Neap

Trial Examination 2023

VCE Mathematical Methods Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 12 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (4 marks)

Let $f:\left(\frac{1}{2},\infty\right)\to R, f(x)=\frac{1}{4x-2}$.

a. Find $f'(x)$. 1 mark

b. i. Find an antiderivative of $f(x)$. 1 mark

ii. Express $\int_1^5 f(x)dx$ in the form $\log_e\left(\sqrt{a}\right)$, where a is an integer. 2 marks

Question 2 (2 marks)

The derivative of the function $f(x)$ has the rule $f'(x) = 3\sin(2x)$.

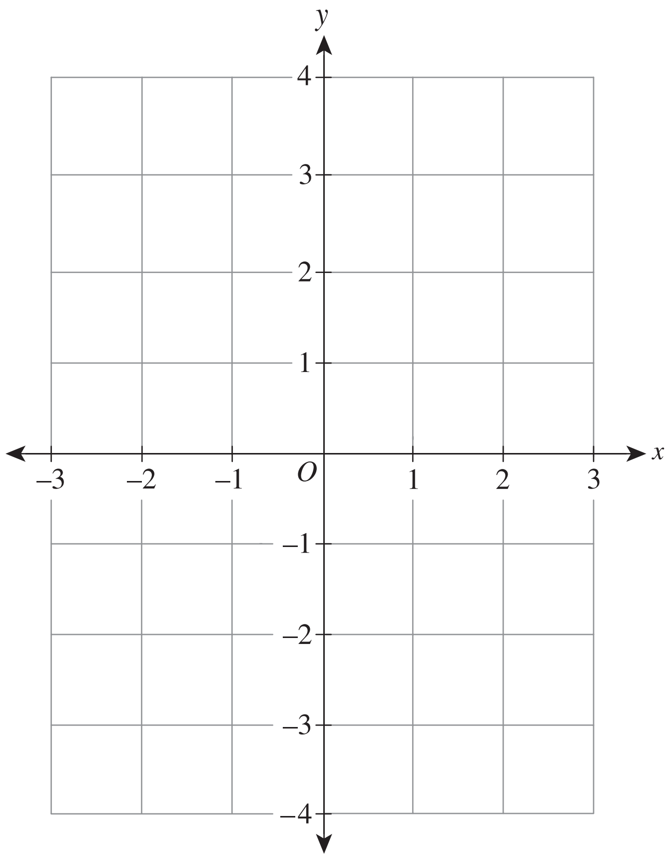
Given that $f\left(\frac{\pi}{3}\right) = 1$, find $f(x)$.

Question 3 (4 marks)

Let $f: (-2, 1] \rightarrow R, f(x) = x^3 + 3x^2$.

a. Find the coordinates of the stationary point. 2 marks

b. On the axes below, sketch the graph of $y = f(x)$. Label the endpoints and stationary point with their coordinates. 2 marks



Question 4 (3 marks)

For events A and B, $\Pr(A \cap B') = 0.2$ and $\Pr(A' \cap B) = 0.1$.

Let $\Pr(A \cap B) = k^2$ and $\Pr(A') = 1.6k$.

a. Find an expression for $\Pr(A' \cap B')$ in terms of k. 1 mark

b. Find the value of k. 2 marks

Question 5 (3 marks)

Solve $4\cos^2(3x) - 1 = 0$ for $x \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$.

Question 6 (2 marks)

A transformation maps the graph of $y = f(x)$ to $y = af(bx + c) + d$, where a, b, c and d are positive real numbers. The following algorithm will be used to map any point (x, y) on the graph of $y = f(x)$ to the graph of $y = af(bx + c) + d$.

```
input x, y                 # Line 1
      x_new = _____        # Line 2
      y_new = _____        # Line 3
      print x_new          # Line 4
      print y_new          # Line 5
```

Complete lines 2 and 3 of the algorithm.

MMU34_QB_Ex1_2023

Question 7 (4 marks)

The possible outcomes when a die is rolled are 1, 2, 3, 4, 5 or 6. Each outcome is equally likely.

A die is rolled three times, and the resulting number is recorded each time.

a. Find the probability that the resulting number is the same each time. 1 mark

b. Find the probability that the resulting number is the same exactly two times. 1 mark

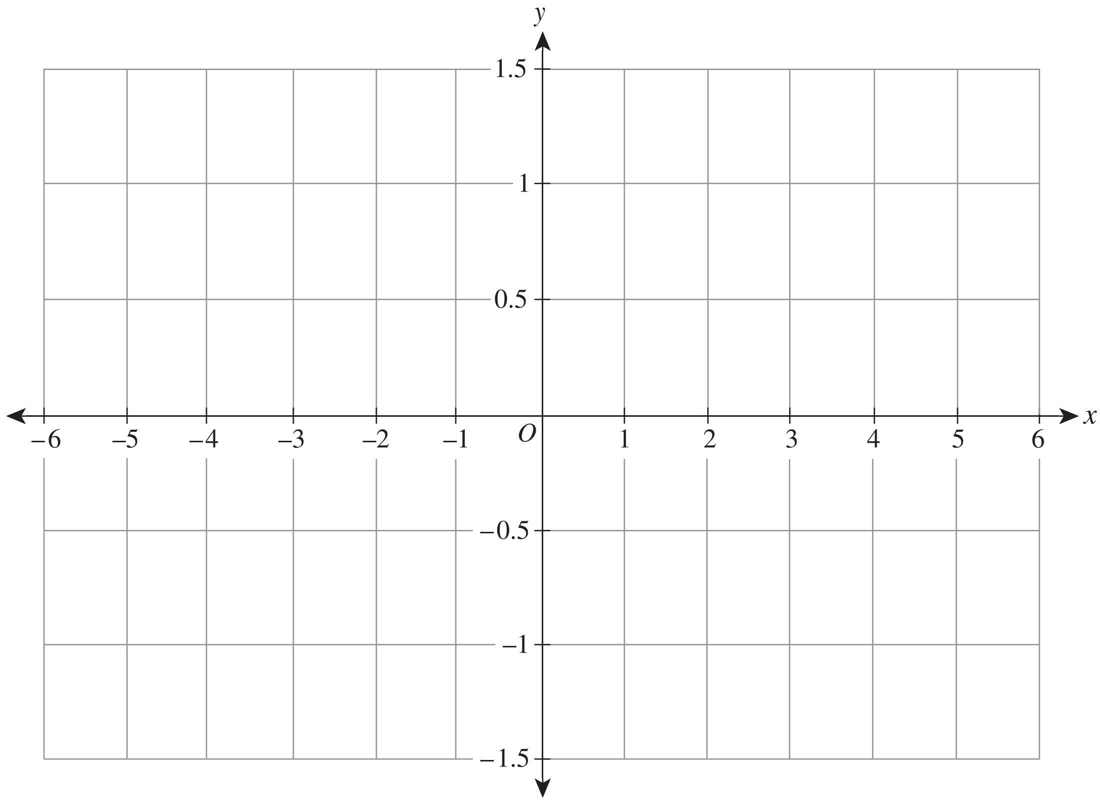
c. What is the probability that the resulting number is greater than 3 each time, given that it is the same exactly two times? 2 marks

Question 8 (12 marks)

Let $f(x) = \log_e(x - 2)$.

a. State the domain of f. 1 mark

b. On the axes below, sketch the graph of $y = f(x)$. Label the axial intercept with its coordinates and the asymptote with its equation. 2 marks



c. Using the trapezium rule with interval widths of 1, approximate the area bounded by the curve $y = f(x)$ and the x-axis from $x = 3$ to $x = 6$. Give your answer in the form $\log_e(a)$, where a is an integer. 3 marks

d. Let $g: (a, \infty) \to R$, $g(x) = x^2 + 4x + 2$ and $h(x) = (f \circ g)(x)$.

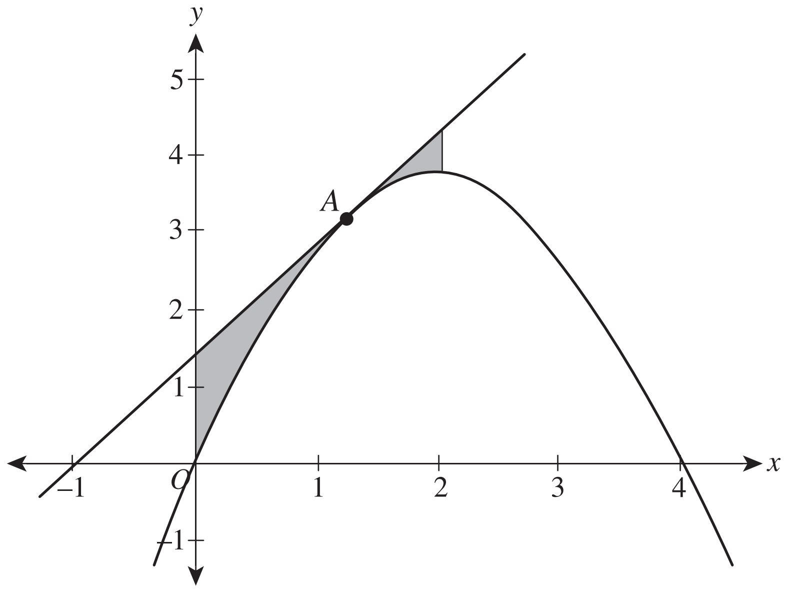
Find the smallest value of a such that $h(x)$ exists. 3 marks

e. State the rule for $h(x)$. 1 mark

f. State the domain and range of $h(x)$. 2 marks

Question 9 (6 marks)

The tangent to the graph of $f(x) = 4x - x^2$ at the point $A\left(a, f(a)\right)$ is shown below.



a. Show that the equation of the tangent is $y = (4 - 2a)x + a^2$. 2 marks

b. Let $S(a)$ be the shaded area bounded by the graph of $y = f(x)$, the y-axis, the line $x = 2$ and the tangent to $f(x)$ at $x = a$, where $0 \le a \le 2$.

Find the rule for $S(a)$. 3 marks

c. Find the value(s) of a such that $S(a)$ is a **maximum**. 1 mark

END OF QUESTION AND ANSWER BOOKLET

Neap

Trial Examination 2023

VCE Mathematical Methods Units 3&4

Written Examinations 1 and 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

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MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	Area $\approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

MMU34_FS_2023

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\operatorname{var}(X) = \sigma^2 = \mathrm{E}\left((X-\mu)^2\right) = \mathrm{E}\left(X^2\right) - \mu^2$
binomial coefficient	$\dbinom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
binomial	$\Pr(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\, dx$	$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}\left(\hat{P}\right) = p$
standard deviation	$\operatorname{sd}\left(\hat{P}\right) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}\left(1-\hat{p}\right)}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}\left(1-\hat{p}\right)}{n}}\right)$

END OF FORMULA SHEET