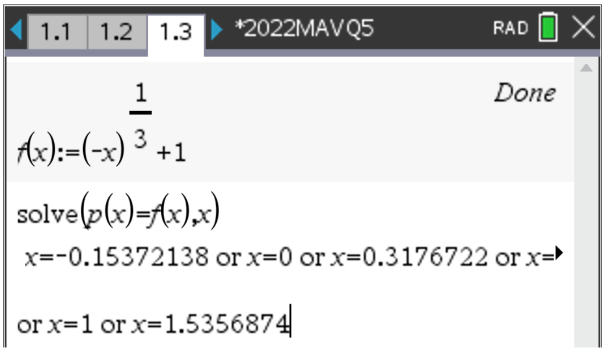
$p^{-1}(x) = \sqrt[3]{-x} + 1 = 1 - \sqrt[3]{x}$ **1A** (either form)

Question 5 (continued)



b. 5 **1A**

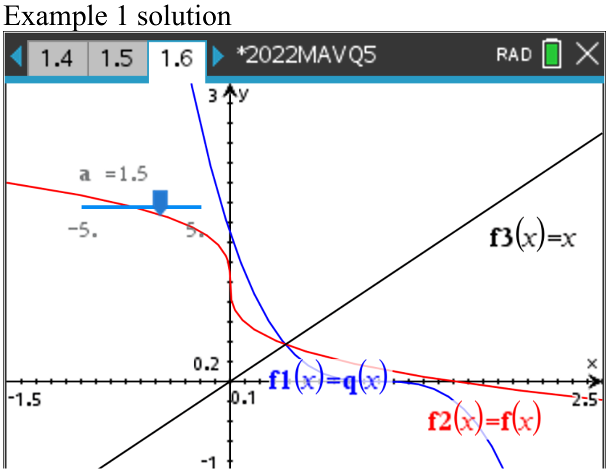




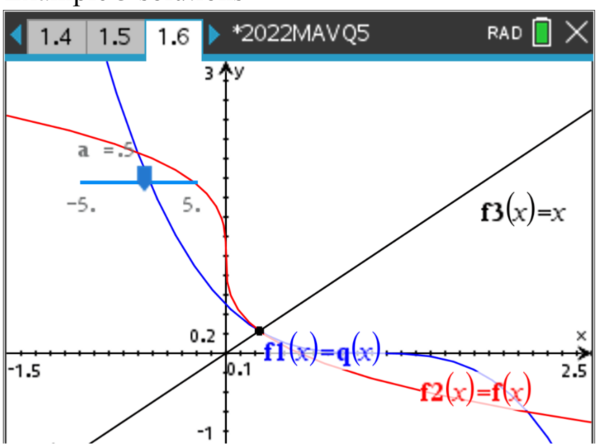
c. $q: R \to R, q(x) = -a(x-1)^3$, $q^{-1}: R \to R, q^{-1}(x) = 1 - \sqrt[3]{\dfrac{x}{a}}$

1, 3 or 5 **1A**

Example 1 solution



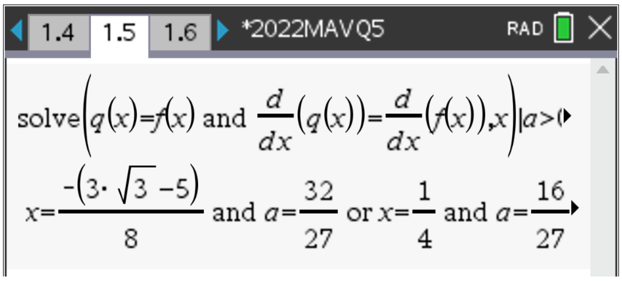
Example 3 solutions

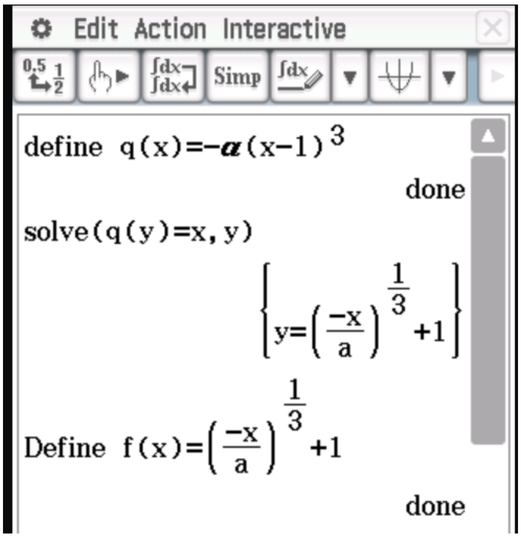


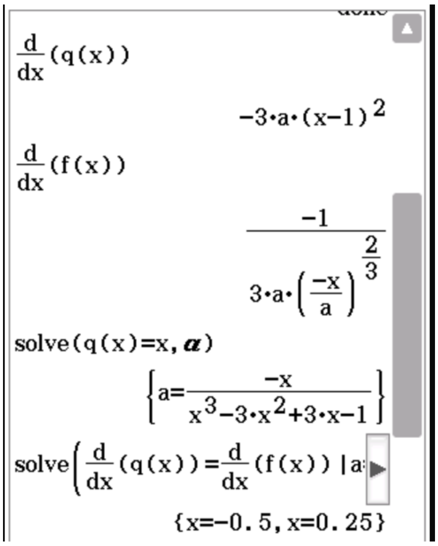
d. Solve $q(x) = q^{-1}(x)$ and $\dfrac{d}{dx}(q(x)) = \dfrac{d}{dx}\left(q^{-1}(x)\right)$ **1M**

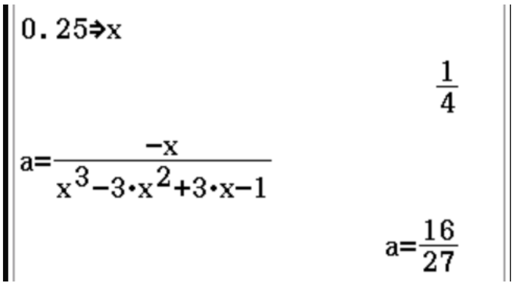
$a = \dfrac{16}{27}$ **1A**

Question 5 (continued)





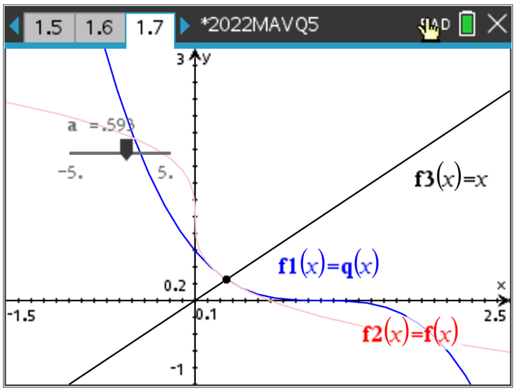


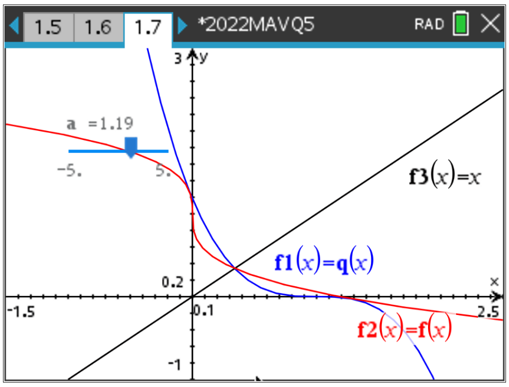


e. Three solutions occur when $0 < a \le \dfrac{16}{27}$ and $a = \dfrac{32}{27}$ **1A**

$a = \dfrac{16}{27}$ $a = \dfrac{32}{27}$

Question 5 (continued)





f. $t: R \to R, t(x) = -(x-1)\left(x^2 + bx + c\right)$

One solution for $x^2 + bx + c = 0$

$b^2 - 4c = 0$

$b = \pm 2\sqrt{c}$, $b \neq -2$ **1A**

Note if $b = -2$, $t(x) = -(x-1)\left(x^2 - 2x + 1\right) = -(x-1)^3$

Two solutions for $x^2 + bx + c = 0$ if one of the factors is $(x-1)$

$x^2 + bx + c = (x-1)(x-c) = x^2 - (c+1)x + c$

$b = -c - 1$, $b \neq -2$ **1A**

END OF SOLUTIONS