

Trial Examination 2023

VCE Mathematical Methods Units 1&2

Written Examination 1

Suggested Solutions

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Question 1 (2 marks)

$$\frac{8p^{-3}q^2}{2p^3q} \div \left(\frac{2p^2q^{-2}}{3p^4q}\right)^2 = \frac{4q}{p^6} \times \frac{9p^8q^2 \times q^4}{4p^4}$$

$$= \frac{9q^7}{p^2}$$
A1

Note: For M1, accept any equivalent.

Question 2 (3 marks) Method 1:

$$\log_{3}(2x+3) + \log_{3}(x-2) = 2$$

$$\log_{3}[(2x+3)(x-2)] = 2$$

$$3^{2} = (2x+3)(x-2)$$

$$9 = 2x^{2} - 4x + 3x - 6$$

$$2x^{2} - x - 15 = 0$$

$$(2x+5)(x-3) = 0$$

$$x = -\frac{5}{2} \text{ or } x = 3$$

A1

$$x = -\frac{3}{2}$$
 or $x = 3$

Method 2:

$$\log_{3}[(2x+3)(x-2)] = \log_{3}(3)^{2}$$

$$3^{2} = (2x+3)(x-2)$$

$$9 = 2x^{2} - 4x + 3x - 6$$
M1
$$2x^{2} - x - 15 = 0$$

$$(2x+5)(x-3) = 0$$

$$x = -\frac{5}{2}$$
 or $x = 3$ A1

For
$$\log_a(b)$$
 to exist, $b > 0$. Therefore, $x = -\frac{5}{2}$ cannot be a solution.
 $x = 3$ A1

Question 3 (2 marks)

Consider
$$\left(\frac{1}{3}\right)^x = 27.$$

 $\left(\frac{1}{3}\right)^x = 3^3$
 $3^{-x} = 3^3$
 $-x = 3$
 $x = -3$
 $\{x: x \in (-\infty, -3]\}$ OR $x \le -3$
M1

Question 4 (3 marks)

The general formula for turning point form is $y = a(x - h)^2 + k$, where (h, k) is the turning point. The turning point (-2, 6) is given in the graph. Therefore:

$$y = a(x+2)^2 + 6$$
 M1

Substituting in the other point in the graph to solve for *a* gives:

$$3 = a(1+2)^{2} + 6$$

$$a(3)^{2} = -3$$

$$9a = -3$$

$$a = -\frac{1}{3}$$

$$y = -\frac{1}{3}(x+2)^{2} + 6$$
A1

a. i.
$$\frac{\pi}{12}$$
: x

π: 180

Using cross-multiplication gives:

$$\pi x = \frac{\pi \times 180}{12}$$
$$x = \frac{30}{2}$$
$$= 15^{\circ}$$
A1

235: *x*

Using cross-multiplication gives:

$$180x = 235\pi$$
$$x = \frac{235\pi}{180}$$
$$= \frac{47\pi}{36}$$
A1

b.

i.
$$\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \mathbf{OR} \frac{\sqrt{2}}{2}$$
 A1

$$\operatorname{ii.} \quad \tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}} \operatorname{OR} - \frac{\sqrt{3}}{3}$$
 A1

- c. i. Given that $\tan(\theta) = \frac{4}{3}$ is in the third quadrant: $x = \frac{1}{3}$ Using Pythagoras' theorem gives: $a^2 + b^2 = c^2$ $3^2 + 4^2 = x^2$ x = 5Therefore, $\sin(\theta) = -\frac{4}{5}$.
 - 5 $\frac{1}{\cos(\theta)} = \frac{1}{\left(-\frac{3}{5}\right)}$ $= 1 \times \left(-\frac{5}{3}\right)$ $= -\frac{5}{3}$
 - ii. Using the answer from **part c.i.**, in the third quadrant:

A1

A1

Note: Consequential on answer to Question 5c.i.

Question 6 (4 marks)

a.
$$\cos(2\pi - x) - \sin\left(\frac{3\pi}{2} - x\right) = \cos(2x - x)$$
$$= \cos(-x)$$
$$= -\cos(x)$$
M1

Using complementary angles gives:

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos(x)$$

Hence, $\cos(2\pi - x) - \sin\left(\frac{3\pi}{2} - x\right) = -\cos(x) - (-\cos(x))$
$$= 0$$
A1

b.
$$\cos(2 \propto) = -\frac{\sqrt{3}}{2}$$

 $2 \propto = \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right)$
 $2 \propto = -\frac{5\pi}{6} \text{ and } \frac{5\pi}{6}$
 $\alpha = -\frac{7\pi}{12}, -\frac{5\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$
A1

Question 7 (3 marks)

When
$$x = -1$$
:
 $y = 2(-1)^3 - 4(-1)^2 + (-1) - 1$
 $= -8$
Therefore, a coordinate is $(-1, -8)$. A1
 $\frac{dy}{dx} = 6x^2 - 8x + 1$
When $x = -1$:
 $\frac{dy}{dx} = 6(-1)^2 - 8(-1) + 1$

$$=15$$
 M1

Therefore, 15 is the gradient of the tangent.

Hence, the equation of the tangent is:

$$y - y_1 = m(x - x_1)$$

$$y - (-8) = 15(x - (-1))$$

$$y + 8 = 15x + 15$$

$$y = 15x + 7$$

A1

Question 8 (3 marks)

$$\int \left(4x + \frac{3}{x^4}\right) dx = \int \left(4x + 3x^{-4}\right) dx$$
$$= \frac{4x^2}{2} + \frac{3x^{-3}}{3} + c$$
$$= 2x^2 + x^{-3} + c$$
M1

Given that y = f(x) passes through the point (-1, 3): $y = 2x^2 + x^{-3} + c$

$$y = 2x^{2} + x^{-3} + c$$

$$3 = 2(-1)^{2} + \left(\frac{1}{-1}\right)^{3} + c$$

$$c = 3 - 2 - 1$$

$$= 0$$

Hence:

$$f(x) = 2x^2 + x^{-3}$$
 A1

Question 9 (7 marks)

b.

a.
$$h(x) = a(x+2)(x-1)(x-3)$$

Given that $h(0) = -6$:
 $-6 = a(0+2)(0-1)(0-3)$
 $-6 = a(2)(-1)(-3)$
 $-6 = 6a$
 $a = -1$
 $h(x) = -1(x+2)(x^2 - 4x + 3)$
 $= -(x^3 - 4x^2 + 3x + 2x^2 - 8x + 6)$
 $= -(x^3 - 2x^2 - 5x + 6)$
 $= -x^3 + 2x^2 + 5x - 6$
A1
b. i. $h'(x) = -3x^2 + 4x + 5$

A1

ii.
$$h'(x) = 0$$
 needs to be solved.

i.
$$h'(x) = 0$$
 needs to be solved.
 $h'(x) = -3x^2 + 4x + 5$
 $-3x^2 + 4x + 5 = 0$

Using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-3)(5)}}{2(-3)}$$

$$= \frac{-4 \pm \sqrt{16 + 60}}{-6}$$

$$= \frac{-4 \pm \sqrt{76}}{-6}$$

$$= \frac{4 \pm 2\sqrt{19}}{6}$$

$$= \frac{2 \pm \sqrt{19}}{3}$$
x-intercepts = $\left(\frac{2 - \sqrt{19}}{3}, 0\right)$ and $\left(\frac{2 + \sqrt{19}}{3}, 0\right)$ A1

Note: Consequential on answer to Question 9b.i.



correct shape A1 correct x- and y-intercepts A1 Note: Consequential on answers to **Questions 9b.i.** and **9b.ii**.

Question 10 (7 marks)

iii.

a. i.
$$Pr(B) = 0.7$$
 A1
ii. $Pr(A \cap B) = 0.5$ A1

$$\textbf{iii.} \quad \Pr(A \cup B)' = 0.2 \tag{A1}$$

iv.
$$Pr(A' | B') = \frac{Pr(A' \cap B')}{Pr(B')}$$
$$= \frac{0.2}{0.3}$$
$$= \frac{2}{3}$$
A1





1 mark per two correct data entries.

ii.Events A and B are not mutually exclusive. The Venn diagram has an intersection,
and mutually exclusive events do not have an intersection.A1