Neap

Trial Examination 2023

VCE Mathematical Methods Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 20 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are not drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

$1 + 3\log_7(2)$ expressed in the form $\log_a(b)$ is

A. $\log_7(8)$

B. $\log_7(15)$

C. $\log_7(16)$

D. $\log_7(42)$

E. $\log_7(56)$

Question 2

For what value(s) of k does $x^2 + 2kx + (k + 2) = 0$ have two real, irrational solutions?

A. $k = -1$ and $k = 2$

B. $k < -1 \cup k > 2$

C. $k \le -1 \cup k \ge 2$

D. $-1 \le k \le 2$

E. $-1 < k < 2$

Question 3

$y = \frac{1}{2}x^2 - 2x + 5$ expressed in the form $y = a(x - h)^2 + k$ is

A. $\frac{1}{2}(x-2)^2 + 3$

B. $\frac{1}{2}(x-2)^2 - 3$

C. $2(x-2)^2 + 3$

D. $2(x-2)^2 - 3$

E. $\frac{1}{2}(2x-2)^2 + 3$

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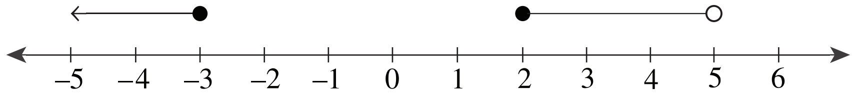
Question 4

Consider the following graph.



Using interval notation, the information in the graph can be described as

A. $x = \in [-3, 2]$

B. $x = \in [-\infty, -3] \cap (2, 5]$

C. $x = \in [-\infty, -3] \cup [2, 5)$

D. $x = \in (-\infty, -3] \cup [2, 5)$

E. $x = \in (-\infty, -3] \cup (2, 5]$

Question 5

The function $f(x) = 2\sin(x)$ undergoes the following transformations.

- translation of 2 units in the negative direction of the y-axis
- translation of 1 unit in the negative direction of the x-axis

The resultant function, $g(x)$, is

A. $g(x) = 2\sin(x - 2) - 1$

B. $g(x) = 2\sin(x + 2) + 1$

C. $g(x) = 2\sin(x + 1) - 2$

D. $g(x) = 2\sin(x - 1) - 2$

E. $g(x) = \sin(x + 1) + 2$

Question 6

If $\sin(x°) = 0.725$, what is $\cos(x°)$?

A. 0.275

B. 0.474

C. 0.526

D. 0.689

E. 0.811

Question 7

If $\cos(\pi + \theta) = -a$, $\cos(2\pi - \theta)$ is equal to

A. $-a$

B. a

C. $-\sin(\theta)$

D. $\sin(\theta)$

E. $\cos(\theta)$

Question 8

The function $g(x) = \log_3(3x - 1)$ has a

A. horizontal asymptote at $x = \frac{1}{3}$.

B. vertical asymptote at $x = \frac{1}{3}$.

C. horizontal asymptote at $y = -\frac{1}{3}$.

D. vertical asymptote at $y = -\frac{1}{3}$.

E. vertical asymptote at $y = \frac{1}{3}$.

Question 9

The graph of the function $f(x) = x^{\frac{1}{3}} + 3$ has a

A. local maximum at (0, 3).

B. point of inflection at (3, 0).

C. point of inflection at (0, 3).

D. stationary point at (3, 0).

E. local minimum at (0, 3).

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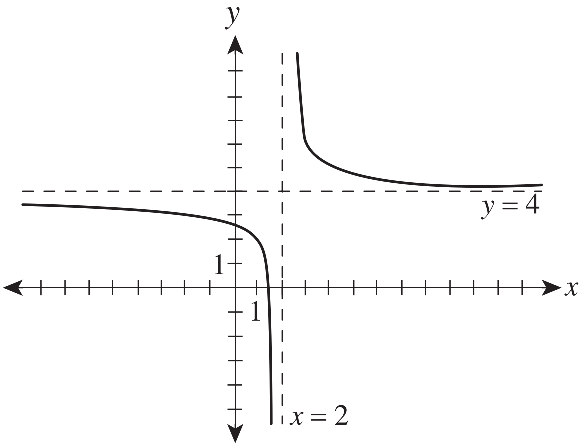
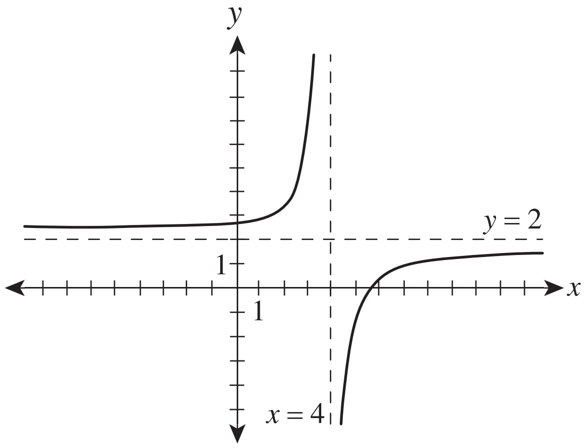
Question 10

Which one of the following graphs best represents $y = -\frac{3}{(x-2)} + 4$?

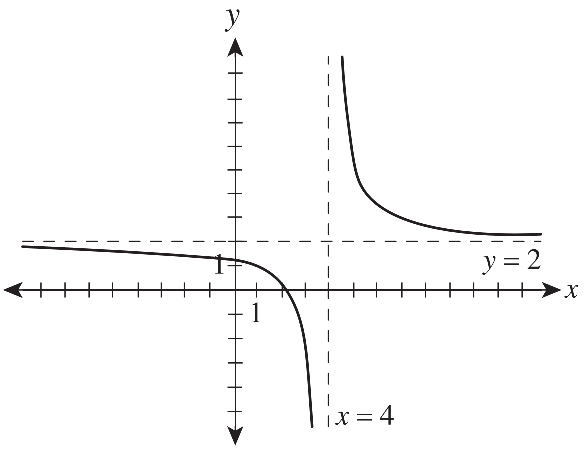
A.



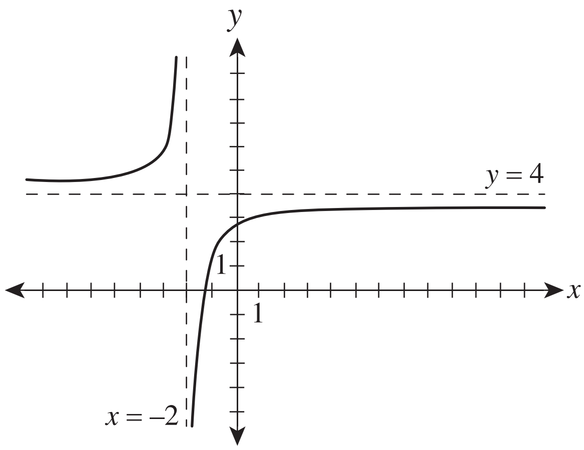
B.



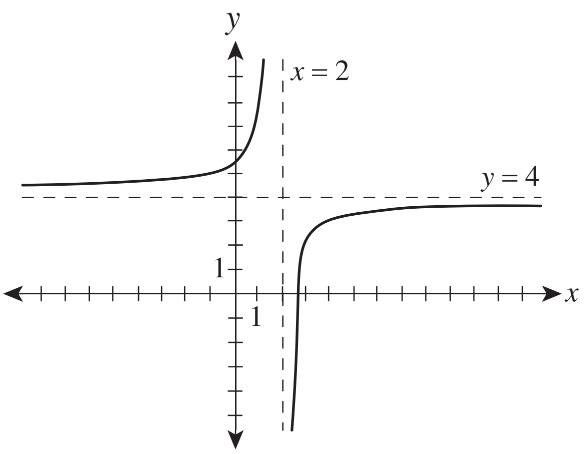
C.



D.

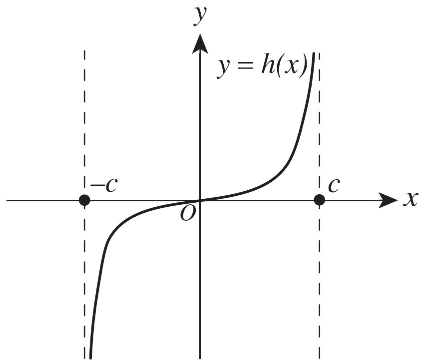


E.



Use the following information to answer Questions 11 and 12.

The graph of the function $h: (-c, c) \to \mathbb{R}, h(x) = \tan\left(\frac{x}{3}\right)$ is as follows.



Question 11

The value of c is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{3\pi}{2}$

D. 3π

E. 6π

Question 12

The average rate of change of h between $x = 0$ and $x = \frac{\pi}{8}$ is

A. –0.335

B. 5.818×10^{-3}

C. 0.335

D. 1.055

E. 6.148

Question 13

Let $f(x) = 2x^{\frac{1}{2}} - 3x^{-3}$.

The value of $f'(3)$ is

A. 0.6884

B. 0.6885

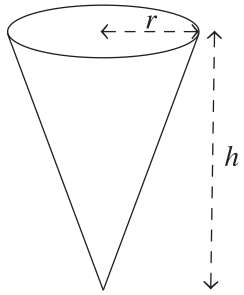
C. 1.8432

D. 3.3529

E. 3.3530

Question 14

Water is being poured into an inverted cone at a constant rate of 15 cm^3/s. The height h in centimetres of the water at any time is equal to two times the radius r cm. The cone is illustrated in the diagram below.



At what rate, in cubic centimetres per second, is the level of water rising when the height of the water is 10 cm?

A. $\frac{9}{80\pi}$ or $\frac{5\pi}{3}$

B. $\frac{9}{5\pi}$ or $\frac{3}{5\pi}$

C. $\frac{25\pi}{3}$ or $\frac{3\pi}{5}$

D. $\frac{400\pi}{3}$ or $\frac{5\pi}{3}$

E. $\frac{500\pi}{3}$ or $\frac{3\pi}{5}$

Question 15

A large kettle contains water at an initial temperature of 96°C. The water cools slowly, and after 2.5 hours, the temperature of the water has dropped to 49°C.

The temperature T°C of the water in the kettle is described by the equation $T = Ae^{-kt} + 10$ where t is the number of hours that the water cools for.

What are the values of A and k?

A. $A = -80$ and $k = 0.29$

B. $A = 0.29$ and $k = -80$

C. $A = 0.32$ and $k = 86$

D. $A = 86$ and $k = 0.07$

E. $A = 86$ and $k = 0.32$

Question 16

If $\Pr(A \cap B) = \frac{1}{5}, \Pr(B) = \frac{1}{4}$ and A and B are independent events, then $\Pr(A)$ is equal to

A. $\frac{1}{20}$

B. $\frac{9}{20}$

C. $\frac{11}{20}$

D. $\frac{16}{20}$

E. $\frac{19}{20}$

Question 17

If L and M are mutually exclusive events, and $\Pr(L) = 0.25$ and $\Pr(M) = 0.45$, then $\Pr(L \cup M)'$ is

A. 0.11

B. 0.19

C. 0.25

D. 0.30

E. 0.45

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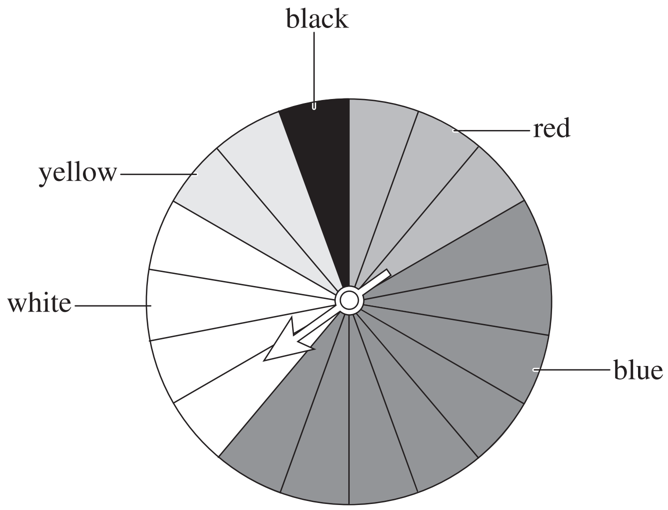
Question 18

The following diagram illustrates a spinner that has 18 segments.



When the spinner is spun, the probability of landing on a black or yellow segment is

A. $\frac{1}{18} \times \frac{1}{9}$

B. $\frac{1}{18}$ or $\frac{1}{9}$

C. $\frac{1}{18} + \frac{1}{9}$

D. $\frac{4}{9} + \frac{1}{9}$

E. $1 - \frac{1}{18} - \frac{1}{9}$

Question 19

A table tennis team of six players is to be selected from 15 people. There are eight females and seven males to choose from.

If there must be three females and three males selected, how many different combinations are possible?

A. 91

B. 546

C. 1960

D. 7560

E. 70 560

Question 20

Twenty books are placed in a row on a bookshelf. Three of these books are components of a series and must be placed together in any order.

The number of ways that the 20 books can be arranged is

A. $18! + 3!$

B. $18! \times 3!$

C. $\frac{18!}{3!}$

D. $20! \times 3!$

E. $\frac{20!}{3!}$

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (13 marks)

The point $P(-3, c)$ lies on the graph of $3x - 2y - 5 = 0$.

a. Find the value of c. 1 mark

b. **i.** Rearrange $3x - 2y - 5 = 0$ to make y the subject. 1 mark

ii. Using the answer to **part b.i.**, find the equation of the line that is perpendicular to $3x - 2y - 5 = 0$ and passes through the point (3, 2). 2 marks

Consider the parabola $y = -3(x - 3)^2 + 4$.

c. **i.** Expand and simplify the equation $y = -3(x - 3)^2 + 4$. 2 marks

ii. Find the axis of symmetry for this parabola. 1 mark

d. Find the point(s) where the line $3x - 2y - 5 = 0$ intersects the parabola $y = -3(x - 3)^2 + 4$.
Give your answer(s) correct to two decimal places. 2 marks

e. Let $P(x) = -2x^3 - 4x^2 - dx + 3$.

i. If $P(x)$ is divided by $(x + 2)$, the remainder is 1.
Find d. 2 marks

ii. Find $P(-b)$. 2 marks

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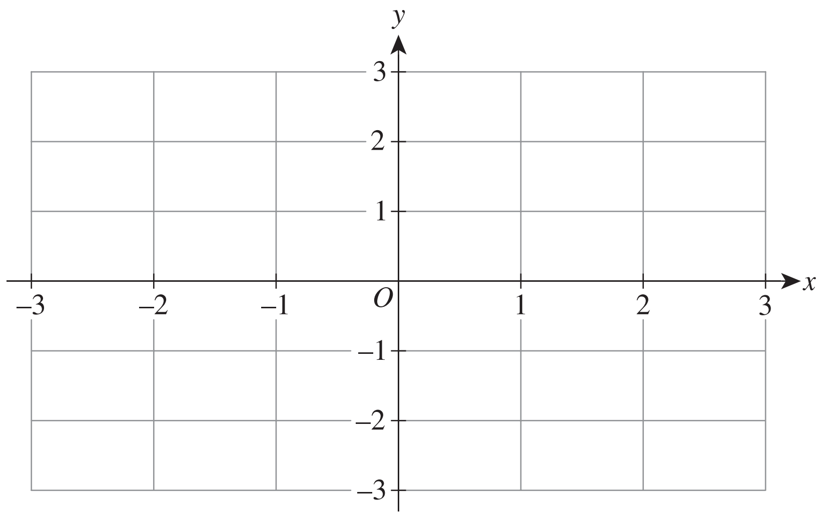
Question 2 (13 marks)

Let f be the function $\left\{f:\mathbb{R}\to\mathbb{R}, f(x)=-\frac{2}{x^2}+3\right\}$.

a. **i.** State the equations of the asymptotes. 2 marks

ii. State the x-intercepts in coordinate form. Give your answers correct to two decimal places. 1 mark

iii. On the axes below, sketch the graph of $y=f(x)$. Label all key features and, where appropriate, give coordinates correct to two decimal places. 2 marks



b. The domain of $f(x)$ is restricted to $\{x : x \in [-2,4]\}$.

i. The equation of $f(x)$ undergoes the following transformations to become $g(x)$.

- reflection in the x-axis
- translation of 2 units in the positive direction of the x-axis

State $g(x)$ using correct function notation. 3 marks

ii. State the range of $g(x)$. 2 marks

c. Find the inverse function $h(x)$ of $g(x)$ using correct function notation. State the domain of $h(x)$. 3 marks

Question 3 (19 marks)

Let $f(x) = 4x^3 + 3x^2 - x - 2$.

a. Find $f'(x)$. 1 mark

b. **i.** Let $g(x) = \int f(x)dx$.

Given that $\int f(-1)dx = 2$, find $g(x)$. 3 marks

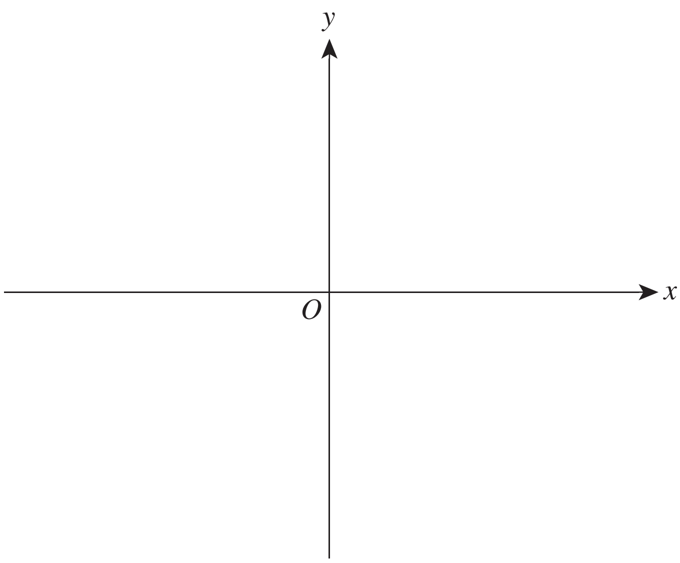
ii. Find $\int_1^2 f(x)dx$. 2 marks

c. **i.** Find the x-intercepts for the graph of $g(x)$ in coordinate form. Where appropriate, give coordinates correct to three decimal places. 2 marks

ii. Find the coordinates of any stationary points of $g(x)$, correct to three decimal places, and state the nature of these points. 4 marks

iii. The domain of $g(x)$ is restricted to $x \in [-1, 1.5)$.

On the axes below, sketch the graph of $y = g(x)$. Label all key features and, where appropriate, give coordinates correct to three decimal places. 4 marks



iv. State the values of x for which the graph of $y = g(x)$ has a negative gradient. 2 marks

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v. Find the instantaneous rate of change of $g(x)$, correct to two decimal places, when $x = 1.1$. 1 mark

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Question 4 (15 marks)

A student rolls two dice simultaneously. Each die has six faces (sides) numbered 1 to 6.

a. **i.** Find the probability that the sum of the values of the uppermost faces is 6. 1 mark

ii. Find the probability that the sum of the values of the uppermost faces is less than 6. 1 mark

b. The student obtains a third die and rolls each die one after the other.

i. How many three-digit outcomes are possible? 1 mark

ii. How many three-digit outcomes are possible if no number is repeated? 1 mark

iii. Find the probability that the student obtains three even numbers. 1 mark

iv. Find the probability that the student obtains three even numbers where no number is repeated. 1 mark

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v. What is the probability of the student obtaining the numbers 2, 3 and 6 **in order**? 1 mark

vi. What is the probability of the student obtaining the numbers 2, 3 and 6 **in any order**? 1 mark

c. The student decides to roll one die and toss a coin. The coin has a heads side and a tails side.

i. Find the total number of possible combinations. 1 mark

ii. Find the total number of possible combinations for when an even number is obtained from the die and heads is obtained from the coin. 1 mark

iii. Calculate the probability that the student obtains an even number from the die and tails from the coin. 1 mark

d. The student decides to roll one die 200 times. The results are shown in the following table.

Number on the die	1	2	3	4	5	6
How many times obtained?	20	10	35	40	30	65

i. Find the probability of obtaining a value that is less than 3. 1 mark

ii. Find the probability of obtaining a 4 or a 6. 1 mark

iii. Find the probability of obtaining a 4 and a 6 **in order**. 1 mark

iv. If the die is rolled twice, find the probability that a 6 is obtained on the second roll, given that an even number was obtained on the first roll. 1 mark

END OF QUESTION AND ANSWER BOOKLET

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Trial Examination 2023

VCE Mathematical Methods Units 1&2

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: ______________________________

Teacher's Name: ______________________________

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like this example: A B C D E

Use pencil only

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

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Trial Examination 2023

VCE Mathematical Methods Units 1&2

Written Examinations 1 and 2

Formula Sheet

Instructions

This formula sheet is provided for your reference. A question and answer booklet is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	Area $\approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

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Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}\left((X - \mu)^2\right) = \mathrm{E}\left(X^2\right) - \mu^2$
binomial coefficient	$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\, dx$	$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}\left(\hat{P}\right) = p$
standard deviation	$\mathrm{sd}\left(\hat{P}\right) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}\left(1-\hat{p}\right)}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}\left(1-\hat{p}\right)}{n}}\right)$

END OF FORMULA SHEET