THE HEFFERNAN GROUP
P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
info@theheffernangroup.com.au
www.theheffernangroup.com.au

Student Name……………………………………

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

2023

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 9 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where a numerical answer is required, an exact value must be given unless otherwise directed.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on pages 12 and 13 of this exam.

Question 1 (3 marks)

a. Let $y = (x^2 + 3x + 2)^5$.

Find $\frac{dy}{dx}$. 1 mark

b. Let $h(x) = \frac{\tan(x)}{3x}$.

Evaluate $h'\left(\frac{\pi}{3}\right)$. 2 marks

Question 2 (3 marks)

a. Find $\int \frac{1}{3x+2}\,dx$, where $x > -\frac{2}{3}$. 1 mark

b. Find $g(x)$, given that $g\left(\frac{1}{4}\right) = 5$ and $g'(x) = \frac{1}{(2x-1)^2}$. 2 marks

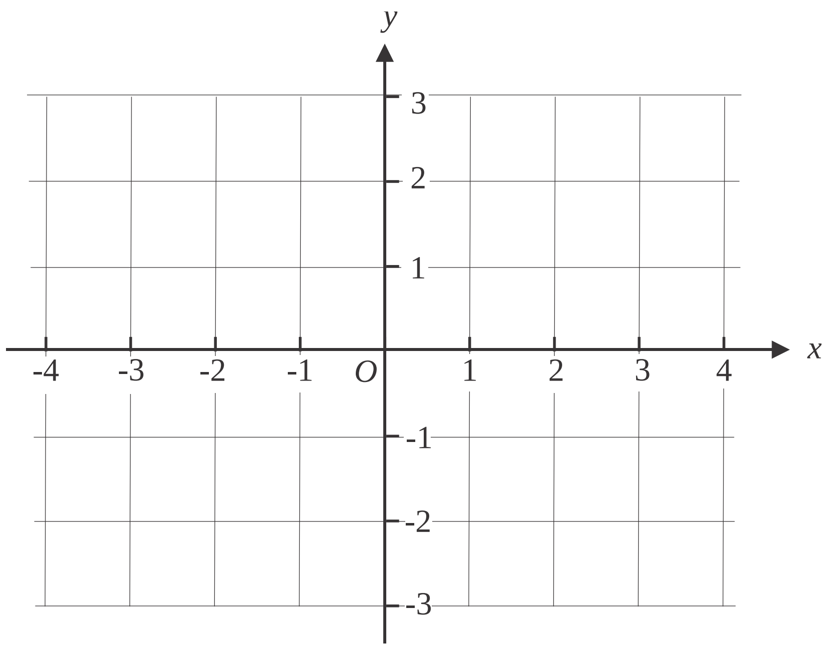
Question 3 (3 marks)

Solve $\sqrt{3} - \cos(2x) = \cos(2x)$ for $x \in R$.

Question 4 (5 marks)

Let $f: R\backslash\{-2\} \rightarrow R,\ f(x)=\dfrac{1}{x+2}-1$.

a. Sketch the graph of *f* on the axes below. Label any asymptotes with their equations, and axis intercepts with their coordinates. 3 marks



b. Find the rule and domain of the inverse function f^{-1}. 2 marks

Question 5 (3 marks)

Let the random variable X be normally distributed with mean 5.3 and standard deviation 0.2.
Let Z be the standard normal variable.

a. Find a such that $\Pr(X < 4.9) = \Pr(Z > a)$. 1 mark

b. Given that $\Pr(Z > 1) = k$, find $\Pr(X > 5.1 \mid X < 5.3)$ in terms of k. 2 marks

Question 6 (4 marks)

A random variable X has a probability density function f given by

$$f(x)=\begin{cases} k\sqrt{x} & 1\le x\le 4 \\ 0 & \text{elsewhere} \end{cases}$$

where k is a positive, real number.

a. Show that $k=\frac{3}{14}$. 2 marks

b. Find $\mathrm{E}(X)$. 2 marks

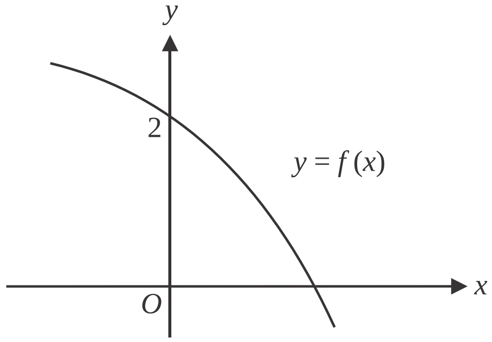
Question 7 (3 marks)

The point P lies on the straight line $y = \frac{1}{2}x + 3$.
The length of the line segment OP, from the origin O to P, is a minimum.

Find the coordinates of P.

Question 8 (7 marks)

Consider the function with rule $f(x) = 3 - e^{kx},\ k > 0$.
Part of the graph of *f* is shown below.



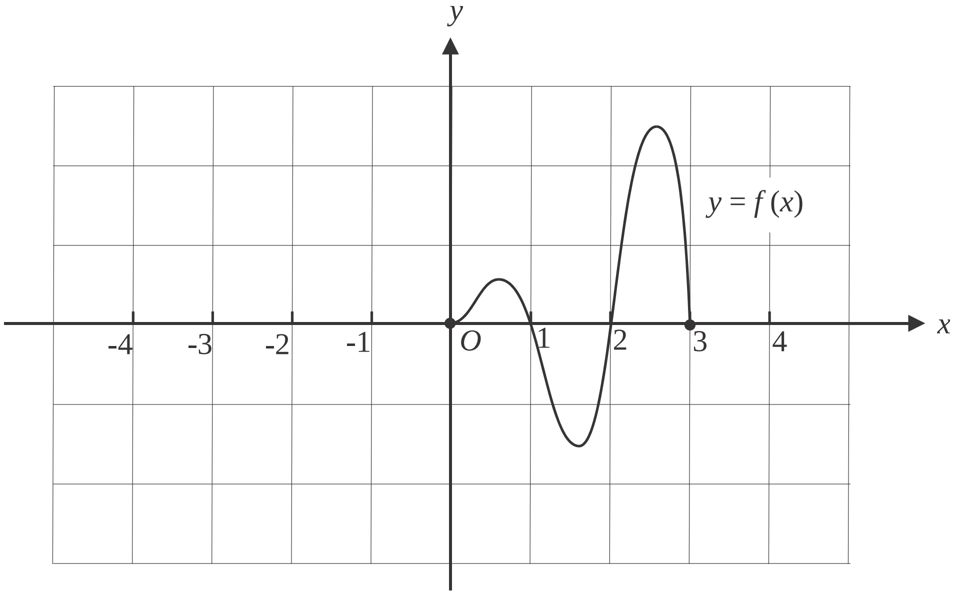
a. Show that the *x*-intercept of the graph of *f* is $\frac{1}{k}\log_e(3)$. 1 mark

b. Find the average value of *f* between $x = 0$ and $x = \frac{1}{k}\log_e(3)$. Express your answer in the form $a - \frac{b}{\log_e(a)}$, where *a* and *b* are positive integers. 4 marks

c. Find the values of k for which the average rate of change of f between $x = 0$ and $x = 1$ is negative. 2 marks

Question 9 (9 marks)

Let $f:[0,3]\to R,\ \ f(x)=x\sin(\pi x)$. The graph of *f* is shown below.



a. Find the gradient of *f* at the point where $x=\frac{1}{4}$. Express your answer in the form $\frac{\sqrt{a}(\pi+b)}{c}$ where *a*, *b* and *c* are positive integers. 2 marks

b. Given that $\frac{d}{dx}(x\cos(\pi x))=\cos(\pi x)-\pi x\sin(\pi x)$, find the area bounded by the graph of *f* and the *x*-axis over the interval $x\in[0,1]$. 2 marks

c. If $\int_1^2 f(x)\,dx = -3\int_1^2 f(x-1)\,dx$ and $\int_1^3 f(x)\,dx = 2\int_0^1 f(x)\,dx$, find the area enclosed by *f* and the *x*-axis. 3 marks

d. The graph of *f* is dilated by a factor of $\frac{1}{3}$ from the *y*-axis and then reflected in the *y*-axis to become the graph of *g*.

i. Find the domain of *g*. 1 mark

ii. Find the rule for *g*. 1 mark

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ \ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos\ (ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area\approx\frac{x_n-x_0}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+...+2f(x_{n-2})+2f(x_{n-1})+f(x_n)\right]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$