

VCE Mathematical Methods Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

Question 1 E

E is correct.

Method 1:

$$\begin{aligned} 1 + 3\log_7(2) &= \log_7(7) + \log_7(2)^3 \\ &= \log_7(7 \times 2^3) \\ &= \log_7(56) \end{aligned}$$

A is incorrect. This option ignores the 1.

B is incorrect. This option uses $\log_7(7 + 2^3)$.

C is incorrect. This option adds the 1 to the 3.

D is incorrect. This option evaluates $\log_7(2)^3$ as $\log_7(6)$.

Method 2:

Using a CAS calculator gives:

$1+3 \cdot \log_7(2)$	2.0686216
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Testing option **E** gives:

$\log_7(56)$	2.0686216
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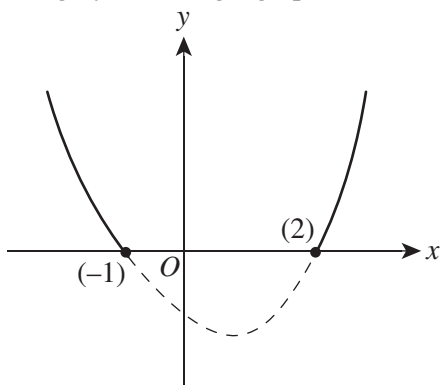
Question 2 B**Method 1:**

Solving when the discriminant equals zero gives:

$$\begin{aligned} \Delta &= b^2 - 4ac = 0 \\ (2k)^2 - 4(1)(k+2) &= 0 \\ 4k^2 - 4k - 8 &= 0 \\ k^2 - k - 2 &= 0 \\ (k-2)(k+1) &= 0 \end{aligned}$$

$$k = -1 \text{ or } k = 2$$

Roughly sketching a graph of k with the discriminant gives:



From the graph, the required values for k are when $\Delta > 0$. Therefore, $k < -1$ and $k > 2$.

Method 2:

Using a CAS calculator gives:

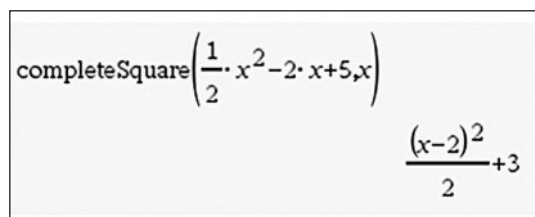
$\text{solve}((2 \cdot k)^2 - 4 \cdot 1 \cdot (k+2) > 0, k)$	$k < -1 \text{ or } k > 2$
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Question 3 A**Method 1:**

$$\begin{aligned}
 y &= \frac{1}{2}x^2 - 2x + 5 \\
 &= \frac{1}{2}(x^2 - 4x + 10) \\
 &= \frac{1}{2}(x^2 - 4x + 4 - 4 + 10) \\
 &= \frac{1}{2}[(x - 2)^2 + 6] \\
 &= \frac{1}{2}(x - 2)^2 + 3
 \end{aligned}$$

Method 2:

Using a CAS calculator gives:



completeSquare($\frac{1}{2} \cdot x^2 - 2 \cdot x + 5, x$)

$$\frac{(x-2)^2}{2} + 3$$

Question 4 D

D is correct. A solid circle indicates that the number is included in the interval and an open circle indicates that the number is not included in the interval.

A is incorrect. This option is the interval that is not included.

B is incorrect. This option has incorrect interval notation and uses an intersection symbol rather than a union symbol.

C is incorrect. This option has the incorrect interval notation for $-\infty$.

E is incorrect. This option has incorrect interval notation.

Question 5 C

The translation of 2 units in the negative direction of the y -axis moves the graph down 2 units (-2). The translation of 1 unit in the negative direction of the x -axis is represented by the opposite symbol in the equation ($+1$). Hence, **C** is correct.

Question 6 D

D is correct.

Method 1:

Using $\sin^2(x) + \cos^2(x) = 1$ gives:

$$0.725^2 + \cos^2(x) = 1$$

$$\cos(x) = 0.689$$

Method 2:

The question is presented in degrees, therefore, using a CAS calculator gives:

```
solve((0.725)^2+z^2=1,z)
z=-0.68874887 or z=0.68874887
```

A is incorrect. This option uses an incorrect identity relationship ($\sin(x) + \cos(x) = 1$).

B is incorrect. This option gives $\cos^2(x)$.

C is incorrect. This option gives $\sin^2(x)$.

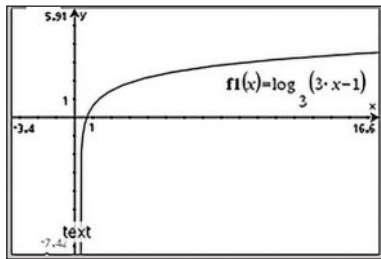
E is incorrect. This option gives $\sin^{-1}(0.725)$.

Question 7 B

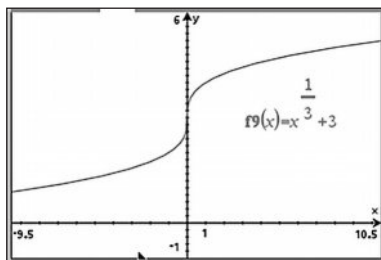
Using symmetry properties of trigonometry, $\cos(\pi + \theta)$, which is in the third quadrant, is equivalent to the positive value of a in the fourth quadrant.

Question 8 B

For the function to exist, $3x - 1 > 0$. Therefore, there is an asymptote at $x = \frac{1}{3}$. Alternatively, this can be found using a CAS calculator.

**Question 9 C**

Using a CAS calculator gives:

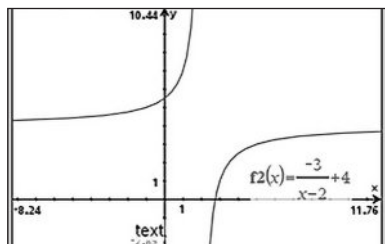


A point of inflection can be observed at $(0, \frac{1}{3})$.

Question 10 E

E is correct. The graph has asymptotes at $x = 2$ and $y = 4$.

Using a CAS calculator gives:



A is incorrect. This option does not reflect the graph in the y -axis.

B and C are incorrect. These options have asymptotes at $x = 4$ and $y = 2$.

D is incorrect. This option has the vertical asymptote at $x = -2$.

Question 11 C

C is correct. The period of a $y = a \tan(n\theta)$ is given by the formula $P = \frac{\pi}{n}$. In this instance, $n = \frac{1}{3}$.

Therefore, $P = \frac{\pi}{\frac{1}{3}} = 3\pi$. The value for c is half of this; hence, $c = \frac{3\pi}{2}$.

A and B are incorrect. These options use incorrect calculations for the period.

D is incorrect. This option is the period of the graph. Alternatively, in this option, the period is calculated using the formula for sine or cosine and then dividing the result by two, which is incorrect.

E is incorrect. This option states the period using the formula for the period of sine or cosine.

Question 12 C

C is correct.

Using a CAS calculator gives:



A is incorrect. This option gives the answer in the incorrect quadrant.

B is incorrect. This option calculates the answer in degrees, not radians.

D is incorrect. This option uses $\tan\left(\frac{\pi}{8}\right)$ rather than $\tan\left(\frac{\pi}{24}\right)$.

E is incorrect. This option uses $\tan\left(\frac{3\pi}{8}\right)$ rather than $\tan\left(\frac{\pi}{24}\right)$.

Question 13 B

B is correct. $f'(3)$ can be found using a CAS calculator.

$$\frac{d}{dx}(2 \cdot x^2 - 3 \cdot x^{-3}) \quad \frac{1}{\sqrt{x}} + \frac{9}{x^4}$$

$$\frac{1}{\sqrt{3}} + \frac{9}{3^4} \quad 0.68846138$$

A is incorrect. This option does not round the answer to four decimal places correctly.

C is incorrect. This option enters the data into the calculator incorrectly.

$$\frac{1}{3^2} + \frac{9}{3^4} \quad 1.8431619$$

D and **E** are incorrect. These options are for $f(3)$ instead of $f'(3)$.

Question 14 B

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dh} = \frac{\pi r^2}{3}$$

Given that $h = 2r$, $r = \frac{h}{2}$. Substituting this value into the equation above gives:

$$\frac{dV}{dh} = \frac{\pi \left(\frac{h}{2}\right)^2}{3} = \frac{\pi h^2}{12}$$

Finding $\frac{dh}{dt}$ when $h = 10$ cm gives:

$$\begin{aligned} \frac{dh}{dt} &= \frac{dh}{dV} \times \frac{dV}{dt} \\ &= \frac{12}{\pi h^2} \times 15 \\ &= \frac{12}{\pi 10^2} \times 15 \\ &= \frac{180}{100\pi} \\ &= \frac{9}{5\pi} \end{aligned}$$

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Alternatively:

Given that $h = 2r$, $r = \frac{h}{2}$. Additionally, $V = \frac{1}{3}\pi r^2 h$. Depending on which variable is to be derived, the volume must be written in terms of that variable.

$\frac{dh}{dt}$ must be found. Using the chain rule, $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$.

$\frac{dV}{dh}$ is needed, so the volume must be found in terms of height with no radius:

$$\begin{aligned} V &= \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h \\ &= \frac{1}{12}\pi h^3 \end{aligned}$$

$$\frac{dV}{dh} = \frac{1}{4}\pi h^2$$

$$\frac{dh}{dV} = \frac{4}{\pi h^2}$$

Using $\frac{dV}{dt} = 15$ and $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ gives:

$$\begin{aligned} \frac{dh}{dt} &= \frac{4}{\pi h^2} \times 15 \\ &= \frac{60}{\pi h^2} \end{aligned}$$

When $h = 10$:

$$\begin{aligned} \frac{dh}{dt} &= \frac{60}{\pi \times 10^2} \\ &= \frac{3}{5\pi} \end{aligned}$$

Question 15 E

When $t = 0$, $T = 96^\circ\text{C}$. Substituting this into the given formula and solving for A gives:

$$T = Ae^{-kt} + 10$$

$$96 = Ae^0 + 10$$

$$A = 86^\circ$$

Alternatively, using a CAS calculator gives:

$\text{solve}(96=a \cdot e^0 + 10, a)$	$a=86.$
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When $t = 2.5$ hours, $T = 49^\circ\text{C}$. Substituting this into the given formula and solving for k gives:

$$T = 86e^{-kt} + 10$$

$$49 = 86e^{-k \times 2.5} + 10$$

$$k = 0.32$$

Alternatively, using a CAS calculator gives:

$\text{solve}(49=86 \cdot e^{-k \cdot 2.5} + 10, k)$	$k=0.31631426$
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Question 16 D

D is correct. For events A and B to be independent, $\Pr(A) \times \Pr(B) = \Pr(A \cap B)$. Hence:

$$\Pr(A) \times \frac{1}{4} = \frac{1}{5}$$

$$\Pr(A) = \frac{4}{5}$$

$$= \frac{16}{20}$$

A, **B**, **C** and **E** are incorrect. These options incorrectly calculate $\Pr(A)$ with respect to independence.

Question 17 D

D is correct. For both L and M to be mutually exclusive events, then $\Pr(L \cap M) = 0$. Therefore:

$$\Pr(L \cup M)' = 1 - \Pr(L) - \Pr(M)$$

$$= 1 - 0.25 - 0.45$$

$$= 0.30$$

A and **B** are incorrect. These options treat L and M as independent events. Option **A** gives $\Pr(L \cap M)$. Option **B** gives $1 - \Pr(L \cap M)$.

C is incorrect. This option gives $\Pr(L)$.

E is incorrect. This option gives $\Pr(M)$.

Question 18 C

C is correct.

$$\Pr(\text{black or yellow}) = \frac{1}{18} + \frac{2}{18} = \frac{1}{18} + \frac{1}{9}$$

A is incorrect. This option gives $\Pr(\text{black and yellow})$.

B is incorrect. This option is not additive for the two outcomes.

D is incorrect. This option uses the probability for blue rather than black.

E is incorrect. This option gives $\Pr(\text{black or yellow})'$.

Question 19 C

C is correct. The required calculation is ${}^8C_3 \times {}^7C_3$.

Using a CAS calculator gives:

$nCr(8,3)$	56.
$nCr(7,3)$	35.
$56 \cdot 35$	1960.

A is incorrect. This option gives ${}^8C_3 + {}^7C_3$.

B and E are incorrect. These options calculate permutations instead of combinations.

D is incorrect. This option calculates factorials instead of combinations.

Question 20 B

The required permutation calculation is $18! \times 3!$.

SECTION B**Question 1** (13 marks)

- a. Substituting $P(-3, c)$ into $3x - 2y - 5 = 0$ gives:

$$3(-3) - 2c - 5 = 0$$

$$-2c = 14$$

$$c = -7$$

A1

Using a CAS calculator gives:

$\text{solve}(3 \cdot -3 - 2 \cdot c - 5 = 0, c)$	$c = -7$
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- b. i. Rearranging to make y the subject gives:

$$3x - 2y - 5 = 0$$

$$2y = 3x - 5$$

$$y = \frac{3}{2}x - \frac{5}{2} \text{ OR } \frac{3x - 5}{2}$$

A1

Using a CAS calculator gives:

$\text{solve}(3 \cdot x - 2 \cdot y - 5 = 0, y)$	$y = \frac{3 \cdot x - 5}{2}$
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- ii. The gradient of $y = \frac{3}{2}x - \frac{5}{2}$ is $\frac{3}{2}$.

The gradient of the perpendicular line is $-\frac{2}{3}$.

M1

Substituting the gradient and the point $(3, 2)$ into $y - y_1 = m(x - x_1)$ gives:

$$y - 2 = -\frac{2}{3}(x - 3)$$

$$y - 2 = -\frac{2}{3}x + 2$$

$$y = -\frac{2}{3}x + 4 \text{ OR equivalent}$$

A1

Using a CAS calculator gives:

$\text{solve}\left(y - 2 = \frac{-2}{3} \cdot (x - 3), y\right)$	$y = 4 - \frac{2 \cdot x}{3}$
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Note: Consequential on answer to Question 1b.i.

c. i. $y = -3(x-3)^2 + 4$
 $= -3(x^2 - 6x + 9) + 4$ M1
 $= -3x^2 + 18x - 27 + 4$
 $= -3x^2 + 18x - 23$ A1

Using a CAS calculator gives:

$$\text{expand}(-3 \cdot (x-3)^2 + 4) \quad -3 \cdot x^2 + 18 \cdot x - 23.$$

ii. The axis of symmetry for a parabola in the form $y = ax^2 + bx + c$ is given by $x = -\frac{b}{2a}$.

Therefore:

$$x = -\frac{b}{2a}$$

$$x = -\frac{18}{2(-3)}$$

$$x = 3$$

A1

Alternatively, using the turning point (3, 4), the axis of symmetry is at $x = 3$.

Note: Consequential on answer to Question 1c.i.

d. Solving simultaneous equations using a CAS calculator gives:

$$\text{solve}\left(\begin{cases} 3 \cdot x - 2 \cdot y - 5 = 0 \\ y = -3 \cdot (x-3)^2 + 4 \end{cases}, \{x, y\}\right)$$

$x = 1.8960874$ and $y = 0.34413115$ or $x = 3.6039126$

$$\text{solve}\left(\begin{cases} 3 \cdot x - 2 \cdot y - 5 = 0 \\ y = -3 \cdot (x-3)^2 + 4 \end{cases}, \{x, y\}\right)$$

$x = 1.8960874$ and $y = 0.34413115$ or $x = 3.6039126$ and $y = 2.9058688$

Hence, the points are (1.90, 0.34) and (3.60, 2.91).

correct first point A1
correct second point A1

- e. i. Given that $(x + 2)$ is a factor of $P(x)$:

$$P(-2) = -2(-2)^3 - 4(-2)^2 - d(-2) + 3 = 1$$

M1

$$16 - 16 + 2d + 3 = 1$$

$$2d = -2$$

$$d = -1$$

A1

Using a CAS calculator gives:

$\text{solve}(-2 \cdot (-2)^3 - 4 \cdot (-2)^2 - d \cdot -2 + 3 = 1, d)$ $d = -1.$
--

- ii. Using the answer to **part e.i.** gives:

$$P(-b) = -2(-b)^3 - 4(-b)^2 - b + 3$$

M1

$$= 2b^3 - 4b^2 - b + 3$$

A1

Using a CAS calculator gives:

$-2 \cdot (-b)^3 - 4 \cdot (-b)^2 + b + 3$ $2 \cdot b^3 - 4 \cdot b^2 + b + 3.$

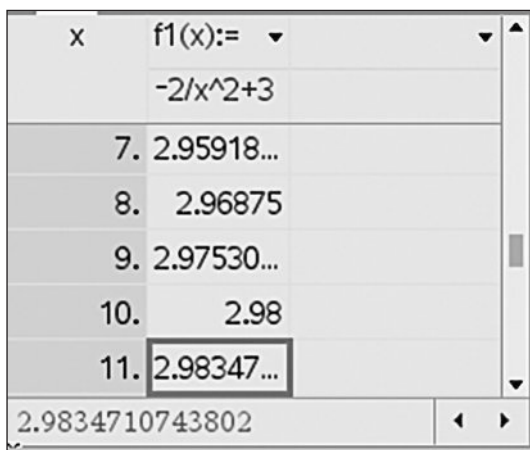
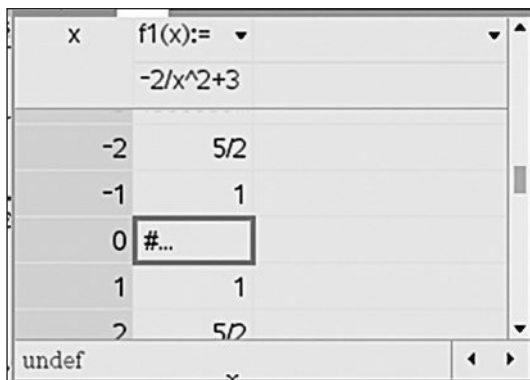
Note: Consequential on answer to Question 1e.i.

Question 2 (13 marks)

- a. i. Given that $\left\{f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = -\frac{2}{x^2} + 3\right\}$, the asymptotes are at $x = 0$ and $y = 3$.

correct vertical asymptote A1
correct horizontal asymptote A1

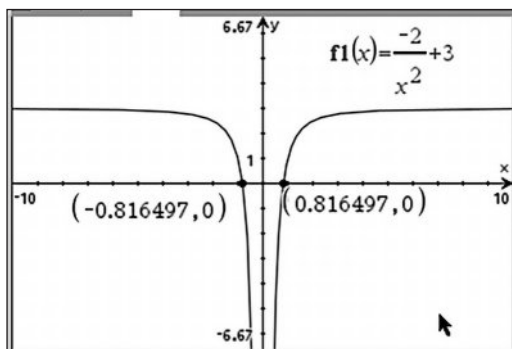
Using a CAS calculator gives:



When $x = 0$, $f(x)$ is undefined. Therefore, there is an asymptote at $x = 0$.

Scrolling through the y -values, it can be seen that y approaches 3, but never equals 3. Hence, there is an asymptote at $y = 3$.

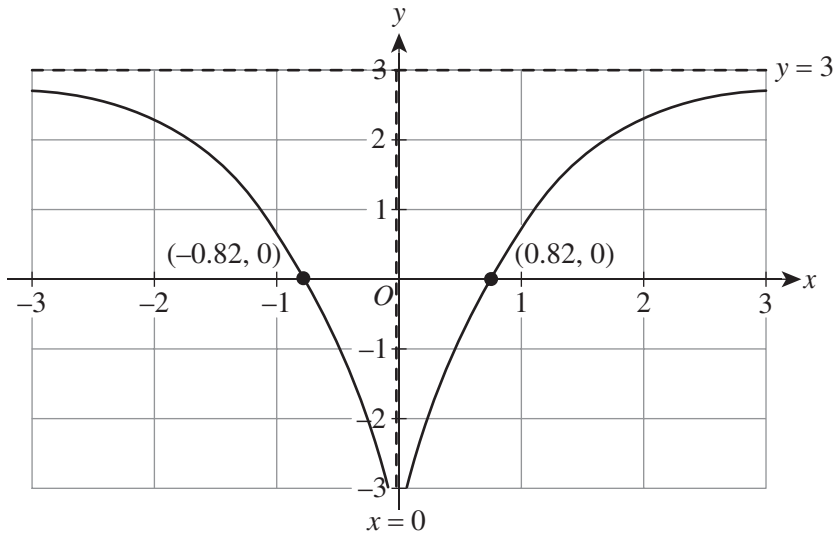
- ii. The x -intercepts can be found using the trace or zero feature of a CAS calculator.



Hence, the x -intercepts are $(-0.82, 0)$ and $(0.82, 0)$.

A1

iii.



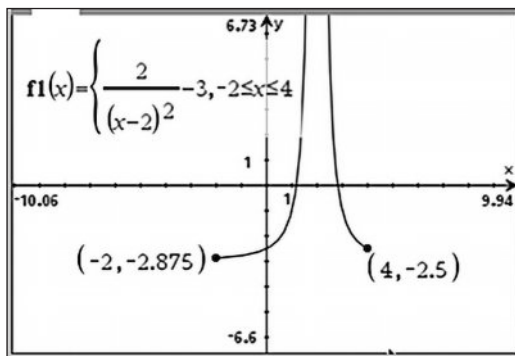
correct shape A1
correct asymptotes and x-intercepts (with x-intercepts given to two decimal places) A1
Note: Consequential on answers to Questions 2a.i. and 2a.ii.

b. i. Reflection in the x -axis gives $g_1(x) = \frac{2}{x^2} - 3$. A1

Translation of g_1 2 units in the positive direction of the x -axis gives $g(x) = \frac{2}{(x-2)^2} - 3$. A1

Therefore, $\left\{ g: [-2, 4] \rightarrow \mathbb{R}, g(x) = \frac{2}{(x-2)^2} - 3 \right\}$. A1

ii. Sketching $g(x)$ with the restricted domain on a CAS calculator gives the range of the function.



Converting -2.875 to an exact value gives:

-2.875	$-\frac{23}{8}$
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Hence, the range is $\left[-\frac{23}{8}, \infty \right)$.

correct values A1
correct brackets A1
Note: Consequential on answer to Question 2b.i.

- c. Consider $y = \frac{2}{(x-2)^2} - 3$. Swapping the x - and y -values and rearranging for y gives:

$$x = \frac{2}{(y-2)^2} - 3$$

M1

$$x + 3 = \frac{2}{(y-2)^2}$$

$$(y-2)^2 = \frac{2}{(x+3)}$$

$$y - 2 = \left(\frac{2}{x+3} \right)^{\frac{1}{2}}$$

$$y = \left(\frac{2}{x+3} \right)^{\frac{1}{2}} + 2$$

$$h(x) = \left(\frac{2}{x+3} \right)^{\frac{1}{2}} + 2$$

$$\text{domain: } \left[-\frac{23}{8}, \infty \right)$$

A1

$$\text{Hence, } \left\{ h: \left[-\frac{23}{8}, \infty \right) \rightarrow \mathbb{R}, h(x) = \left(\frac{2}{x+3} \right)^{\frac{1}{2}} + 2 \right\}.$$

A1

*Note: Consequential on answers to **Questions 2b.i.** and **2b.ii.***

Question 3 (19 marks)

a. $f'(x) = 12x^2 + 6x - 1$

A1

Using a CAS calculator gives:

$$\frac{d}{dx}(4 \cdot x^3 + 3 \cdot x^2 - x - 2) \quad 12 \cdot x^2 + 6 \cdot x - 1$$

b. i. $g(x) = \int f(x) dx$
 $= \int (4x^3 + 3x^2 - x - 2) dx$
 $= x^4 + x^3 - \frac{x^2}{2} - 2x + c$

M1

Using a CAS calculator gives:

$$\int (4 \cdot x^3 + 3 \cdot x^2 - x - 2) dx \quad x^4 + x^3 - \frac{x^2}{2} - 2 \cdot x$$

$$\int f(-1) dx = 2 \text{ indicates that } g(-1) = 2.$$

Using a CAS calculator gives:

$$\text{solve}\left(1 - 1 - \frac{1}{2} + 2 + c = 2, c\right) \quad c = \frac{1}{2}$$

$$c = \frac{1}{2}$$

A1

Hence, $g(x) = x^4 + x^3 - \frac{x^2}{2} - 2x + \frac{1}{2}$.

A1

ii. $\int_1^2 f(x) dx = \left[x^4 + x^3 - \frac{x^2}{2} - 2x \right]_1^2$ **OR** equivalent

$$= \left(2^4 + 2^3 - \frac{2^2}{2} - 2(2) \right) - \left(1^4 + 1^3 - \frac{1^2}{2} - 2(1) \right)$$

$$= 16 + 8 - 2 - 4 + \frac{1}{2}$$

$$= 18\frac{1}{2} \text{ or } \frac{37}{2}$$

M1

A1

Using a CAS calculator gives:

Define $f(x) = 4 \cdot x^3 + 3 \cdot x^2 - x - 2$	Done
$\int_1^2 f(x) dx$	$\frac{37}{2}$

Note: Consequential on answer to **Question 3b.i.**

- c. i. Using a CAS calculator gives:

$$\text{Define } g(x) = x^4 + x^3 - \frac{x^2}{2} - 2 \cdot x + \frac{1}{2} \quad \text{Done}$$

$$\text{solve}(g(x)=0,x) \quad x=0.24415112 \text{ or } x=1.$$

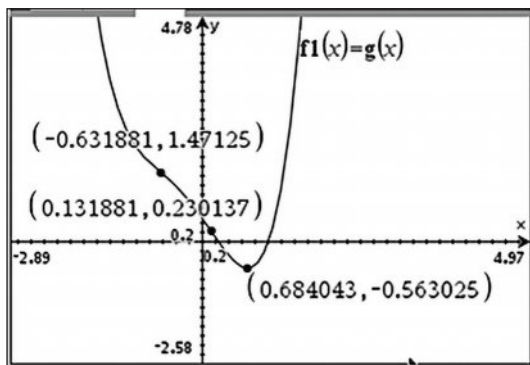
Hence, the x -intercepts are $(0.244, 0)$ and $(1, 0)$.

correct first intercept A1

correct second intercept A1

*Note: Consequential on answer to **Question 3b.i.***

- ii. Sketching the graph on a CAS calculator gives:



Therefore:

- point of inflection at $(-0.632, 1.471)$
- point of inflection at $(0.132, 0.230)$
- minimum at $(0.684, -0.563)$

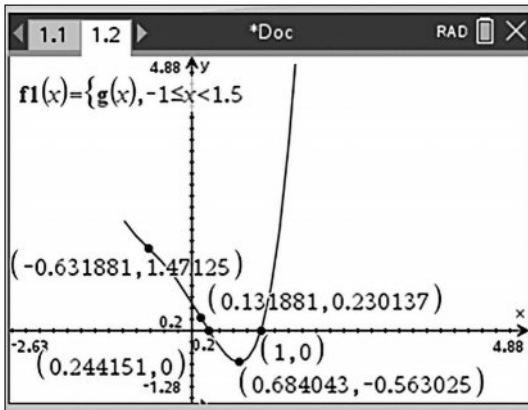
correct coordinates A3

Note: Award 1 mark for each stationary point.

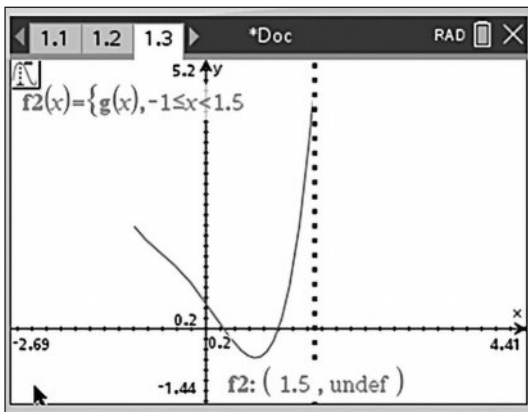
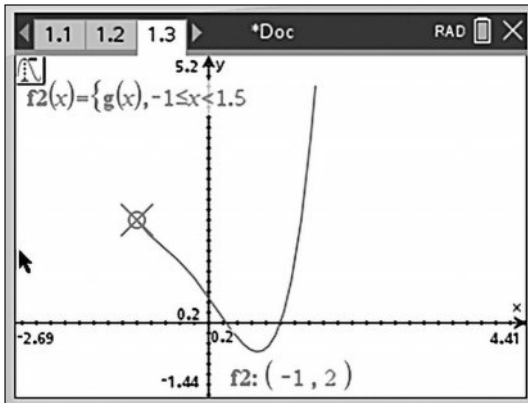
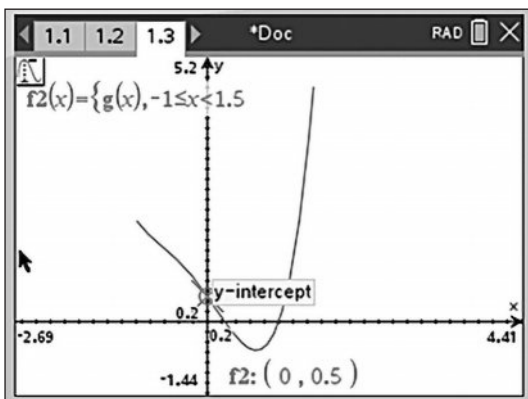
correct natures A1

Note: Award 1 mark for all three natures.

iii. Using a CAS calculator to show the x -intercepts and stationary points gives:

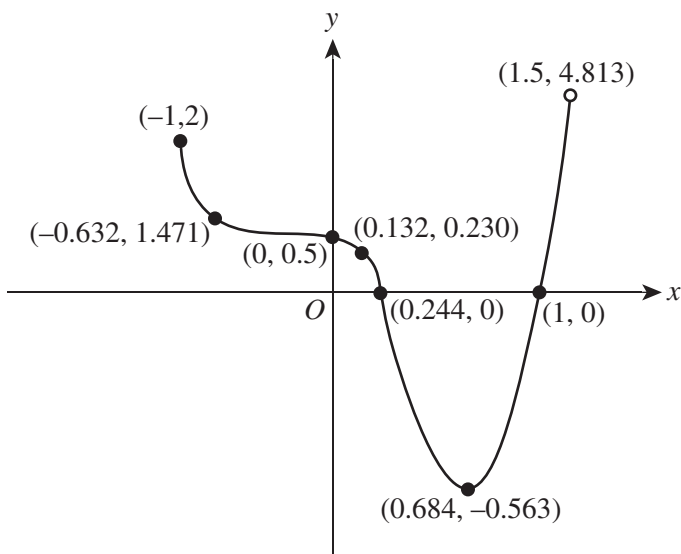


Using the trace function to show the end points and y -intercept gives:

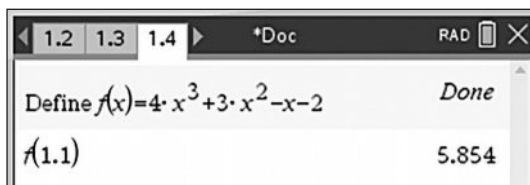


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*correct shape A1**correct x- and y-intercepts A1**correct stationary points A1**correct endpoints with $(-1, 2)$ as a filled-in circle and $(1.5, 4.813)$ as an open circle A1**Note: Consequential on answers to **Questions 3c.i. and 3c.ii.***

- iv. $x \in [-1, -0.632) \cup (-0.632, 0.648)$ A2
- v. Using a CAS calculator, the instantaneous rate of change is the derivative of $g(1.1)$, which is equivalent to $f'(1.1)$.



Hence, the instantaneous rate of change is 5.85.

A1

Question 4 (15 marks)

- a. i.** Pr(sum of the values of the uppermost faces is 6)

A table can be drawn to show the sum for each combination.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Hence, Pr(sum of the values of the uppermost faces is 6) = $\frac{5}{36}$.

A1

- ii.** Pr(sum of the values of the uppermost faces is less than 6)

A table can be drawn to show the sum for each combination.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Hence, Pr(sum of the values of the uppermost faces is less than 6) = $\frac{10}{36} = \frac{5}{18}$.

A1

- b. i.** $6 \times 6 \times 6 = 216$

A1

- ii.** $6 \times 5 \times 4 = 120$

A1

- iii.** Pr(three even numbers) = $\frac{3 \times 3 \times 3}{6 \times 6 \times 6}$
 $= \frac{1}{8}$

A1

- iv.** Pr(three even numbers where no number is repeated) = $\frac{3 \times 2 \times 1}{6 \times 6 \times 6}$
 $= \frac{1}{36}$

A1

$$\begin{aligned} \text{v. } \Pr(2, 3 \text{ and } 6 \text{ in order}) &= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{216} \end{aligned}$$

A1

$$\begin{aligned} \text{vi. } \Pr(2, 3 \text{ and } 6 \text{ in any order}) &= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times 3! \\ &= \frac{1}{36} \end{aligned}$$

A1

Using a CAS calculator gives:

$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 3!$	$\frac{1}{36}$
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$$\text{c. i. } {}^6C_1 \times {}^2C_1 = 12$$

A1

$nCr(6,1)$	6
$nCr(2,1)$	2
$6 \cdot 2$	12

$$\text{ii. } 3 \times 1 = 3$$

A1

$$\begin{aligned} \text{iii. } \Pr(\text{even number and tails}) &= \frac{3 \times 1}{6 \times 2} \\ &= \frac{1}{4} \end{aligned}$$

A1

- d. The probabilities can be calculated using the table provided in the question.

Number on the die	1	2	3	4	5	6
How many times obtained?	20	10	35	40	30	65
Pr(x)	$\frac{20}{200} = \frac{1}{10}$	$\frac{10}{200} = \frac{1}{20}$	$\frac{35}{200} = \frac{7}{40}$	$\frac{40}{200} = \frac{1}{5}$	$\frac{30}{200} = \frac{3}{20}$	$\frac{65}{200} = \frac{13}{40}$

i. $\Pr(\text{obtaining a value that is less than 3}) = \frac{1}{10} + \frac{1}{20}$
 $= \frac{3}{20}$ A1

ii. $\Pr(\text{obtaining a 4 or a 6}) = \frac{1}{5} + \frac{13}{40}$
 $= \frac{21}{40}$ A1

iii. $\Pr(\text{obtaining a 4 and a 6 in order}) = \frac{1}{5} \times \frac{13}{40}$
 $= \frac{13}{200}$ A1

iv. $\Pr(6|\text{even}) = \frac{\frac{13}{40}}{\frac{1}{20} + \frac{1}{5} + \frac{13}{40}}$
 $= \frac{\frac{13}{40}}{\frac{23}{40}}$
 $= \frac{13}{23}$ A1

Using a CAS calculator gives:

$\frac{13}{40}$	$\frac{13}{23}$
<hr/>	
$\frac{1}{20} + \frac{1}{5} + \frac{13}{40}$	