Mathematical Methods

Written Examination 2

2024 Insight Year 12 Trial Exam Paper

Worked Solutions

This book presents:

- worked solutions
- mark allocations
- tips.

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Answers to multiple-choice questions

Question	Answer
1	B
2	D
3	B
4	A
5	A
6	D
7	C
8	B
9	B
10	D
11	D
12	C
13	D
14	D
15	A
16	B
17	B
18	C
19	B
20	A

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SECTION A

Question 1

Answer: B

Explanatory notes

The amplitude is the size of the coefficient of cosine: 3.

The period is 2π divided by the coefficient of x: $\frac{2\pi}{2\pi}=1$.

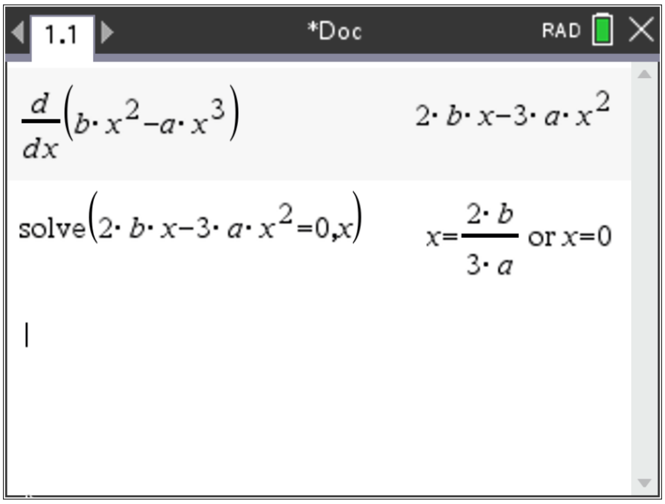
The period can also be observed by sketching the graph.

Question 2

Answer: D

Explanatory notes

The best way to answer questions like this is to use the 'Solve' functionality on a CAS.



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Question 3

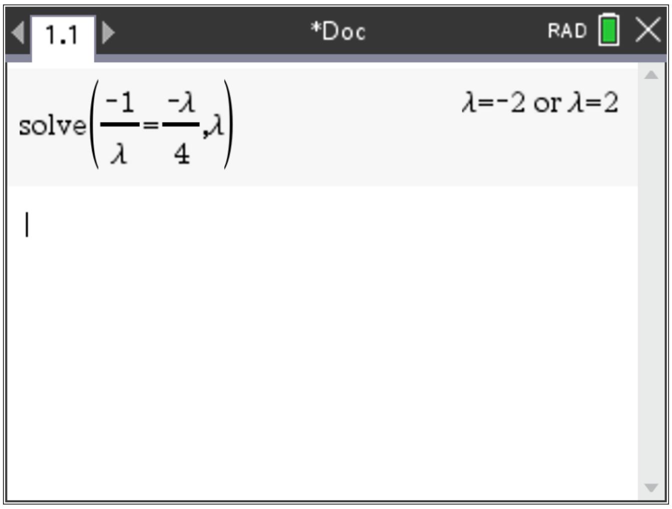
Answer: B

Explanatory notes

The system of equations will have infinite solutions when the equations represent the same line.

The two equations can be rearranged to give $y=-\frac{1}{\lambda}x+3$ and $y=-\frac{\lambda}{4}x+3$.

Since the y-intercepts are already the same, these two equations are the same line when their gradients are the same: $-\frac{1}{\lambda}=-\frac{\lambda}{4}$.

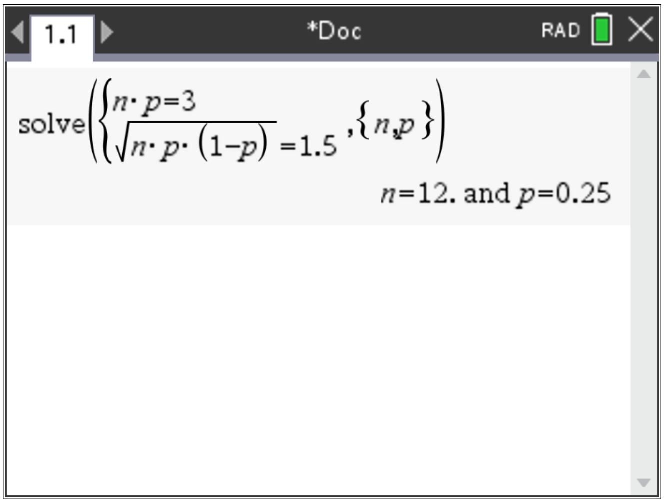


Question 4

Answer: A

Explanatory notes

For $X \sim \mathrm{Bi}(n, p)$, the mean is np and the standard deviation is $\sqrt{np(1-p)}$.



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Question 5

Answer: A

Explanatory notes

The domain of a sum/difference function is the intersection of the domains of each section of the function. $\log_e(5-x)$ has a domain of $(-\infty, 5)$ and $\sqrt[3]{x+1}$ has a domain of R. Therefore, the overall domain is $(-\infty, 5)$.

Question 6

Answer: D

Explanatory notes

$y = x^{\frac{1}{3}}$ is the only graph which does not converge to a limit as $x \to \infty$ or $x \to -\infty$.

This can be seen by sketching the five graphs or by considering what happens to the five graphs as $x \to \infty$ or $x \to -\infty$.

Question 7

Answer: C

Explanatory notes

The sum of the probabilities must equal 1.

$$0.15 + 0.1 + m + m^2 = 1$$

Solving this gives $m = -1.5$ or $m = 0.5$. Since m is a probability, it must be 0.5.

Therefore, the mean is

$$\begin{aligned} \mathrm{E}(X) &= 0 \times 0.15 + 1 \times 0.1 + 2 \times 0.5 + 3 \times 0.25 \\ &= 1.85 \end{aligned}$$

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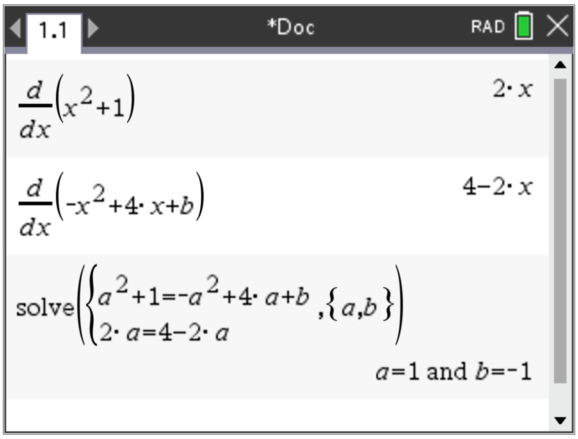
Question 8

Answer: B

Explanatory notes

The function is smooth and continuous when the function values and derivative values are equal at the change-over point.

That is, $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x)$ and $\lim\limits_{x \to a^-} f'(x) = \lim\limits_{x \to a^+} f'(x)$.



Question 9

Answer: B

Explanatory notes

$$
\begin{aligned}
\int_{-3}^{3} -f(x)\,dx &= -\int_{-3}^{3} f(x)\,dx \\
&= -\left(\int_{-3}^{-1} f(x)\,dx + \int_{-1}^{3} f(x)\,dx\right) \\
&= -\left(-\int_{-1}^{-3} f(x)\,dx + \int_{-1}^{3} f(x)\,dx\right) \\
&= -\left(-7+1\right) \\
&= -(-6) \\
&= 6
\end{aligned}
$$

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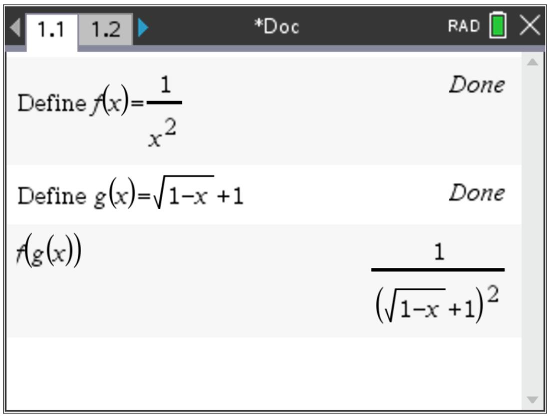
Question 10

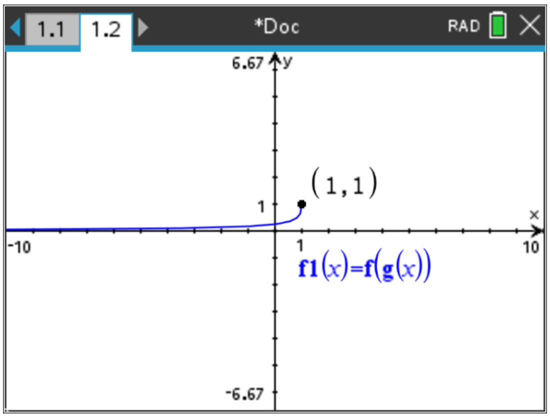
Answer: D

Explanatory notes

The range of a graph is very difficult to work out without sketching.

Sketch the graph of $y=(f\circ g)(x)$.

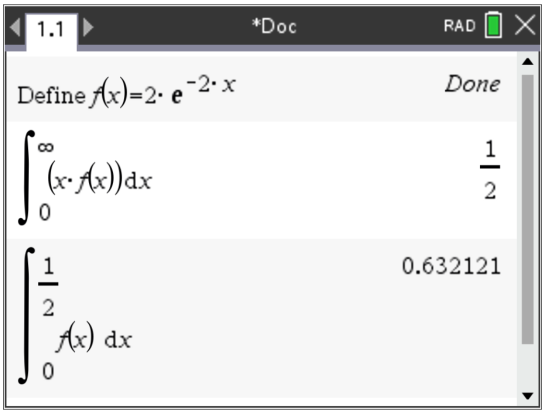




Question 11

Answer: D

Explanatory notes



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Question 12

Answer: C

Explanatory notes

$$
\begin{aligned}
f(x)+g(x) &= -\log_e(1-x) \\
\log_e(x+1)+g(x) &= -\log_e(1-x) \\
g(x) &= -\log_e(1-x)-\log_e(x+1) \\
g(x) &= -\left[\log_e(1-x)+\log_e(x+1)\right] \\
g(x) &= -\left[\log_e(1-x^2)\right] \\
g(x) &= \log_e\left(\frac{1}{1-x^2}\right)
\end{aligned}
$$

Question 13

Answer: D

Explanatory notes

The width of a confidence interval is twice the size of the margin of error: $z\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$.

To halve the size of the margin of error, we need $\frac{z}{2}\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$, which is equivalent to $z\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{4n}}$. Therefore, the sample size needs to be four times greater: $120\times 4=480$.

Question 14

Answer: D

Explanatory notes

The trapezium approximation formula can be used here.

$$
\begin{aligned}
Area &\approx \frac{4-1}{2(3)}\left[\left(1+1\right)+2\left(1+\sqrt{2}\right)+2\left(1+\sqrt{3}\right)+\left(1+\sqrt{4}\right)\right] \\
&\approx 7.6
\end{aligned}
$$

Tip

- *The trapezium approximation formula can be found on the formula sheet. In the formula, n is the number of trapeziums to be used, x_0 is the lower terminal of integration and x_n is the upper terminal of integration.*

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Question 15

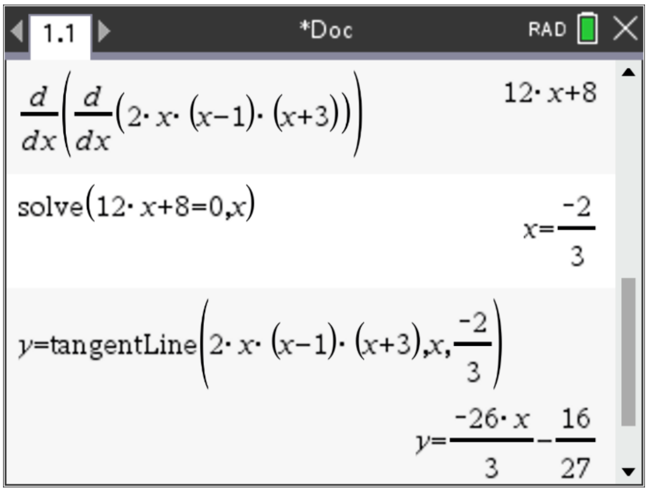
Answer: A

Explanatory notes

For cubics with two stationary points, the non-stationary point of inflection is the only point where the tangent does not cross the graph at any other point. This can be deduced from looking at the graph of a cubic with two stationary points.

Therefore, the x-value of this tangent is found by solving for when the second derivative equals zero.

The y-intercept of this tangent is $-\frac{16}{27}$.

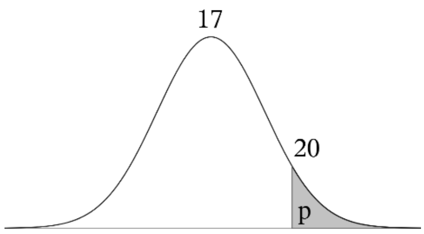


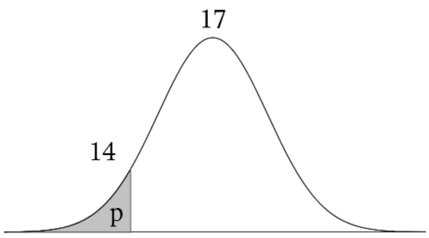
Question 16

Answer: B

Explanatory notes

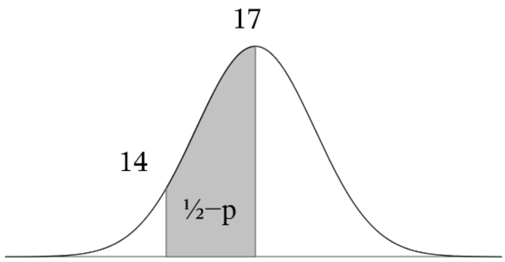
By symmetry, if $\Pr(X > 20) = p$ then $\Pr(X < 14) = p$ too.





Also, $\Pr(X \le 17) = \dfrac{1}{2}$.

Therefore, $\Pr(14 < X \le 17) = \dfrac{1}{2} - p$.



Tip

- *Draw a diagram of the normal distribution for questions such as these. Plot the points equidistant from the mean and use the symmetry of the curve.*

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Question 17

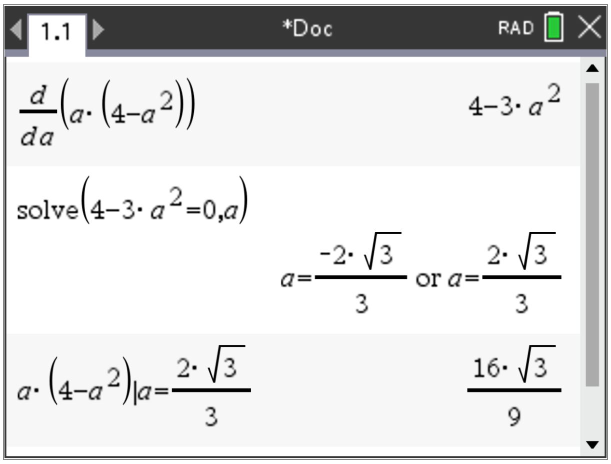
Answer: B

Explanatory notes

The top right vertex of the rectangle has coordinates $\left(a, 4-a^2\right)$ where $a \in (0,2)$.

Therefore, the area of the rectangle is $A = a\left(4-a^2\right)$.

The maximum value of A is found by solving for when $\frac{dA}{da} = 0$.



Tip

- *Always read the question carefully and check which value is required. You could do all the working out correctly for this question but select A instead of B because you stopped after finding the value of a which gives the maximum area, rather than continuing on to find the actual maximum area.*

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Question 18

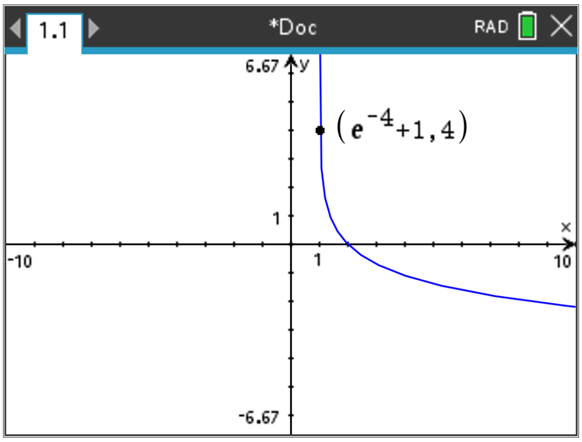
Answer: C

Explanatory notes

$(f \circ g)(x)$ is defined if the range of g is a subset (or equal to) the domain of f.

Therefore, the maximum range of g could be $(-\infty, 4]$.

Sketching the graph of $y = g(x)$ and making sure its range fits this set leads to a maximal domain of g of $\left[e^{-4}+1, \infty\right)$.

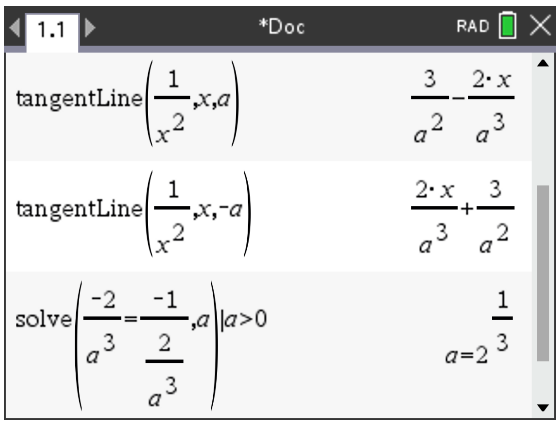


Question 19

Answer: B

Explanatory notes

Find the tangents at $x = a$ and $x = -a$. Then set the gradient of one to be the negative reciprocal of the other and solve for a.



Tip

- *Use your CAS to find the equation of a tangent line. This will help you avoid doing a lot of unnecessary algebra by hand.*

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Question 20

Answer: A

Explanatory notes

The derivative of $\left[f(x)\right]^2$ is $2f(x)f'(x)$.

Using the quotient rule, the derivative of $\dfrac{\left[f(x)\right]^2}{g(x)}$ is $\dfrac{2f(x)f'(x)g(x)-\left[f(x)\right]^2 g(x)}{\left[g(x)\right]^2}$.

Substituting in 1 for x gives the gradient of the tangent as -6.

The equation of the tangent at $x=1$ is therefore $y-\dfrac{\left[f(1)\right]^2}{g(1)}=-6\left(x-1\right)$, which simplifies to $6x+y=8$.

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SECTION B

Question 1a.

Worked solution

Solving $f(x)=0$ gives $x=0$ or $x=1$.

$\therefore (0,0)$ and $(1,0)$

Mark allocation: 1 mark

- 1 mark for both correct coordinates

Question 1b.

Worked solution

Solving $f'(x)=0$ gives $x=0$ or $x=\dfrac{2}{3}$.

$f(0)=0$ and $f\left(\dfrac{2}{3}\right)=\dfrac{4}{27}$.

$\therefore (0,0)$ and $\left(\dfrac{2}{3},\dfrac{4}{27}\right)$

Mark allocation: 1 mark

- 1 mark for both correct coordinates

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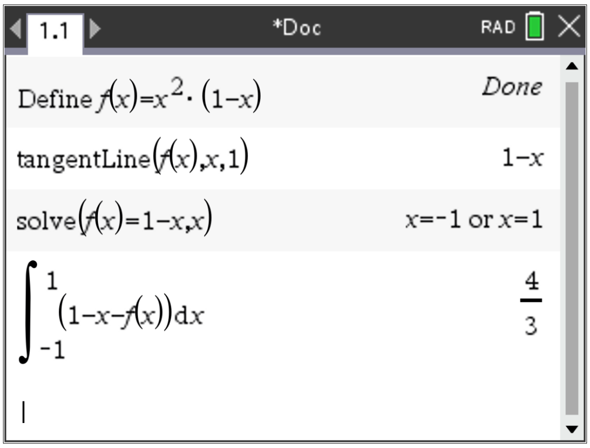
Question 1c.

Worked solution

The tangent at $x=1$ is $y=1-x$.

Solving $f(x)=1-x$ gives two points of intersection at $x=1$ and $x=-1$.

The area bound by the graphs is $\int_{-1}^{1}\left[(1-x)-f(x)\right]dx=\frac{4}{3}$.



Mark allocation: 3 marks

- 1 mark for the equation of the tangent line
- 1 mark for finding the x-values of the intersections

Note: seeing these values as the terminals of the integral is sufficient to earn this mark.

- 1 mark for the final answer: $\frac{4}{3}$

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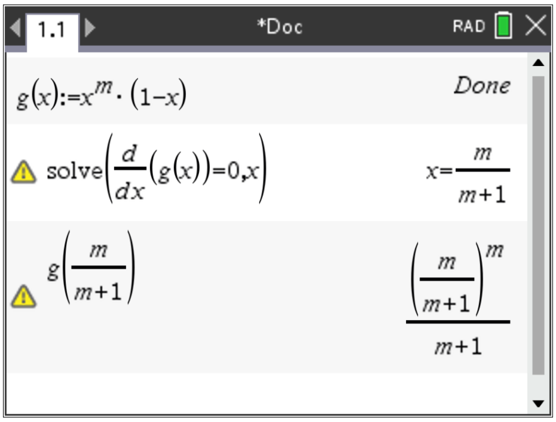
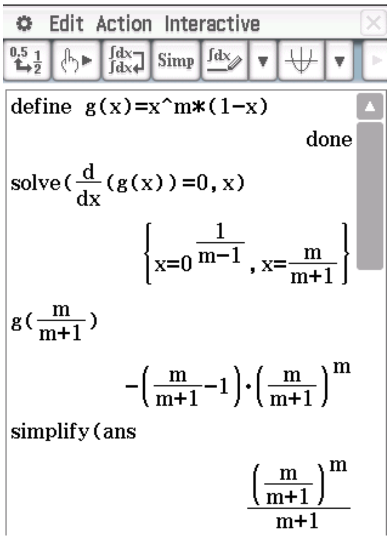
Question 1d.

Worked solution

The maximum turning point occurs when $g'(x)=0$. Solving this gives $x=\dfrac{m}{m+1}$, which gives a corresponding function value of $\dfrac{m^m}{(m+1)^{m+1}}$.

Therefore, $\left(\dfrac{m}{m+1},\dfrac{m^m}{(m+1)^{m+1}}\right)$.





Mark allocation: 2 marks

- 1 mark for finding the x-value of the maximum turning point
- 1 mark for writing the correct final answer in coordinate form

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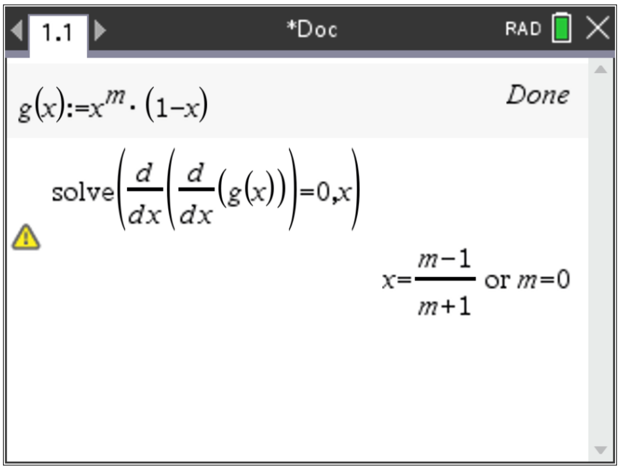
Question 1e.

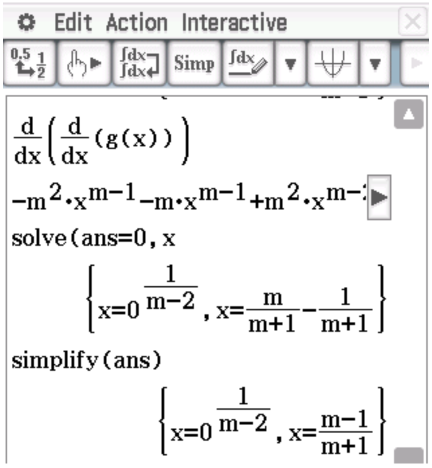
Worked solution

$x = \frac{m-1}{m+1}$

The maximum gradient will occur when the second derivative is equal to zero.

Solving $g''(x) = 0$ gives $x = \frac{m-1}{m+1}$.





Mark allocation: 2 marks

- 1 mark for setting the second derivative of g to zero
- 1 mark for the correct final answer

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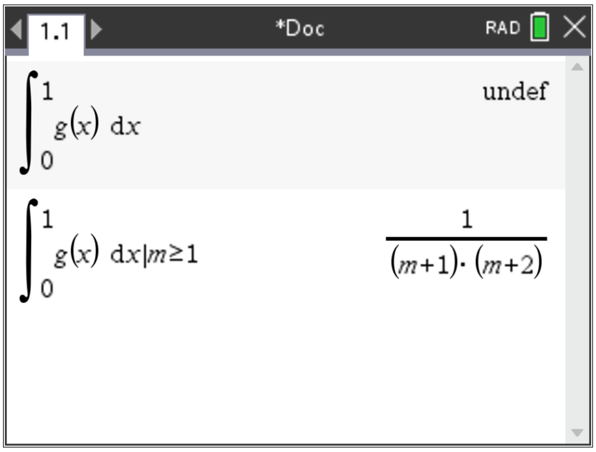
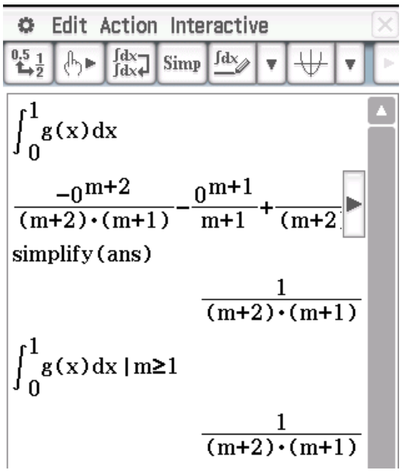
Question 1f.

Worked solution

The area is given by $\int_0^1 g(x)\,dx = \dfrac{1}{(m+1)(m+2)}$.





Mark allocation: 2 marks

- 1 mark for the correct integral expression
- 1 mark for the correct final answer

Note: an answer of $\dfrac{1}{m+1} - \dfrac{1}{m+2}$ is equivalent and is therefore also accepted.

Tip

- *The TI and Classpad both struggle with this question. If your CAS is giving an undefined or unexpected result, then it may be an issue with the parameter domain. Try to execute the calculation again with an appropriate restriction on the parameter. Since m is a positive integer, we must have $m \geq 1$.*

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Question 1g.

Worked solution

m is even.

The gradient at $(1, 0)$ is always negative, so the tangent will touch the graph again as long as the degree of the polynomial is odd. The highest power of g can be seen by expanding the function: $g(x) = x^m - x^{m+1}$. Therefore, the degree is odd when m is even.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2a.

Worked solution

14 m

$D(4) = 14$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2b.

Worked solution

$D'(3) = \frac{\pi\sqrt{2}}{6}$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2c.

Worked solution

14.35 m

$\frac{1}{8-0}\int_0^8 D(t)\,dt \approx 14.35$

Mark allocation: 2 marks

- 1 mark for using the average value formula
- 1 mark for the correct answer

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Question 2d.

Worked solution

19.70

The maximum depth occurs when $t = 12$. Since the graph is symmetric about $t = 12$, the depth at $t = 10.5$ and $t = 13.5$ are the same. Therefore, the depth is above this height for three hours during the day: $D(10.5) = D(13.5) \approx 19.70$.

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 2e.

Worked Solutions

9.24

$\text{solve}(d(t)=19,t)|0\le t\le 24$

$t=9.23936$ or $t=14.7606$

Mark allocation: 1 mark

- 1 mark for the correct answer

Tip

- *When solving circular functions on your CAS, you will be given general solutions. Restrict the domain in order to find specific solutions.*

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Question 2f.

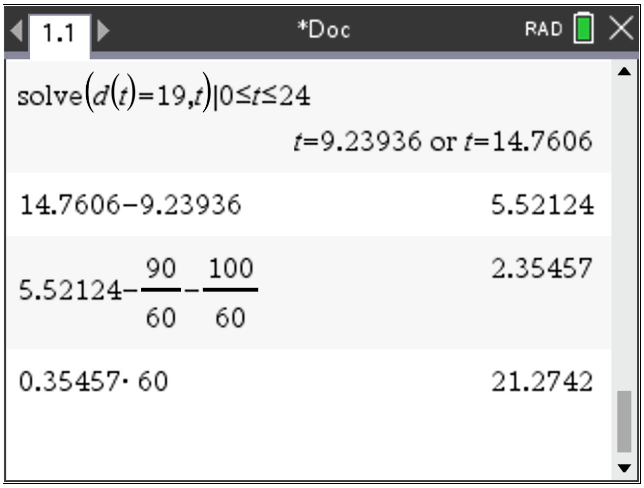
Worked solution

2 hours, 21 minutes.

The ship can enter the harbour at $t \approx 9.24$ and must leave the harbour by $t \approx 14.76$.

This gives a maximum time in the harbour of approximately 5.52 hours.

Subtracting the 90 minutes and 100 minutes gives a maximum time to unload of 2.3546 hours, which is equivalent to 2 hours, 21 minutes.



Mark allocation: 2 marks

- 1 mark for finding the maximum time in the harbour or the maximum time available to unload (5.52 or 2.35)
- 1 mark for the correct answer in the specified form

Tip

- *Use the unrounded values in your CAS, rather than the rounded values that you write down on the page. This will help avoid compounding rounding errors.*

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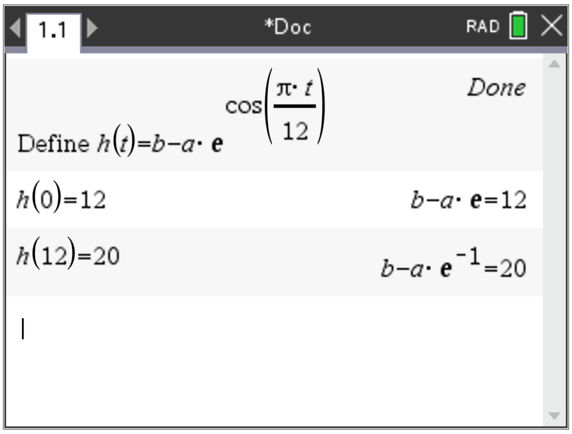
Question 2g.

Worked solution

$B - Ae = 12$

and

$B - Ae^{-1} = 20$



Mark allocation: 1 mark

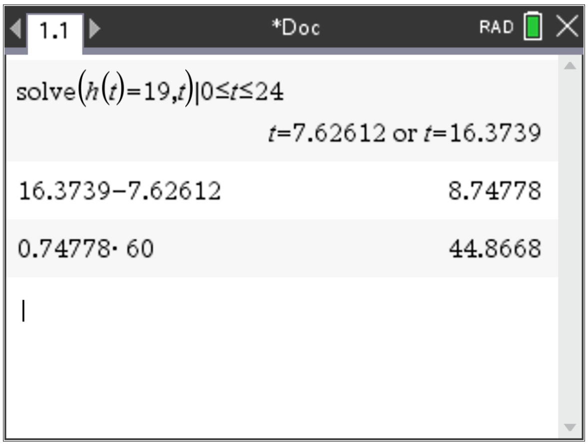
- 1 mark for writing both equations

Note: writing the equations $H(0) = 12$ and $H(12) = 20$ is sufficient for earning this mark.

Question 2h.

Worked solution

8 hours, 45 minutes



Mark allocation: 1 mark

- 1 mark for the correct answer

Tip

- *Sometimes the CAS finds it difficult to solve inequalities. Solve for when the function is equal to a value and then use the graph to determine the horizontal values for which the function is above or below the value.*

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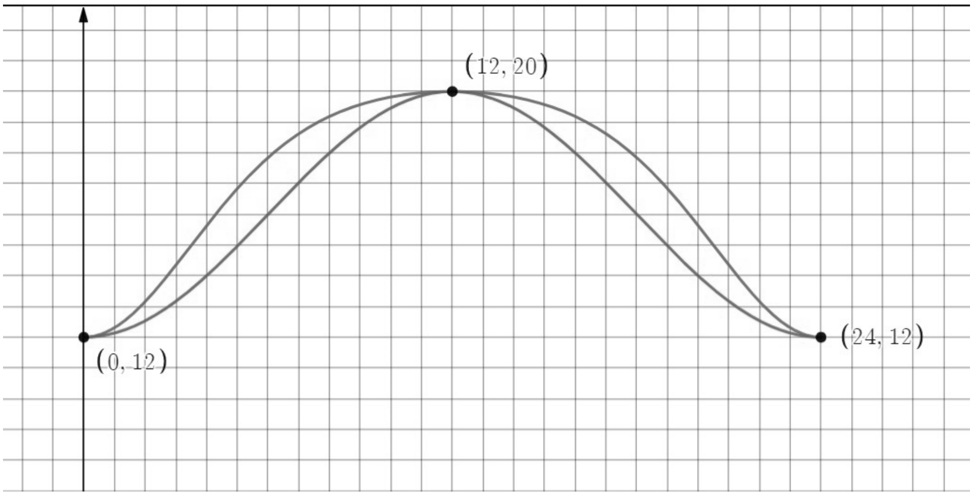
Question 2i.

Worked Solutions



Mark allocation: 2 marks

- 1 mark for the correct shape of the graph, including the correct domain
- 1 mark for showing the graph intersecting with $y = D(t)$ at the three labelled points

Question 2j.

Worked Solutions

The oceanographer's model is always above the original model, so the difference between them is given by $H(t) - D(t)$.

The maximum difference will occur when $\frac{d}{dt}\left(H(t) - D(t)\right) = 0$.

Solving this equation gives $t = 0,\ 5.38064,\ 12,\ 18.6194,\ 24$.

Trivially, the minimums occur at 0, 12, and 24, since they are when the models are equal.

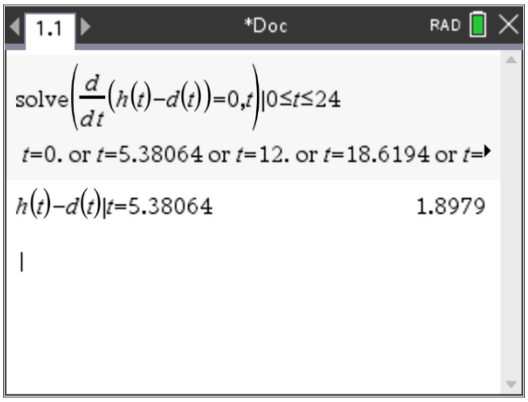
Therefore, the maximums occur at $t = 5.38064 \text{ and } 18.6194$.

Since the graphs are symmetric, these two values will give the same difference.

Therefore, the maximum difference is $H(5.38064) - D(5.38064) = 1.90$.

Mark allocation: 2 marks

- 1 mark for the setting $\frac{d}{dt}\left(H(t) - D(t)\right) = 0$
- 1 mark for the correct answer

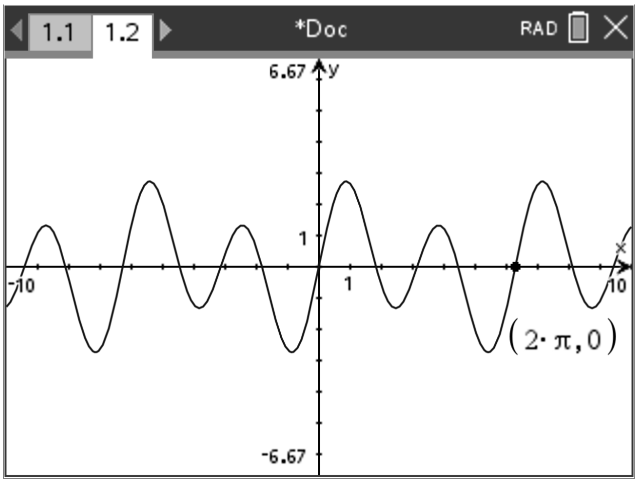


Question 3a.

Worked solution

2π

This can be observed from the graph.



Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3b.

Worked solution

$x = n\pi,\ n \in Z$

Mark allocation: 1 mark

- 1 mark for the correct answer

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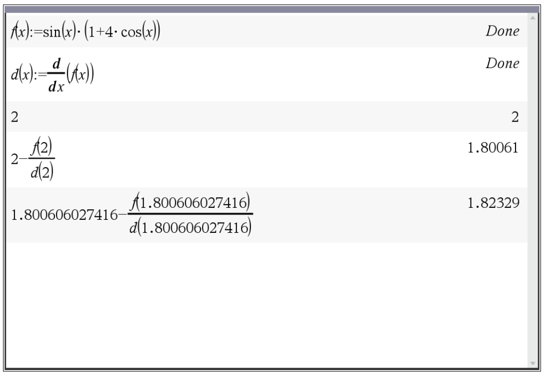
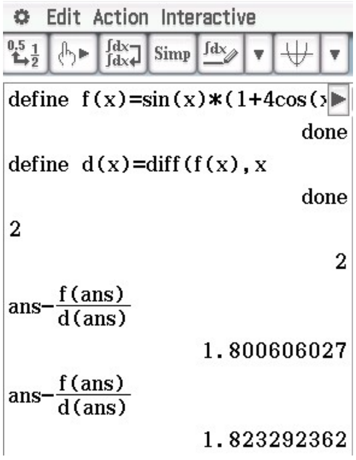
Question 3c.

Worked Solutions

Applying Newton's method with $x_0 = 2$ gives $x_1 = 1.801$ and $x_2 = 1.823$.





Mark allocation: 1 mark

- 1 mark for both correct answers

Tip

- *To implement Newton's method efficiently on a CAS, you can type in the value of x_0 and press Enter. Then, typing in $\text{Ans} - \frac{\boldsymbol{f}(\text{Ans})}{\boldsymbol{d}(\text{Ans})}$ (where $\boldsymbol{d}$ has already been defined as the derivative of f) will allow you to find each successive estimate for x by simply pressing Enter (or execute) repeatedly.*

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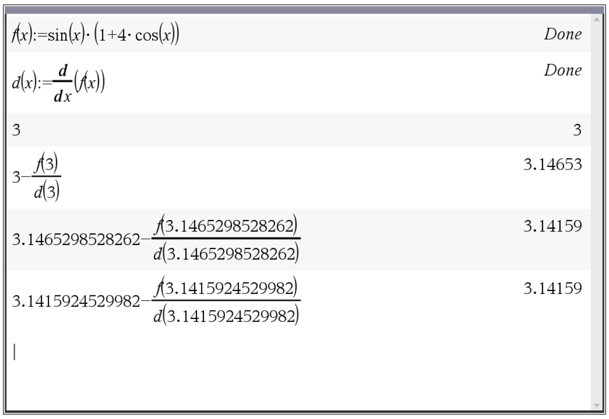
Question 3d.

Worked solution

Newton's method would converge to another x-intercept, $(\pi, 0)$.



Mark allocation: 1 mark

- 1 mark for a correct explanation

Question 3e.

Worked solution

$$\cos(x) = \frac{-1 \pm \sqrt{1^2 - 4(8)(-4)}}{2(8)}$$
$$= \frac{-1 \pm \sqrt{129}}{16}$$

Mark allocation: 1 mark

- 1 mark for a correct method (e.g. quadratic formula) leading to the required result

Question 3f.

Worked solution

$$x = \cos^{-1}\left(\frac{-1 + \sqrt{129}}{16}\right)$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 3g.

Worked solution

From the graph, it can be observed that $y = f(x)$ is above $y = g(x)$ from $x = \frac{\pi}{4}$ to $\frac{5\pi}{4}$.

Therefore, the area is $\int_{\pi/4}^{5\pi/4} \left[f(x) - g(x)\right] dx = 2\sqrt{2}$.

Mark allocation: 2 marks

- 1 mark for an appropriate integral
- 1 mark for the correct final answer

Question 3h.

Worked Solutions

$a = -1$ and $b = \frac{\pi}{2}$

$$f(ax+b) = \sin(ax+b)\big(1+4\cos(ax+b)\big)$$
$$g(x) = \cos(x)\big(1+4\sin(x)\big)$$

$\therefore\ \sin(ax+b) = \cos(x)$ and $\cos(ax+b) = \sin(x)$

There is no single, direct translation that transforms both sine into cosine and cosine into sine, so we must have $a = -1$. Then the smallest value of b which works is $b = \frac{\pi}{2}$.

Mark allocation: 2 marks

- 1 mark for $a = -1$
- 1 mark for $b = \frac{\pi}{2}$

Question 4a.

Worked solution

$\Pr(X < 600) \approx 0.2023$

Mark allocation: 1 mark

- 1 mark for the correct answer

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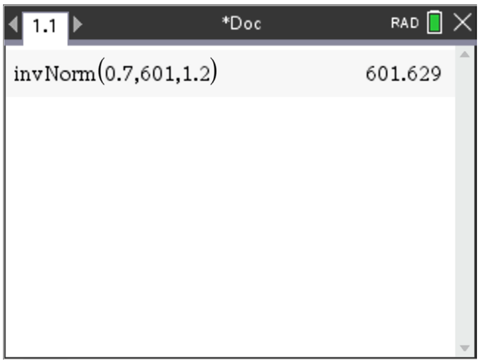
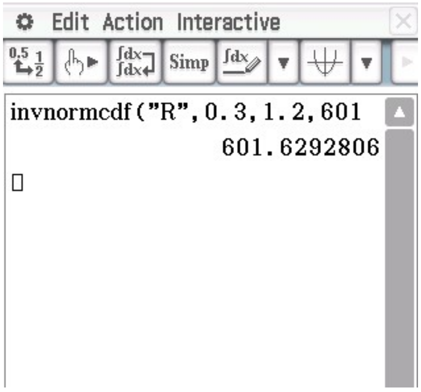
Question 4b.

Worked Solutions

$\Pr(X > a) = 0.3$

$a = 601.63$





Mark allocation: 1 mark

- 1 mark for the correct answer

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Question 4c.

Worked solution

First, the standard score for the tenth percentile must be found.

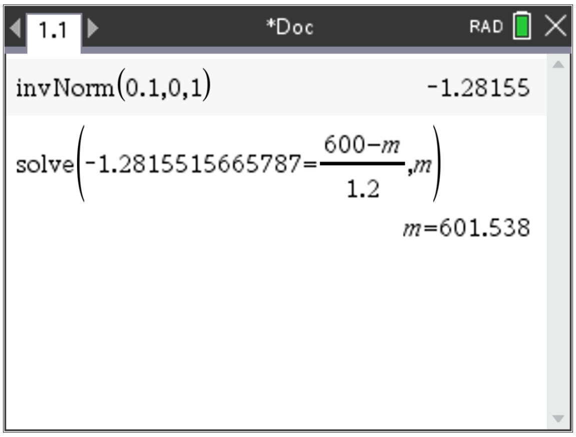
$\Pr(Z < z) = 0.1$

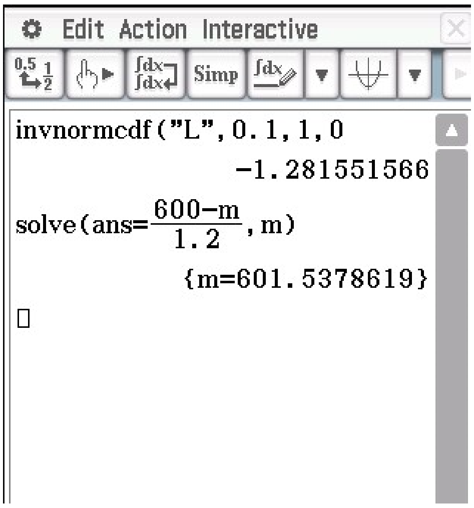
$z = -1.28155$

Then the standard score can be used to find the new mean.

$$-1.28155 = \frac{600 - m}{1.2}$$

Solving this equation gives a new mean of 601.54.





Mark allocation: 2 marks

- 1 mark for the correct method
- 1 mark for the correct answer

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Question 4d.

Worked solution

20

$Y \sim \mathrm{Bi}(200, 0.1)$

$$\begin{aligned} E(Y) &= np \\ &= 0.1 \times 200 \\ &= 20 \end{aligned}$$

Mark allocation: 1 mark

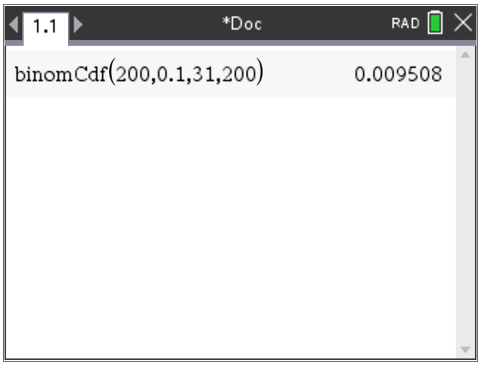
- 1 mark for the correct answer

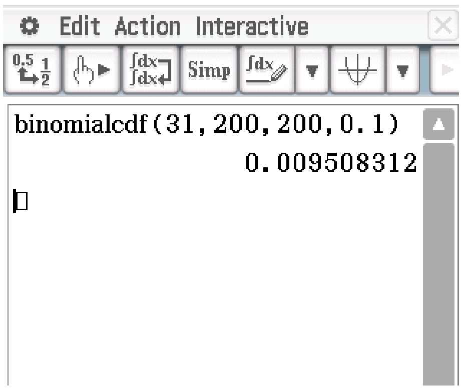
Question 4e.

Worked solution

$15\% \times 200 = 30$

$$\begin{aligned} \Pr(Y > 30) &= \Pr(Y \geq 31) \\ &= 0.0095 \end{aligned}$$





Mark allocation: 2 marks

- 1 mark for correctly determining that 15% of 200 is 30
- 1 mark for the correct answer

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Question 4f.

Worked solution

0.3

$$\begin{aligned}\Pr(R \geq 2) &= \Pr(R = 2) + \Pr(R = 3)\\ &= 0.15 + 0.15\\ &= 0.3\end{aligned}$$

Mark allocation: 1 mark

- 1 mark for the correct answer

Question 4g.

Worked solution

$\frac{1}{5}$

$$\begin{aligned}\Pr(2 \text{ in first year and } 2 \text{ in second year } 2 \mid 4 \text{ total}) &= \frac{\Pr(2 \text{ in year and } 2 \text{ in year } 2)}{\Pr(4 \text{ total})}\\ &= \frac{\Pr(2,2)}{\Pr(1,3) + \Pr(2,2) + \Pr(3,1)}\\ &= \frac{0.15 \times 0.15}{0.3 \times 0.15 + 0.15 \times 0.15 + 0.15 \times 0.3}\\ &= \frac{1}{5}\end{aligned}$$

Mark allocation: 2 marks

- 1 mark for recognising the correct conditional probability
- 1 mark for the correct answer

Question 4h.

Worked solution

0.35

$$\begin{aligned}\hat{p} &= \frac{0.2455 + 0.4545}{2}\\ &= 0.35\end{aligned}$$

Mark allocation: 1 mark

- 1 mark for the correct answer

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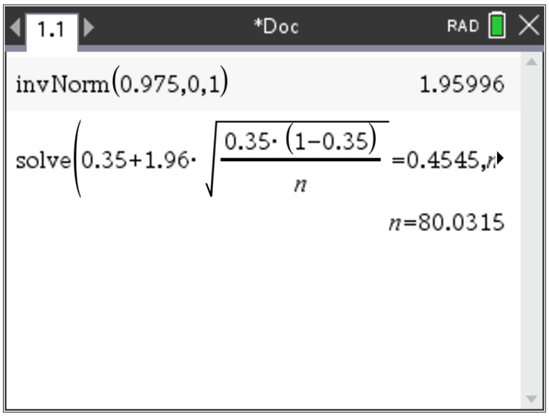
Question 4i.

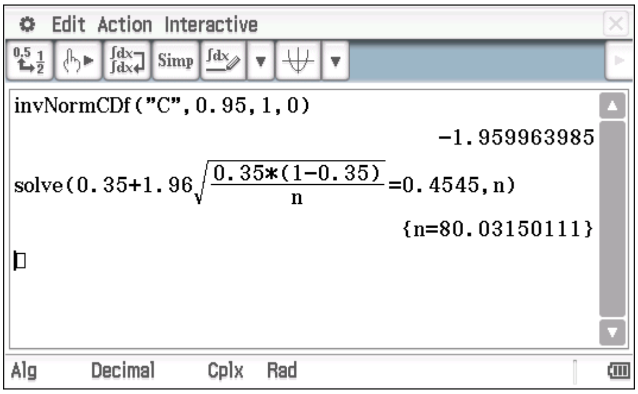
Worked solution

$$\text{upper limit} = \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\therefore\ 0.4545 = 0.35 + 1.96\sqrt{\frac{0.35(1-0.35)}{n}}$$

Solving for n gives $n = 80$.





Mark allocation: 2 marks

- 1 mark for the correct method
- 1 mark for the correct answer

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Question 5a.

Worked solution

$(0,0)$ and $\left(1, \frac{a}{e}\right)$

Mark allocation: 1 mark

- 1 mark for both correct answers

Question 5b.

Worked solution

$a \in (1, e]$

Due to the shape of the graph, we require its y-value at the endpoint, $x = 1$, to be below the graph of $y = x$ in order for the graph to intersect with its inverse, thus creating a bounded region.

Hence,

$$\frac{a}{e} \le 1$$
$$a \le e$$

Since $a \in (1, \infty)$, we have $a \in (1, e]$.

Mark allocation: 2 marks

- 1 mark for a correct method
- 1 mark for the correct answer, including the lower limit of the set

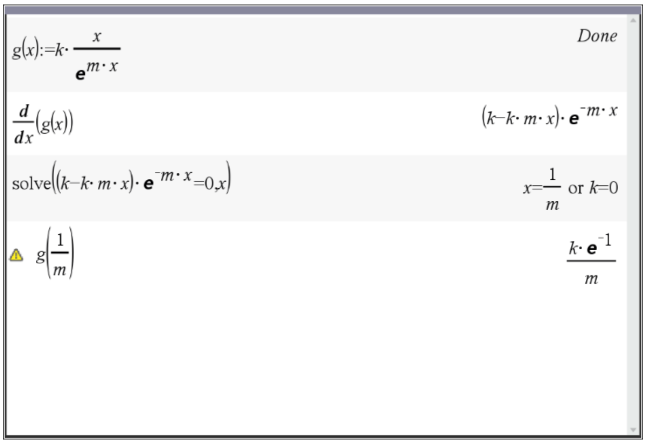
Question 5c.

Worked solution

$\left(\frac{1}{m}, \frac{k}{me}\right)$

Solving $g'(x) = 0$ for x gives $x = \frac{1}{m}$.

$$g\left(\frac{1}{m}\right) = \frac{k}{me}$$



Mark allocation: 2 marks

- 1 mark for setting the derivative of g to 0
- 1 mark for the correct coordinates

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Question 5d.

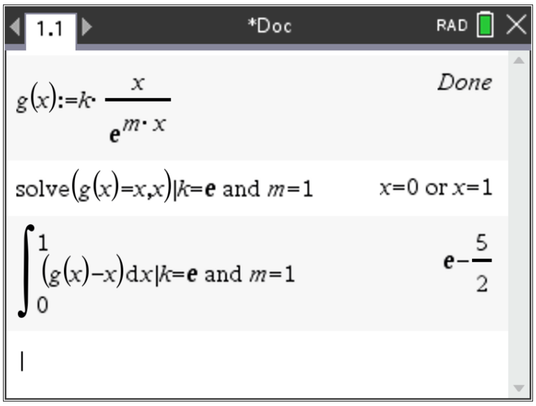
Worked solution

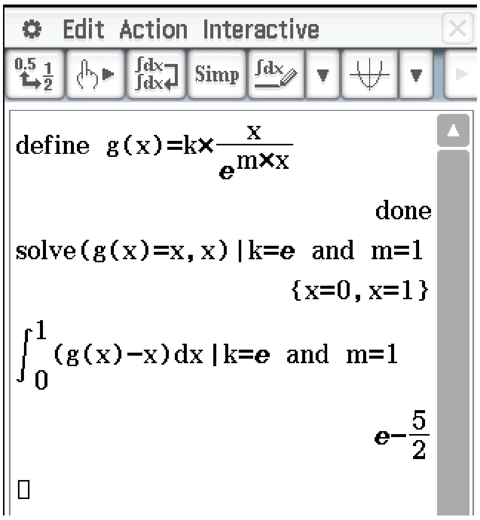
$e-\frac{5}{2}$

Solving $g(x)=x$ for x when $k=e$ and $m=1$ gives $x=0$ and $x=1$.

The graph of $y=g(x)$ is above the graph of $y=x$.

Hence, $\int_0^1 \left(g(x)-x\right)dx = e-\frac{5}{2}$ when $k=e$ and $m=1$.





Mark allocation: 2 marks

- 1 mark for finding the x-values of the two intersection points
- 1 mark for the correct answer

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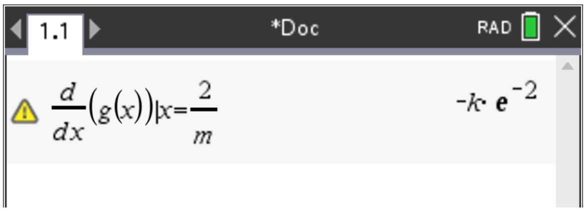
Question 5e.

Worked solution

$-\dfrac{k}{e^2}$



Mark allocation: 1 mark

- 1 mark for the correct answer

Question 5f.

Worked solution

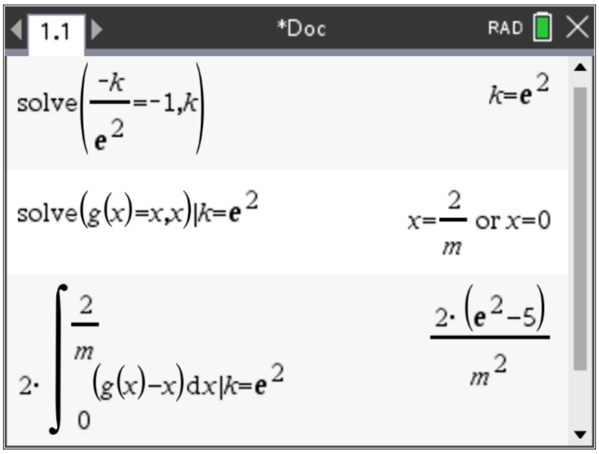
The minimum gradient is $-\dfrac{k}{e^2}$ (from **5e.**).

Solving $-\dfrac{k}{e^2}=-1$ gives $k=e^2$.

The area bound by the four graphs is twice the area bound by $y=g(x)$ and $y=x$.

Solving $g(x)=x$ gives $x=0$ and $x=\dfrac{2}{m}$.

$\displaystyle\int_0^{2/m}\left(g(x)-x\right)dx=\frac{2\left(e^2-5\right)}{m^2}$ when $k=e^2$.



Mark allocation: 3 marks

- 1 mark for finding that $k=e^2$
- 1 mark for an appropriate integral
- 1 mark for the correct answer

Tip

- *This question involves several parameters. It will be useful to sketch the graph(s) with a few different values of k and m. This will help you get a feel for the graphs that are possible and their general shape.*

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