NAME: ______________________________

VCE® Mathematical Methods

UNITS 3 & 4 Practice Written Examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	**20**	**20**	**20**
B	**5**	**5**	**60**
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, an approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 25 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions
- Write your **student name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is **correct** or that **best answers** the question. A correct answer scores 1; an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers. No marks will be given if more than one answer is completed for any question. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The period and amplitude of $f(x) = 2\cos cos\,(3x - 2) + \pi$ are respectively:

A. $3, -2$

B. $\frac{2\pi}{3}, 2$

C. $\frac{3}{2\pi}, 2$

D. $3, 2$

E. $\frac{2\pi}{3}, -2$

Question 2

The range of $g: R \rightarrow [-2, 4], g(x) = x^2 + 2x - 8$ is:

A. $[0, 16]$

B. $(0, 16)$

C. $[-1, 8]$

D. $[-9, 16]$

E. $(0, 4]$

F.

Question 3

The equation of the normal to $y = x(x - 2)^2$ at $x = 0$ is:

A. $x = 0$

B. $y = 0$

C. $y = 4x$

D. $y = \frac{1}{4}x$

E. $y = -\frac{1}{4}x$

Question 4

The instantaneous rate of change of $f(x) = x^2 + bx + c,\ b, c \in R$ at $x = 1$ is 4.

The average value of $f(x)$ between $x = 1$ and $x = 3$ is $\frac{34}{3}$. $f(2)$ is therefore:

A. -11

B. -3

C. 3

D. 4

E. 11

Question 5

Consider the system of simultaneous linear equations below:

$mx + 3y = 6$

$2x + (m - 1)y = 2$

Where $m \in R$

What are the values of the value(s) of *m* for which the system of equations has no solutions?

A. $m \neq -2$

B. $m \neq 3$

C. $m \in \{-2, 3\}$

D. $m \in R \backslash \{-2, 3\}$

E. $m = 3$

Question 6

A bag contains four red balls, five blue balls and three green balls. Three balls are randomly selected, with replacement.

What is the probability that at least two of the balls selected were blue, given that at least one of them was blue?

A. 0.4693

B. 0.3762

C. 0.8015

D. 0.3015

E. 0.5855

Question 7

The probability that a team wins a game is 0.30.

How many games would the team need to play so the probability that it won at least one game was greater than 0.9?

A. 3

B. 5

C. 7

D. 8

E. 11

Question 8

A probability density function is shown below:

$$f(x) = \{ \frac{1}{\sqrt{x}} \qquad 2 \le x \le a \ 0 \qquad elsewhere$$

What is the value of a?

A. $\frac{4\sqrt{2}+9}{4}$

B. $\frac{1}{4}$

C. $2\sqrt{2} + 3$

D. $\frac{9-4\sqrt{2}}{4}$

E. $4 - \sqrt{2}$

Question 9

The Mathematical Methods scores at a particular school are normally distributed with a mean of 65.

If 50% of the students achieved a result within ten marks of the mean, the standard deviation of this distribution is closest to:

A. 3.85

B. 6.74

C. 13.49

D. 14.83

E. 20

Question 10

The time required for a person to set up a tent is normally distributed with a mean of 8.5 minutes and a standard deviation of 2.0 minutes.

What is the probability that in a group of three people, each will set up their tent in under 7 minutes?

A. 0.0116

B. 0.0755

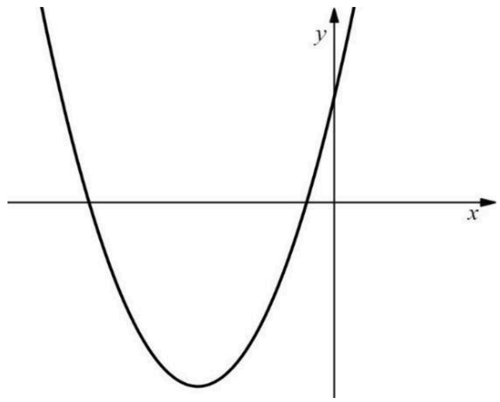
C. 0.2266

D. 0.5374

E. 0.6798

Question 11

The graph of $f(x)$ is shown below. $f(x)$ is the derivative of $F(x)$.



A possible rule for $F(x)$ is:

A. $F(x) = (x - 1)(x - 3)^2$

B. $F(x) = (x + 1)(x - 3)^2$

C. $F(x) = -(x - 1)(x + 3)^2$

D. $F(x) = (x - 1)(x + 3)^2$

E. $F(x) = -(x - 1)(x - 3)^2$

Question 12

Consider the equation shown below:

$$f(x) = ax^3 + bx^2 + cx \quad a, b, c \in R\backslash\{0\}$$

Given $f(x)$ has two x-intercepts and a turning point located at $\left(\frac{4}{3}, 256\right)$, the values of a, b and c are:

A. $a = -27,\ b = 216,\ c = -432$

B. $a = 27,\ b = 216,\ c = 432$

C. $a = 27,\ b = -216,\ c = 432$

D. $a = -27,\ b = -216,\ c = -432$

E. $a = 27,\ b = -216,\ c = -432$

Question 13

$f(x) = a\sqrt{x - 4} \quad a \in R$

$g(x) = \frac{1}{10}x^2 - 1$

Given $f(x)$ and $g(x)$ join smoothly at a point, P, what is the value of a correct to two decimal places?

A. $a = 1.53$

B. $a = 1.44$

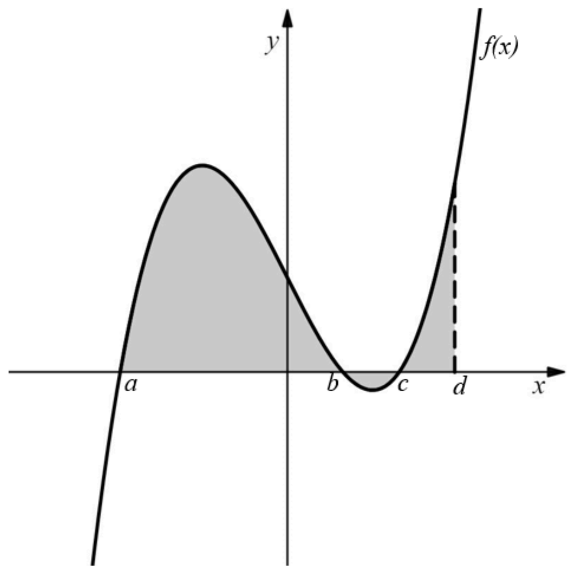
C. $a = 4.61$

D. $a = -4.61$

E. $a = -1.44$

Question 14

Part of the graph of $f(x)$ is shown below.



The total area of the shaded sections could be obtained by evaluating which one of the following definite integrals?

A. $-\int_b^a f(x)dx + \int_b^c f(x)dx + \int_c^d f(x)dx$

B. $-\int_b^c f(x)dx - \int_c^d f(x)dx - \int_b^a f(x)dx$

C. $\int_a^b f(x)dx - \int_b^c f(x)dx - \int_c^d f(x)dx$

D. $\int_a^d f(x)dx - \int_b^c f(x)dx$

E. $-\int_b^c f(x)dx + \int_c^d f(x)dx - \int_b^a f(x)dx$

Question 15

Consider $f(x) = 2\cos cos\,(4x) + 1$

If $g(x)$ is obtain by reflection of $f(x)$ about the *x*-axis, followed by dilation by a factor of 2 from the *y*-axis, followed by a translation of $\frac{\pi}{2}$ units to the right, what is the equation of $g(x)$?

A. $g(x) =- 2\cos cos\left(2x - \frac{\pi}{2}\right) + 1$

B. $g(x) =- 4\cos cos\left(4x - \frac{\pi}{2}\right) - 1$

C. $g(x) =- 2\cos cos\left(2\left(x - \frac{\pi}{2}\right)\right) - 1$

D. $g(x) =- 4\sin sin\,(4x) - 1$

E. $g(x) = 2\sin sin\,(2x) + 1$

Question 16

Consider two functions, $f(x)$ and $g(x)$. Given $f(x) = x^2 - 2x + 1$ and $g(x) = 2x - 1$, which of the following is true?

A. $2f(g(0)) < g(f(-1))$

B. $f(g(-1)) > g(f(-1)) > f(g(0))$

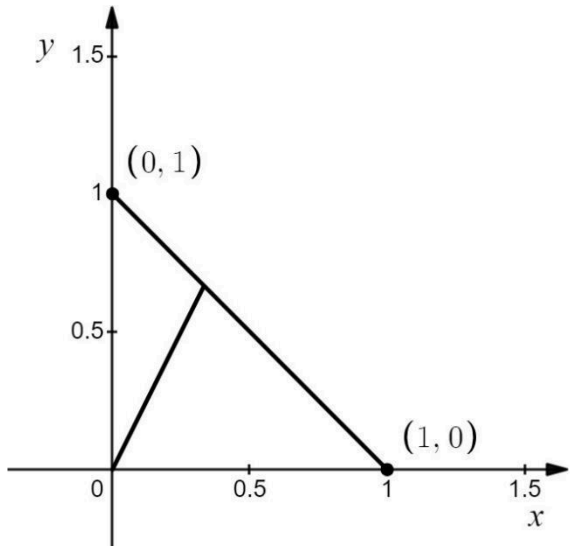
C. $g(f(3)) > f(g(0)) > f(g(2))$

D. $f(g(-1)) = g(f(-1))$

E. $f(g(0))\times f(g(2)) < f(g(-1))$

Question 17

A line segment, l_1, has endpoints located at (0,1) and (1,0). A second line segment, l_2, has one endpoint at the origin and the other endpoint at the point of intersection with l_1. This is shown in the figure below.



What is the average length of l_2?

A. $\frac{\sqrt{2}}{8}\left(3 + 2\sqrt{2}\right) + \frac{1}{2}$ $units$

B. $\frac{\sqrt{2}}{2}$ $units$

C. $\frac{1}{2}$ $units$

D. $\frac{\sqrt{2}}{4}\left(3 + 2\sqrt{2}\right) + \frac{\sqrt{2}}{2}$ $units$

E. $\frac{\sqrt{2}}{4}\left(3 + 2\sqrt{2}\right) - \frac{1}{2}$ $units$

Question 18

A preliminary survey showed that 8 out of 11 people preferred the proposed new colour scheme for the school. A larger survey was then performed, and the width of the 95% confidence interval for the population proportion was 0.114.

Rounded up to the nearest person, how many people were surveyed in this larger survey?

A. 41

B. 59

C. 118

D. 164

E. 235

Question 19

A confidence interval for the population proportion for a survey of 542 people was $(0.642,\ 0.708)$. What was the value of Z?

A. 1.28

B. 1.44

C. 1.64

D. 1.96

E. 2.58

Question 20

For which of the following pairs of functions is $f \circ g$ defined but $g \circ f$ not defined?

A. $f(x) = \sqrt{x},\ g(x) = tan(x)$

B. $f(x) = (x),\ \ g(x) = x^2$

C. $f(x) = e^x, g(x) = cos(x)$

D. $f(x) = x^{-2}, g(x) = (x)$

E. $f(x) = \sin sin\,(x)\,,\ g(x) = \sqrt{x}$

SECTION B – Short Answer Questions

Question 1 (12 marks)

A family of curves is defined by the equation:

$$f(x) = a\sqrt{x} + \frac{b}{x}, \;\; a, b \in R^{+}$$

a. What is the implied domain of $f(x)$? 1 mark

b. Show that $f(x)$ has no x-intercepts. 2 marks

c. Consider the case where $a = 1$ and $b = 1$.

(i) Determine the exact value of the x-coordinate of the turning point. 1 mark

(ii) Show that the turning point is local minimum. 1 mark

(iii) Determine the exact value of the y-coordinate of the turning point. 1 mark

(iv) What is the equation of the vertical asymptote? 1 mark

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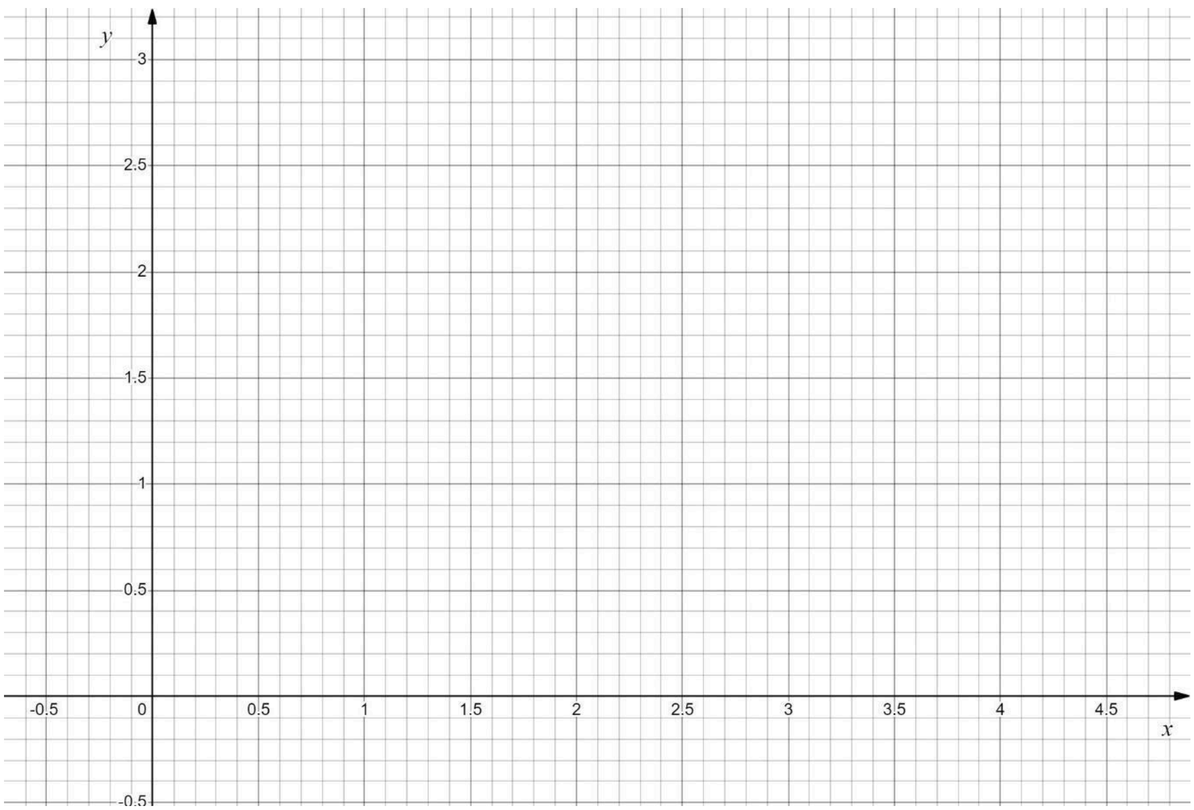
(v) Use addition of ordinates and the information developed in the preceding parts to sketch the graph of:

$$g: R \to (0, 4],\ g(x) = \sqrt{x} + \frac{1}{x}$$

Include the equations of any horizontal and vertical asymptotes, as well as the coordinates of any axes intercepts and turning point. 3 marks



d. Under what conditions will the turning point of $f(x)$ always occur at $x = 2$? 2 marks

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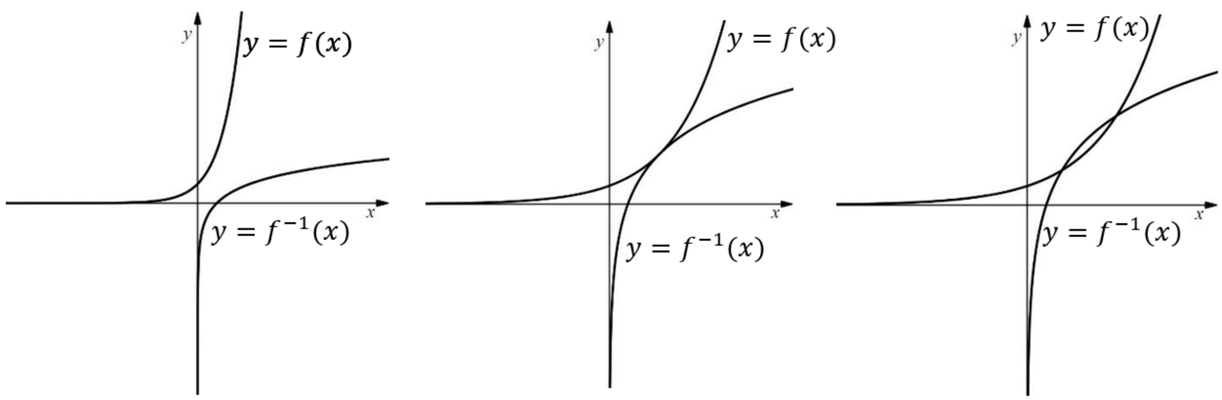
Question 2 (12 marks)

Consider the function $f(x) = e^{bx}, b \in R^{+}$

a. Determine the rule for $f^{-1}(x)$, the inverse of $f(x)$. State the domain and range of $f^{-1}(x)$. 2 marks

The graphs of $f(x)$ and $f^{-1}(x)$ may intersect at 0, 1 or 2 points, depending upon the value of b. These scenarios are shown in the graphs below.



b. What is the gradient of $f(x)$ for when $f(x)$ and $f^{-1}(x)$ intersect once? 1 mark

c. Hence determine the point of intersection of $f(x)$ and $f^{-1}(x)$ for when $f(x)$ and $f^{-1}(x)$ intersect once only. 2 marks

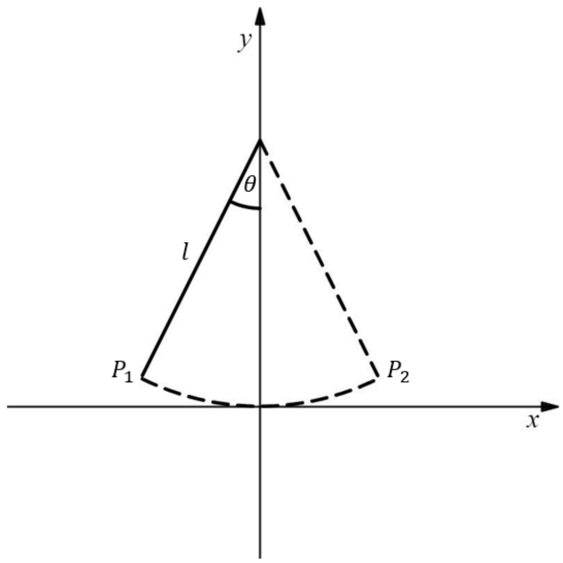
d. Determine the values of b for when $f(x)$ and $f^{-1}(x)$ do not intersect and intersect at 2 points. 2 marks

e. Determine the minimum distance between $f(x)$ and $f^{-1}(x)$, in terms of b, under the condition that $f(x)$ and $f^{-1}(x)$ do not intersect. Hint: use properties of inverses. 3 marks

f. Hence determine the value of b for when this length is a maximum and state the value of this length. 2 marks

Question 3 (15 marks)

A swing of length, l (meters), is shown in the diagram below. When at rest, the swing just touches the ground, which corresponds to the origin in the diagram. The swing is released from an angle, $\theta \left(0 < \theta < \frac{\pi}{2}\right)$, also shown in the diagram. The seat of the swing is initially at P_1, When the swing is released, the seat moves to P_2, tracing an arc of radius l. It then returns to P_1. This motion repeats indefinitely. The time, t, in seconds, required for the seat to move from P_1 to P_2 and back again is referred to as the period.



The period, P (seconds), is given by:

$$P = 2\pi\sqrt{\frac{l}{10}}$$

a. For a given angle, θ, what is the initial height of the swing seat above the ground? 1 mark

b. The height, h (meters), of the seat above the ground, as a function of time, can be modelled by the equation:

$$h(t) = a \cos cos\,(nt) + k, \quad a, n, k \in R$$

a, n and k are model parameters that depend upon l and θ.

(i) Determine the value of a in the model. 1 mark

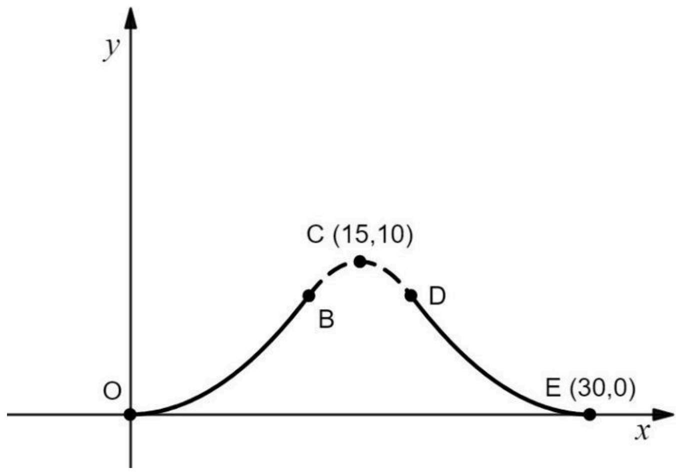
(ii) Determine the value of n in the model. 1 mark

(iii) Determine the value of k in the model. 1 mark

(iv) Hence, express the model in terms of l and θ and t. 1 mark

(v) For a particular swing, l is 5 metres. If the angle of the tangent to the arc described by the swing seat at t = 0 is $\frac{5\pi}{6}$ radians. Determine the initial height of the swing above the ground. 2 marks

c. A roof is to be constructed above the swing. The diagram below shows the profile of the roof.



All distances are in meters. The profile is to be modelled in three sections. The sections are continuous and smooth where they join at points B and D. All sections can be modelled by quadratic functions with turning points located at (0.0), $(15, 10)$ and $(30, 0)$. The profile is symmetrical about the line $x = 15$.

Let a be the dilation factor of the end sections and b be the dilation factor of the middle section.

(i) What is the equation for the section between points O and B? 1 mark

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(ii) What is the equation for the section between points B and D? 1 mark

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(iii) Express a in terms of b such that two of the sections join smoothly at point B. Express the relationship in the form $a = f(b)$. 3 marks

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(iv) If $b = -\frac{1}{5}$, construct a hybrid function, called $p(x)$, that models the profile of the roof from points O to E, inclusive. Include relevant subdomains and any parameter values. 3 marks

Question 4 (14 marks)

a. The time required for a group to finish their booking at a particular restaurant follows a normal distribution with a mean of 45 minutes and a standard deviation of 10 minutes. Round all probabilities to four decimal places.

(i) Determine the probability, that a group will finish their booking in less than 30 minutes. 1 mark

(ii) Determine the probability that of the four groups about to start their booking, at least three will take more than 60 minutes. 2 marks

(iii) When a group finishes a booking at a particular table, it takes 5 minutes to prepare the table for the next booking. The restaurant is open for 4 hours. Determine the probability that 6 groups will be able to complete their booking at a particular table. 3 marks

b. The waiters of the restaurant receive tips for their service. The total amount of tips, X (dollars), a particular waiter receives in an evening follows a probability density function given by:

$$f(x) = \{ax^{\frac{1}{3}} \quad 0 \le x \le c \; 0 \qquad elsewhere$$

The probability that the waiter receives at most \$50 in tips in an evening is 0.4605. The probability that the waiter receives between \$30 and \$70 in tips in an evening is 0.4882.

(i) Determine the value of the parameter a to six decimal places, and hence determine the maximum amount the waiter can receive in tips in an evening, rounded to the nearest dollar. 2 marks

(ii) One evening the waiter received an amount in tips equal to the mean value, while on a different evening the amount received was equal to the median value. What 3 marks is the magnitude of the difference between what was received on these evenings? Round the answer to the nearest 10 c.

c. The owner of the restaurant wanted to determine the level of customer satisfaction with the menu. Of the 16 people questioned in a preliminary survey, 11 indicated that they were satisfied with the menu.

(i) What is the value of $\hat{p}$? 1 mark

(ii) The owner wanted to construct a 95% confidence interval of width 0.10. How many people should be surveyed for this condition to be achieved? Round your answer to the nearest ten people. 2 marks

Question 5 (7 marks)

Consider the equations:

$p(x) = e^{x}$

$q(x) = x^{3}$

The *x*-coordinate, a, of a point of intersection of $p(x)$ and $q(x)$ is located within the interval $1 < x < 2$. $p(x) > q(x)$ when $x < a$ and $p(x) < q(x)$ when $a < x < 2$.

The following pseudocode approximates the point of intersection of $p(x)$ and $q(x)$ for the x-intercept located within the interval $1 < x < 2$.

define $f(x)$:
  return $e^{x} - x^{3}$
$m \leftarrow 1$

$n \leftarrow 2$

$c \leftarrow 1$

while $n - m > 0.001$
$b \leftarrow \frac{n+m}{2}$

  if $f(m) \times f(b) < 0$ then
    $n \leftarrow b$
  else
$m \leftarrow b$

  end if
$c \leftarrow c + 1$

end while
print $b,\ f(b),\ c$

a. What is the purpose of the variable *c*? 1 mark

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b. Perform a desk check by completing the table below (provide exact value or round to three decimal places for non-terminating decimals). 3 marks

c	m	n	$n - m$	b	$f(b)$
1					
2					
3					

c. What will be the value of c when the condition to end the while command is satisfied? 1 mark

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d. Modify the pseudocode, by completing the table below, so that it approximates the other point of intersection of $p(x)$ and $q(x)$ and that the accuracy of the approximation is increased by a factor of ten.

If a line of the code is unchanged, write the letter U in the adjacent cell. 2 marks

Original Pseudocode	Modified Pseudocode
define $f(x)$:	
return $e^x - x^3$	
$m \leftarrow 1$	
$n \leftarrow 2$	
$c \leftarrow 1$	
while $n - m > 0.001$	
$b \leftarrow \frac{n+m}{2}$	
if $f(m) \times f(b) < 0$ then	
$n \leftarrow b$	
else	
$m \leftarrow b$	
end if	
$c \leftarrow c + 1$	
end while	
print $b,\ f(b),\ c$	

END OF EXAMINATION

VCE Mathematical Methods Units 3&4 Multiple-Choice Answer Sheet

NAME: ___________________________________

Question				
1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D
11	A	B	C	D
12	A	B	C	D
13	A	B	C	D
14	A	B	C	D
15	A	B	C	D
16	A	B	C	D
17	A	B	C	D
18	A	B	C	D
19	A	B	C	D
20	A	B	C	D

