

VCE Mathematical Methods Units 3&4

Suggested Solutions

2024 Trial Examination 1

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Question 1 (3 marks) a. $\frac{dy}{dx} = 3(5x^2 - x)^2(10x - 1)$ A1 OR $\frac{dy}{dx} = 3x^2(5x - 1)^2(10x - 1)$ A1 b. $f'(x) = e^x \times \sin(2x) + e^x \times 2\cos(2x)$ $= e^x(\sin(2x) + 2\cos(2x))$ A1 $f'(\frac{\pi}{4}) = e^{\frac{\pi}{4}}(\sin(\frac{\pi}{2}) + 2\cos(\frac{\pi}{2}))$ $= e^{\frac{\pi}{4}}(1 + 0)$ $= e^{\frac{\pi}{4}}$ A1

$$f(x) = \int \frac{4}{x} dx$$

= $4 \log_e(x) + c$ A1
 $f(1) = 2$
 $4 \log_e(1) + c = 2$
 $0 + c = 2$
 $c = 2$
 $\therefore f(x) = 4 \log_e(x) + 2$ A1

Question 3 (6 marks)

а.	When $x = 0$:
	$y = \frac{8}{(0-1)^2} - 2$
	$(0-1)^2$
	= 6
	When $y = 0$:
	8 2 2
	$\frac{8}{(x-1)^2} - 2 = 0$
	8 2
	$\frac{8}{\left(x-1\right)^2}=2$
	$(x-1)^2=4$
	$\begin{bmatrix} x - 1 = 2 \end{bmatrix} \begin{bmatrix} x = 3 \end{bmatrix}$
	$\begin{bmatrix} x - 1 = 2 \\ x - 1 = -2 \Rightarrow \begin{bmatrix} x = 3 \\ x = -1 \end{bmatrix}$
	y
	y ∧ y = 1
	7 + x = 1
	3+
	(-1, 0) (3, 0)
	-7 - 6 - 5 - 4 - 3 - 2 - 1 0 1 2 3 4 5 6 7
	<i>y</i> = -2 _3
	-4
	-5
	1

correct shape A1

correct y-intercept A1

correct x-intercepts A1

correct asymptotes with equations A1

b.
$$-\int_{5}^{7} \left(\frac{8}{(x-1)^{2}}-2\right) dx$$
 A1
$$=-\int_{5}^{7} \left(8(x-1)^{-2}-2\right) dx$$
$$=-\left[-8(x-1)^{-1}-2x\right]_{5}^{7}$$
$$=\left(\frac{8}{6}+14\right)-(2+10)$$
$$=\frac{20}{6}$$
$$=\frac{10}{3}$$
 A1

$$\hat{p} = \frac{80}{400}$$

$$= \frac{1}{5}$$
A1
95% CI = $\left(\hat{p} - x\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + x\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

$$= \left(\frac{1}{5} - 2\sqrt{\frac{\frac{1}{5} \times \frac{4}{5}}{400}}, \frac{1}{5} + 2\sqrt{\frac{\frac{1}{5} \times \frac{4}{5}}{400}}\right)$$

$$= \left(\frac{1}{5} - \frac{4}{100}, \frac{1}{5} + \frac{4}{100}\right)$$

$$= (0.16, 0.24) \text{ OR } \left(\frac{4}{25}, \frac{6}{25}\right)$$
A1

Question 5 (3 marks)

$$\cos(2x) = -\frac{1}{2}$$

$$\begin{bmatrix} 2x = \frac{2\pi}{3} + 2\pi k \\ 2x = \frac{4\pi}{3} + 2\pi k \end{bmatrix}$$

$$K \in \mathbb{Z}$$

$$x = \frac{4\pi}{3} + 2\pi k$$

$$\begin{bmatrix} x = \frac{\pi}{3} + \pi k \\ x = \frac{2\pi}{3} + \pi k \end{bmatrix}$$

$$K \in \mathbb{Z}$$

$$K = \mathbb{Z}$$

A1

A1

Question 6 (6 marks)

b.
$$\frac{f\left(\frac{\pi}{2}\right) - f\left(-\frac{\pi}{3}\right)}{\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)} = \frac{(3+1) - \left(-\frac{3\sqrt{3}}{2} + 1\right)}{\frac{5\pi}{6}}$$
 M1

$$=\frac{\frac{6+3\sqrt{3}}{2}}{\frac{5\pi}{6}}$$
$$=\frac{18+9\sqrt{3}}{5\pi}$$

c.
$$\frac{1}{\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)} \int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} (3\sin(x) + 1) dx$$
 M1

$$= \frac{1}{\frac{5\pi}{6}} \left[-3\cos(x) + x \right]_{-\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{6}{5\pi} \left[\left(0 + \frac{\pi}{2} \right) - \left(-\frac{3}{2} - \frac{\pi}{3} \right) \right]$$
A1

Question 7 (5 marks)

 $=\frac{6}{5\pi}\left(\frac{5\pi}{6}+\frac{3}{2}\right)$

 $=\frac{5\pi+9}{5\pi}$

a.		With blueberries (<i>B</i>)	Without blueberries (<i>B</i> ′)	Total
	With nuts (<i>N</i>)	4 (given)	6	10 <i>(given)</i>
	Without nuts (<i>N'</i>)	8	2	10
	Total	12	8 (given)	20 <i>(given)</i>

valid attempt to find the number of muffins that contain blueberries only M1

Pr(both blueberries only) =
$$\frac{8}{20} \times \frac{7}{19}$$

= $\frac{14}{95}$

A1

b. Pr(3 without nuts | 2 blueberries)

_ Pr(2 blueberries only and 1 neither blueberries nor nuts)		
Pr(2 blueberries and 1 without blueberries)		
$=\frac{\frac{8}{20} \times \frac{8}{20} \times \frac{2}{20} \times 3}{\frac{12}{20} \times \frac{12}{20} \times \frac{8}{20} \times 3}$	M1	
$=\frac{1}{9}$	A1	

Question 8 (6 marks)

a.
$$f'(x) = \frac{3x^2 + 6x}{x^3 + 3x^2}$$

$$= \frac{3x(x+2)}{x^2(x+3)}$$

$$= \frac{3(x+2)}{x(x+3)}$$

$$f'(x) = 0$$

$$x = -2$$

$$\begin{cases} f'(-2^-) > 0 \\ f'(-2^+) < 0 \end{cases}$$
A1

Therefore, the coordinates of the stationary point are $(-2, \log_e(4))$, and the point is a maximum. A1

b.
$$h(x) = g(f(x))$$

 $= e^{\log_e (x^3 + 3x^2)}$
 $= x^3 + 3x^2$ A1
c. $D_h = D_f = (-3, 0)$
 $R_f = (-\infty, \log_e(4)]$
 $e^{-\infty} \to 0$
 $e^{\log_e(4)} = 4$
 $R_h = (0, 4]$ A1

Question 9 (6 marks)

a. Swapping the values of *x* and *y* gives:

Swapping the values of x and y gives.

$$x = \sqrt{8y - y^{2}}$$

$$x^{2} = 8y - y^{2}$$

$$y^{2} - 8y + x^{2} = 0$$
M1
Solving for y gives:

$$y_{1} = \frac{8 - \sqrt{64 - 4x^{2}}}{2}$$

$$= 4 - \sqrt{16 - x^{2}} \le 4$$

$$y_{2} = \frac{8 + \sqrt{64 - 4x^{2}}}{2}$$

$$= 4 + \sqrt{16 - x^{2}} \ge 4$$
Considering $D_{f} = R_{f} - 1$:

$$f^{-1}(x) = 4 - \sqrt{16 - x^{2}}$$
A1

b.
$$y = \sqrt{8x - x^2}$$

 $y^2 = 8x - x^2$
 $x^2 - 8x + 16 + y^2 = 16$
 $(x - 4)^2 + y^2 = 16$ M1
c. $\operatorname{area} = \frac{4 - 0}{2} (f(0) + 2f(2) + f(4))$ M1

$$area = \frac{1}{2} (7(0) + 27(2) + 7(4))$$

$$= 0 + 2\sqrt{12} + 4$$

$$= 4\sqrt{3} + 4$$
A1

d. The value in **part c.** approximates the area of a quarter circle with radius 4.

$$\frac{\pi \times 4^2}{4} \approx 4\sqrt{3} + 4$$
$$\pi \approx \sqrt{3} + 1$$
A1

Note: Consequential on answer to Question 9c.