

Trial Examination 2023

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Neap[®] Education (Neap) Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only for a period of 12 months from the date of receiving them. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

Copyright © 2023 Neap Education Pty Ltd ABN 43 634 499 791 Level 1 223 Hawthorn Rd Caulfield North VIC 3161 Tel: (03) 9639 4318

Question 1 (4 marks)

a.
$$f(x) = (4x - 2)^{-1}$$

 $f'(x) = -(4x - 2)^{-2} \times 4 \text{ OR } \frac{-4}{(4x - 2)^2} \text{ OR } \frac{-1}{(2x - 1)^2}$ A1

b. i.
$$\int \frac{1}{4x-2} dx = \frac{1}{4} \log_e(4x-2) + c \text{ OR } \frac{1}{4} \log_e(2x-1) + c$$
 A1

Note: Responses do not require c in order to obtain full marks.

ii.
$$\int_{1}^{5} f(x)dx = \frac{1}{4} \left[\log_{e} (4x - 2) \right]_{1}^{5} = \frac{1}{4} \left(\log_{e} (18) - \log_{e} (2) \right)$$

$$= \frac{1}{4} \log_{e} (9)$$

$$= \log_{e} \left(\sqrt{3} \right)$$
A1

Question 2 (2 marks)

$$f(x) = \int 3\sin(2x)dx$$

$$= -\frac{3}{2}\cos(2x) + c$$
M1

$$f\left(\frac{\pi}{3}\right) = 1 \Rightarrow -\frac{3}{2}\cos\left(\frac{2\pi}{3}\right) + c = 1$$

$$\left(-\frac{3}{2}\right) \times \left(-\frac{1}{2}\right) + c = 1$$

$$c = \frac{1}{4}$$

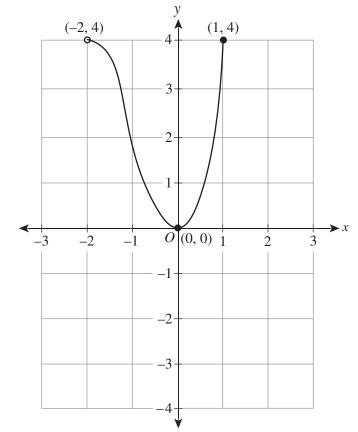
$$f(x) = -\frac{3}{2}\cos(2x) + \frac{1}{4}$$
 A1

Question 3 (4 marks)

a.
$$f'(x) = 3x^{2} + 6x$$

$$f'(x) = 0 \Rightarrow 3x(x+2) = 0 \Rightarrow x = 0 \text{ or } x = -2 \notin D_{f}$$

$$f(0) = 0 \Rightarrow (0,0)$$
A1



correct shape with an inflection point A1 correct endpoints and stationary point with (-2, 4) excluded A1

Question	4	(3	marks)
		< -	

a.

b.

	В	B ′	
A	k^2	0.2	
A'	0.1		1.6k

$$\Pr(A' \cap B') = 1 - \left(k^2 + 0.2 + 0.1\right) = 0.7 - k^2 \text{ OR } \Pr(A' \cap B') = 1.6k - 0.1$$
 A1

Note: Responses do not require a table to obtain full marks.

b.
$$Pr(A') = 1.6k = 0.1 + 0.7 - k^2$$
 M1
 $k^2 + 1.6k - 0.8 = 0$
 $5k^2 + 8k - 4 = 0$
 $k = -2 \text{ or } k = \frac{2}{5}$
 $k = \frac{2}{5}$ A1

Question 5 (3 marks) $\cos^{2}(3x) = \frac{1}{4}$ M1 $3x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\left[\cos(3x) = \frac{1}{2} \Longrightarrow \begin{bmatrix} 3x = \frac{\pi}{3} \\ 3x = -\frac{\pi}{3} \end{bmatrix}$ M1 $x = -\frac{\pi}{9} \text{ or } x = \frac{\pi}{9}$ A1

Question 6 (2 marks)

$$x_{new} = \frac{x - c}{b}$$
 A1

Question 7 (4 marks)

a. Three numbers are obtained.

The first number can be any number; hence, the probability is $\frac{6}{6}$. The second number must be the same as the first; hence, the probability is $\frac{1}{6}$. The third number must be the same as the first; hence, the probability is $\frac{1}{6}$.

$$\frac{6}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$
 A1

b. The first number can be any number; hence, the probability is $\frac{6}{6}$. The second number must be the same as the first; hence, the probability is $\frac{1}{6}$. The third number must be different to the first; hence, the probability is $\frac{5}{6}$. The order of the numbers can be arranged in three ways.

Multiplying all the probabilities by the number of possible ways gives:

$$3 \times \frac{6}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{15}{36} \text{ OR } \frac{5}{12}$$

c. A: all numbers are greater than 3 B: exactly two numbers are the same Determining $A \cap B$:

The first number must be greater than 3; hence, the probability is $\frac{3}{6}$.

The second number must be the same as the first; hence, the probability is $\frac{1}{6}$.

The third number must be greater than 3 but not the same as the previous number; hence,

the probability is $\frac{2}{6}$.

The order of the numbers can be arranged in three ways.

Multiplying all the probabilities by the number of possible ways gives:

$$3 \times \frac{3}{6} \times \frac{1}{6} \times \frac{2}{6}$$

Determining *B*:

The answer from **part b.**
$$\left(\frac{15}{36}\right)$$
 is used.

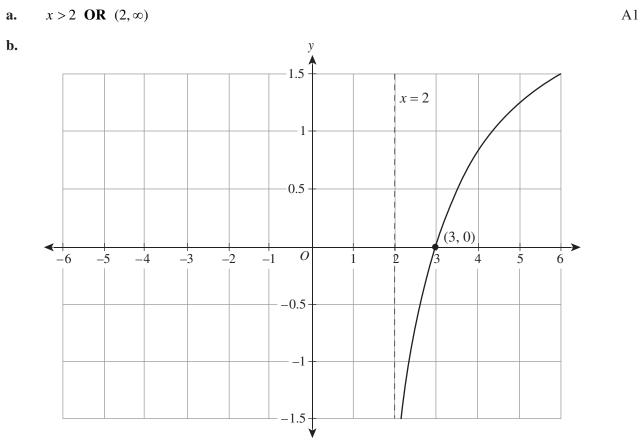
$$Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)}$$

$$= \frac{3 \times \frac{3}{6} \times \frac{1}{6} \times \frac{2}{6}}{\frac{15}{36}}$$

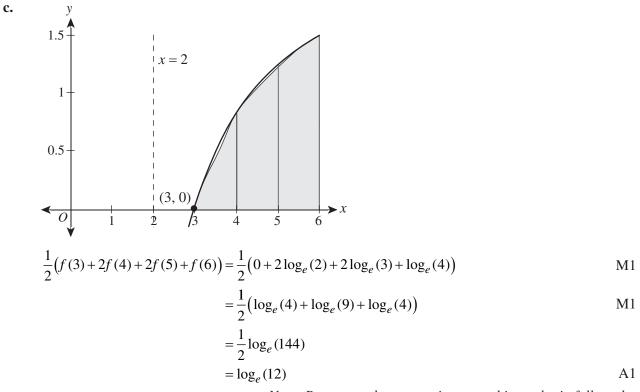
$$= \frac{1}{5}$$
A1

Note: For M1, a correct numerator or denominator is sufficient to obtain the mark.

Question 8 (12 marks)



correct shape A1 *correct x-intercept and vertical asymptote* A1

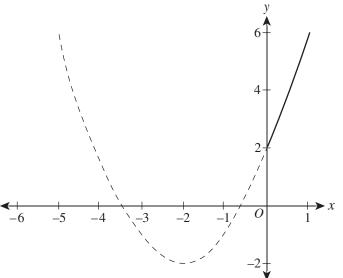


Note: Responses do not require a graphic to obtain full marks.

6

d. $R_g \subseteq D_f = (2, \infty)$ M1

The restricted graph of g(x) from *a* to ∞ such that its range is contained in $(2, \infty)$ is as follows.



♦ M1 Note: Accept any equivalent graphical or non-graphical method. A1

Hence,
$$a = 0$$
.

$$\mathbf{e.} \qquad h(x) = \log_e \left(x^2 + 4x \right) \tag{A1}$$

f.
$$D_h = D_g = (0, \infty)$$

Note: Accept the response from **part d.** for this mark.
For $x > 0$, $x^2 + 4x \in \mathbb{R}^+ \Rightarrow \log_e (x^2 + 4x) \in \mathbb{R}$.

$$R_h = R$$
 A1

Question 9 (6 marks)

a.
$$f'(x) = 4 - 2x$$

 $m = f'(a) = 4 - 2a$
 $y - f(a) = m(x - a)$
 $y - 4a + a^2 = (4 - 2a)(x - a)$
 $y = (4 - 2a)x - 4a + 2a^2 + 4a - a^2$
 $y = (4 - 2a)x + a^2$
M1

b.
$$S(a) = \int_{0}^{2} ((4-2a)x + a^{2} - f(x)) dx$$
 M1
$$= \left[(2-a)x^{2} + a^{2}x - 2x^{2} + \frac{x^{3}}{3} \right]_{0}^{2}$$
 M1
$$= 4(2-a) + 2a^{2} - 8 + \frac{8}{3}$$

$$= 2a^{2} - 4a + \frac{8}{3}$$
 A1

c. S'(a) = 4a - 4

 $S'(a) = 0 \Longrightarrow a = 1$ gives the minimum area. The maximum area occurs for a = 0 or a = 2.

A1