Neap

Trial Examination 2022

VCE Mathematical Methods Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 20 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Sam is currently 27 years older than Oma. In three years' time, Oma's age will be a quarter of Sam's age.

How old are Sam and Oma currently?

	Sam	Oma
A.	29	2
B.	31	4
C.	32	5
D.	33	6
E.	35	8

Question 2

Consider the following simultaneous equations.

$$y = 4x - 3$$
$$y - ax + 2x + b - 1 = 0$$

What are the values of a and b if there is an infinite number of solutions to the simultaneous equations?

A. $a = 6, b = 4$

B. $a = 6, b = -4$

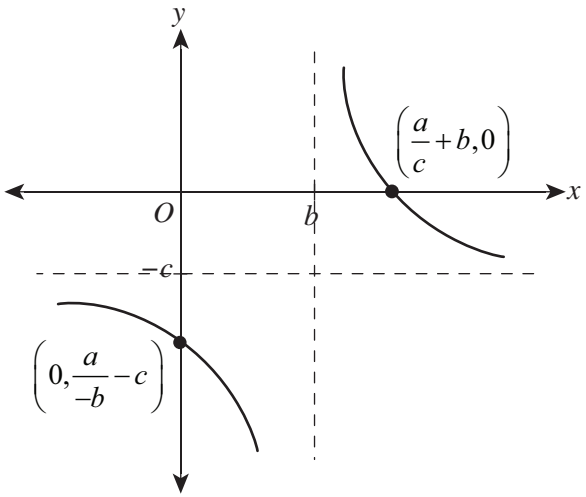
C. $a = 6, b \neq -4$

D. $a \neq 6, b \neq 4$

E. $a \neq 6, b \in \mathbb{R}$

Question 3

The graph of a rectangular hyperbola is shown below.



A possible equation for this function is

A. $y=\frac{a}{x-b}+c$

B. $y=\frac{a}{x-b}-c$

C. $y=\frac{a}{x+b}-c$

D. $y=\frac{a}{x+b}+c$

E. $y=\frac{c}{x-b}-a$

Question 4

The graph of $f(x)=x^{-2}$ undergoes a series of transformations to become the image graph of $h(x)=-\frac{1}{(x+2)^2}+3$.

The transformations that take $f(x)$ to $h(x)$ are

A. dilation of 2 units parallel to the x-axis, reflection in the x-axis and a translation of 3 units in the negative direction of the y-axis.

B. dilation of 2 units parallel to the y-axis, reflection in the x-axis and a translation of 3 units in the positive direction of the y-axis.

C. reflection in the y-axis, a translation of 2 units in the negative direction of the x-axis and a translation of 3 units in the positive direction of the y-axis.

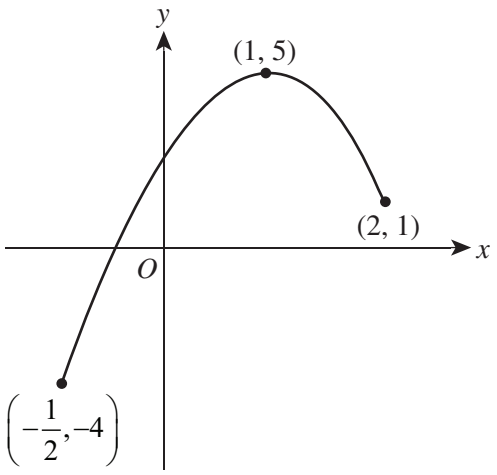
D. reflection in the x-axis, a translation of 2 units in the negative direction of the x-axis and a translation of 3 units in the positive direction of the y-axis.

E. reflection in the x-axis, a translation of 2 units in the positive direction of the x-axis and a translation of 3 units in the positive direction of the y-axis.

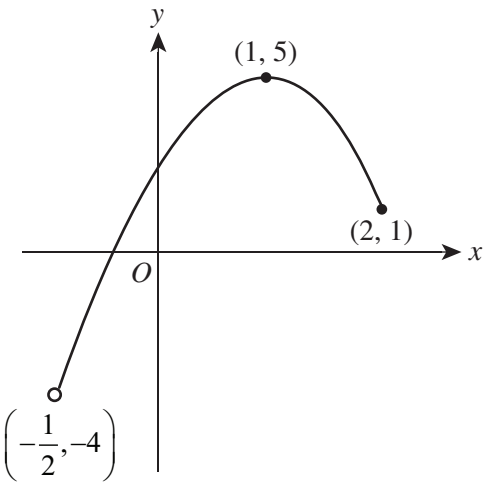
Question 5

Which one of the following graphs represents $y = -4(x-1)^2 + 5$ for $x \in \left[-\frac{1}{2}, 2\right)$?

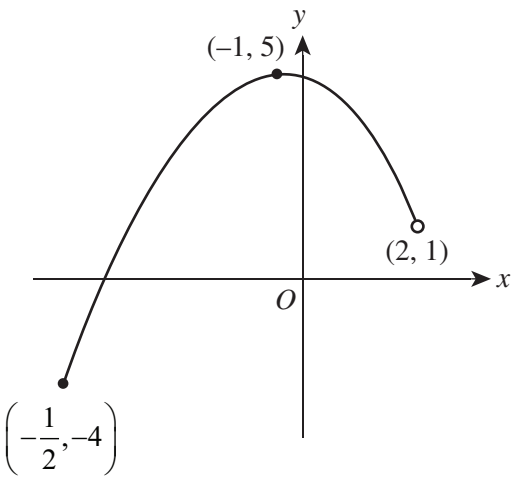
A.



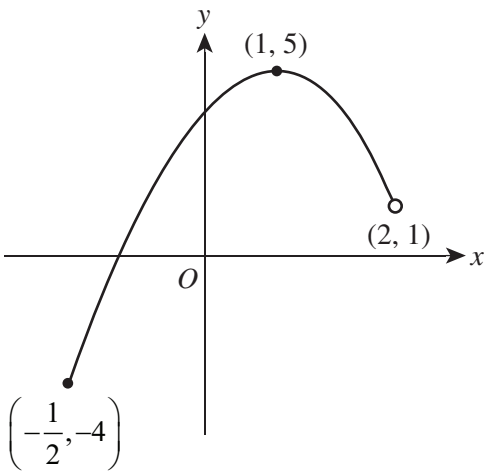
B.



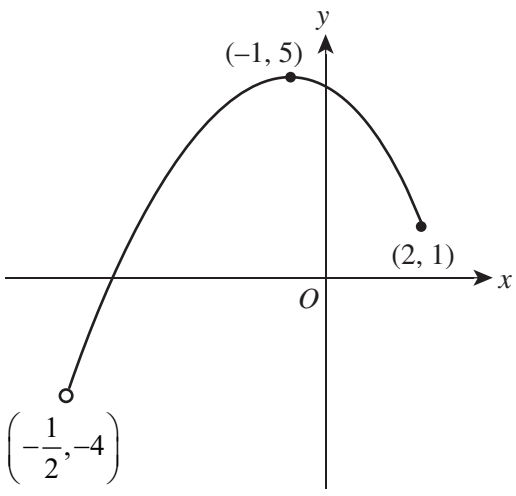
C.



D.



E.



Question 6

Given that $\sin(a) = 0.02$, find all possible values for $\cos(a)$.

A. $\cos(a) = \pm\sqrt{1 - 0.02}$

B. $\cos(a) = \sqrt{1 - 0.02^2}$

C. $\cos(a) = \pm\sqrt{1 - 0.02^2}$

D. $\cos(a) = -\sqrt{1 - 0.02^2}$

E. $\cos(a) = 1 - 0.02^2$

Question 7

The inverse of the graph $f(x) = 2 \times 3^{-x}$ can be written as

A. $f^{-1}(x) = \log_3\left(\frac{x}{2}\right)$

B. $f^{-1}(x) = -\log_3\left(\frac{x}{2}\right)$

C. $f'(x) = -\log_3\left(\frac{x}{2}\right)$

D. $f'(x) = \log_3\left(\frac{x}{2}\right)$

E. $f^{-1}(x) = \log_2\left(\frac{x}{3}\right)$

Question 8

Consider the polynomial $P(x) = 6x^4 + 7x^3 - 18x^2 + 5x$.

Which one of the following is **not** a rational solution of $P(x) = 0$?

A. $-\frac{5}{2}$

B. 0

C. $\frac{1}{3}$

D. 1

E. $\frac{5}{2}$

Question 9

In radians, the angle 210° is

A. $\frac{2\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{5\pi}{6}$

D. $\frac{7}{6}$

E. $\frac{7\pi}{6}$

Question 10

If $\int_{6}^{12} f(x) \cdot dx = 3,$ then $\int_{3}^{6} f(2x) \cdot dx$ is equal to

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 6

E. 12

Question 11

Event A is defined as the set of integers {1, 2, …, 10}. Event B is defined as the set of integers {3, 4, 5}. The universal set is defined as the set of integers {1, 2, …, 12}.

What is the set of $\Pr(A \cup B)$?

A. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

B. {3, 4, 5}

C. {1, 2, 6, 7, 8, 9, 10}

D. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

E. $\varnothing$

Question 12

${}^{7}C_{5}$ is **not** equivalent to

A. $\binom{7}{5}$

B. $\frac{{}^{7}P_{5}}{5!}$

C. $\frac{7\times6\times5\times4\times3}{5\times4\times3\times2\times1}$

D. $\frac{7!}{5!}$

E. $\frac{7!}{5!\times2!}$

Question 13

A sports team of 8 people is to be chosen from a selection of 5 men and 8 women.

The probability that 3 men and 5 women are chosen is

A. $\frac{1}{154\,440}$

B. $\frac{1}{1287}$

C. $\frac{10}{1287}$

D. $\frac{56}{1287}$

E. $\frac{560}{1287}$

Question 14

When fully simplified, the expression $\frac{a^{\frac{3}{4}} \times a^{-\frac{1}{5}}}{a^{-\frac{1}{4}} \times a^{\frac{2}{5}}}$ is

A. $\frac{1}{a^{-\frac{2}{3}}}$

B. $a^{\frac{4}{5}}$

C. $a^{-\frac{4}{5}}$

D. $a^{\frac{2}{5}}$

E. $a^{-\frac{2}{5}}$

Question 15

The period of the graph $y = a\tan(bx)$ is

A. a

B. b

C. $\frac{\pi}{b}$

D. $\frac{2\pi}{a}$

E. $\frac{2\pi}{b}$

Question 16

When fully simplified, the expression $\frac{\log_a(32)}{\log_a(4)}$ is

A. $-\frac{5}{2}$

B. $\frac{5}{2}$

C. $\frac{5}{2}\log_a(2)$

D. $\log_a(8)$

E. $\log_a(128)$

Question 17

Let $f(x) = 3x^3 - 4x^2 - x + 2$.

Which row of the table gives the average rate of change between $x = 0$ and $x = \frac{1}{2}$ and the instantaneous rate of change at $x = \frac{1}{2}$?

	Average rate of change	Instantaneous rate of change
A.	$-\frac{9}{4}$	$-\frac{11}{4}$
B.	$\frac{9}{4}$	$\frac{11}{4}$
C.	$-\frac{11}{4}$	$-\frac{9}{4}$
D.	$\frac{11}{4}$	$\frac{9}{4}$
E.	$\frac{7}{8}$	$-\frac{11}{4}$

Question 18

The uranium-238 isotope is a radioactive material that decays over time. A half-life is the amount of time required for half of such a material to decay. Uranium-238 has a half-life of 4.5 billion years.

The amount of radioactive material, U, present at a particular time, t, in years, is given by the formula

$$U = U_0 \times 2^{-kt}$$

where U_0 is the initial amount of uranium and k is a positive constant unique to uranium.

Find the value of k.

A. 2.22×10^{10}

B. 2.22×10^{-10}

C. -5.55×10^{-11}

D. 5.56×10^{-11}

E. 5.56×10^{11}

Question 19

Let $s(t) = 12t - t^2 + 7$ be the position function of a particle moving in a straight line, where t is the time in seconds and s is the position in metres.

The position of the particle when $t = 11$ is

A. 0.34

B. 7

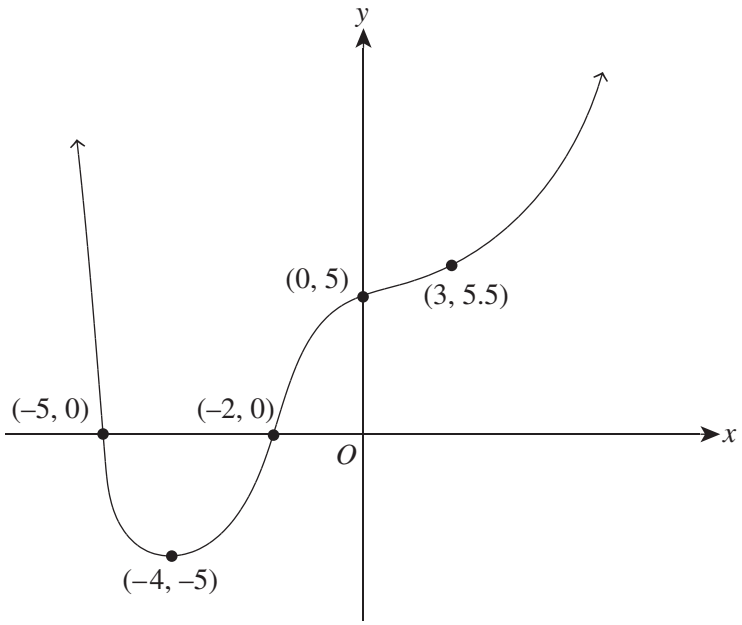
C. 12

D. 18

E. 260

Question 20

Consider the following graph of $y = h(x)$.



Find $\{x : h'(x) < 0\}$.

A. $\{x : x \in (-\infty, -5)\}$

B. $\{x : x \in (-5, -4)\}$

C. $\{x : x \in (-\infty, -4)\}$

D. $\{x : x \in (-4, \infty)\}$

E. $\{x : x \in (-4, 3) \cup (3, \infty)\}$

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (15 marks)

A line segment is made between the two points $A\left(\frac{3}{2}, 5\right)$ and $B\left(-2, -\frac{5}{2}\right)$.

a. Find the midpoint of the line segment. 1 mark

b. Find the distance between points A and B. 2 marks

c. Show that the equation of the line that passes through points A and B is $f(x) = \frac{15}{7}x + \frac{25}{14}$. 3 marks

d. Find the angle that $f(x)$ makes with the positive direction of the x-axis. Give your answer correct to the nearest degree. 1 mark

e. The equation of $f(x)$ undergoes the following transformations to become $g(x)$.

- a translation of $\frac{25}{14}$ in the negative direction of the y-axis
- a dilation of factor $\frac{1}{7}$ from the x-axis

Find $g(x)$. 1 mark

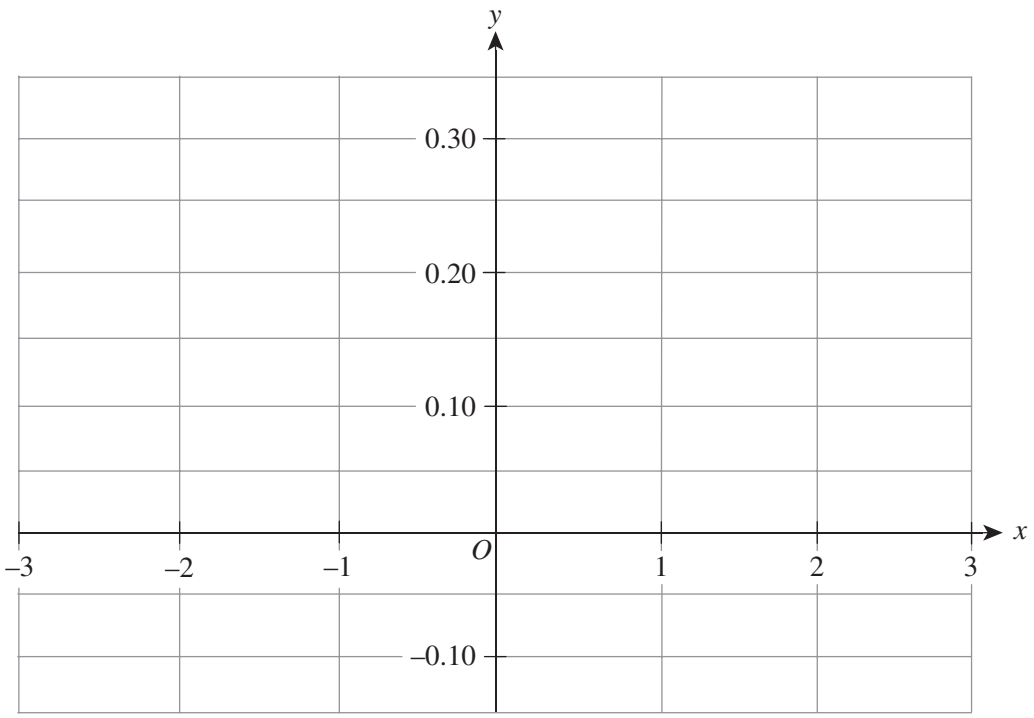
f. **i.** The function $h(x)$ is such that $h(x) = f(x) \times \frac{1}{g(x)}$.

Show that $h(x) = \frac{5}{42x} + \frac{1}{7}$. 2 marks

ii. Find $h(1)$, correct to three decimal places. 1 mark

iii. Solve $h(x) = 0$ for x. Give your answer correct to three decimal places. 1 mark

iv. Hence, sketch the graph of $h(x)$, showing all key features. Where appropriate, write coordinates correct to three decimal places. 3 marks



Question 2 (14 marks)

Of the Year 12 students at a particular school, 48% have their driver's licence, 35% have their own car, and 22% do not have their driver's licence and do not have their own car.

Let L represent the students that have their driver's licence and L' represent those that do not.

Let C represent the students that have their own car and C' represent those that do not.

a. i. Represent this information in the Karnaugh map (two-way table) below. 3 marks

	C	C'	
L			
L'			
			1

ii. Find the probability that a student who is randomly selected has a car and their driver's licence. 1 mark

iii. Find the probability of randomly selecting a student who owns a car given that they do not have their driver's licence. 2 marks

iv. Given that there are 267 Year 12 students, how many students do not have their licence and do not own a car? 1 mark

Of the Year 12 students who have their licence, 45% drive to and from school on a normal school day and 75% drive to and from school on a day where there are extra-curricular activities after school.

Let N represent a normal school day and N' represent a day when there are extra-curricular activities after school.

Let D represent students who drive to school and D' represent students who do not drive to school.

There are extra-curricular activities after school 35% of the time.

b. **i.** In the space below, draw a tree diagram to represent this information. 3 marks

ii. Find the probability that a randomly selected student who has a driver's licence drove to school on a particular day. Give your answer correct to two decimal places. 2 marks

iii. Given that a randomly selected student who has a driver's licence drove to school, find the probability that it was not a normal school day. Give your answer correct to two decimal places. 2 marks

Question 3 (17 marks)

Consider the function defined by $f : \mathbb{R} \to \mathbb{R}, f(x) = 6x^3 - 19x^2 + 10x$.

a. Find $f\left(\frac{1}{3}\right)$ and $f(3)$. 2 marks

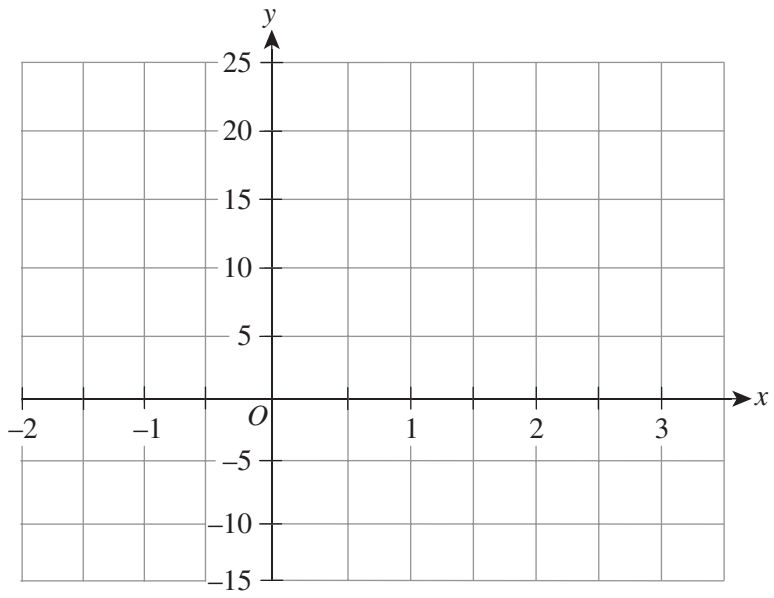
b. State the factors of $f(x)$. 1 mark

A restriction is placed on the domain such that $x \in \left[\frac{1}{3}, 3\right)$.

c. Find the minimum value of $f(x)$, correct to two decimal places. 1 mark

d. State the range of $f(x)$, where appropriate, correct to two decimal places. 1 mark

e. Sketch the graph of $f(x)$, showing the coordinates of all key points. 3 marks



f. State the x-values where $f(x) > 1$, correct to two decimal places. 3 marks

g. **i.** Find $f'(x)$. 1 mark

ii. Find $f'(-1)$. 1 mark

iii. Find the second derivative of $f(x)$. 1 mark

h. Find $\int_{2.6}^{2.8} f(x) \cdot dx$, correct to two decimal places. 1 mark

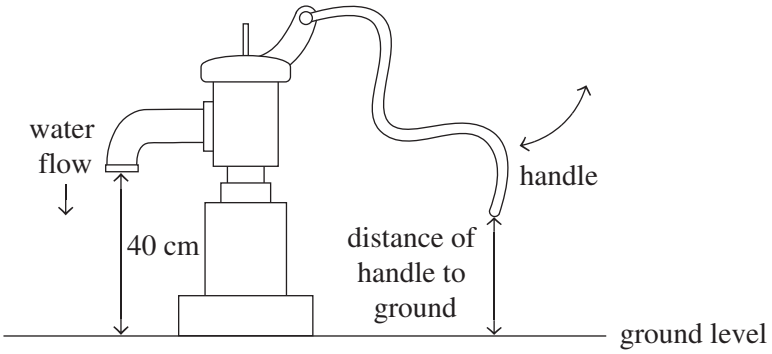
i. The function $f(x)$ is a component of the hybrid function $h(x)$.

$$h(x) = \begin{cases} x - 1, & -3 < x < \frac{1}{3} \\ 6x^3 - 19x^2 + 10x, & \frac{1}{3} \leq x < 3 \\ (x-3)^2 + 21, & x \geq 3 \end{cases}$$

State the value(s) of x for which $h(x)$ shows discontinuity. 2 marks

Question 4 (14 marks)

The diagram below shows a water well that pumps water from deep underground. The water well is controlled by a handle, which follows a curved motion up and down and follows the shape of a sine curve.



The handle is always left in its lowest position when not in use.

The movement and height, h, in cm, above ground of the handle can be modelled by an equation of the form

$$h(t) = a\sin\left(\frac{b\pi t}{c}\right) + d,$$

where t is the time, in seconds, from a neutral position.

The handle has its lowest position above the ground at 0.5 m. Its highest position is at 1.0 m above the ground. For the average person, performing one complete cycle of the movement of the handle takes 8.4 seconds.

a. **i.** State the values of a, b, c and d. 4 marks

ii. What is the domain of the function? 1 mark

iii. Hence, or otherwise, state the equation $h(t)$. 1 mark

b. State the central value of the function $h(t)$. 1 mark

c. Find the coordinates of the first maximum and minimum values of the vertical movement of the handle, correct to one decimal place where appropriate. 2 marks

d. Find the height of the handle when $t = 85$, correct to the nearest centimetre. 1 mark

For each complete cycle of the handle, $1\frac{3}{4}$ L of water is pumped.

e. An average person pumps the handle for 29.4 seconds, collecting the water pumped into a bucket.

How much water is collected in the bucket, correct to the nearest litre? 2 marks

f. Over time, rain, erosion and weathering washes away much of the ground soil and grass around the water pump, causing the ground level to decrease by 15 cm. As a result, the equation that models the movement of the handle has changed and is defined as $g(t)$.

i. What is the new equation of $g(t)$? 1 mark

ii. What is the new distance, in cm, of the spout above the ground? 1 mark

END OF QUESTION AND ANSWER BOOKLET