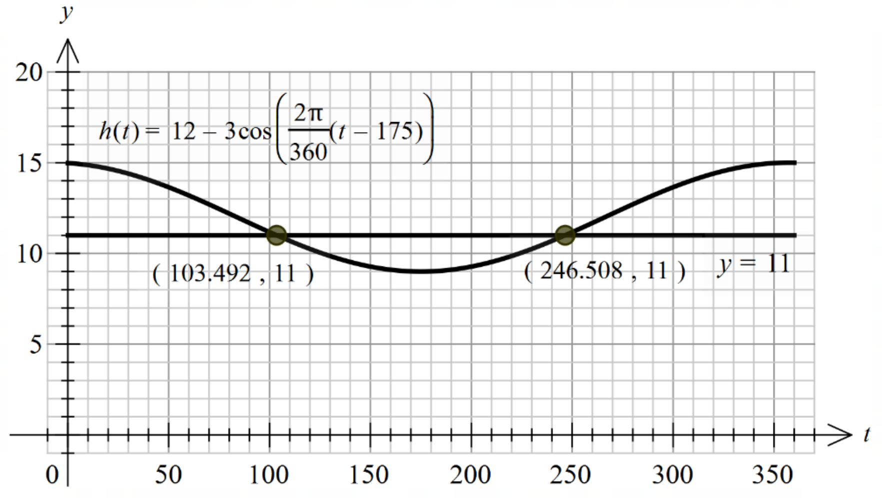
2024 VCE Mathematical Methods 2 (NHT) external assessment report

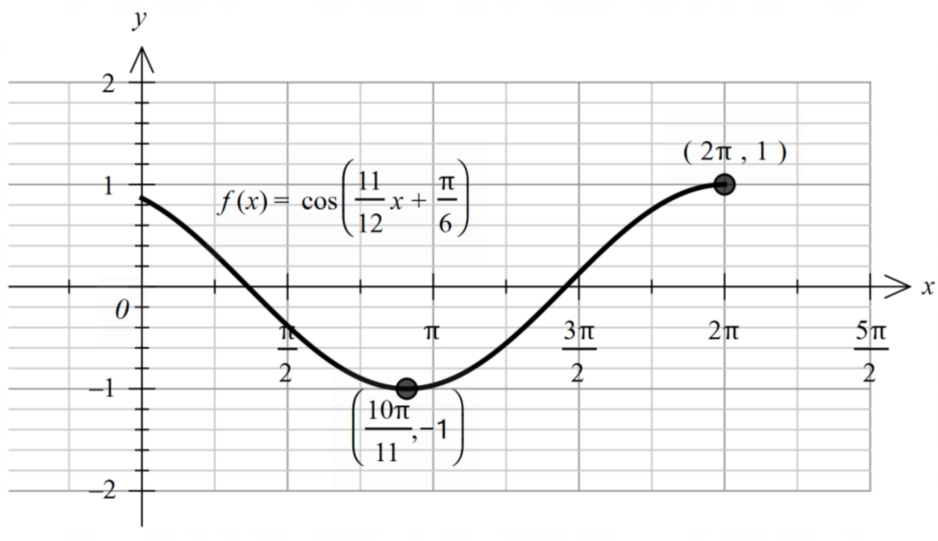
Specific information

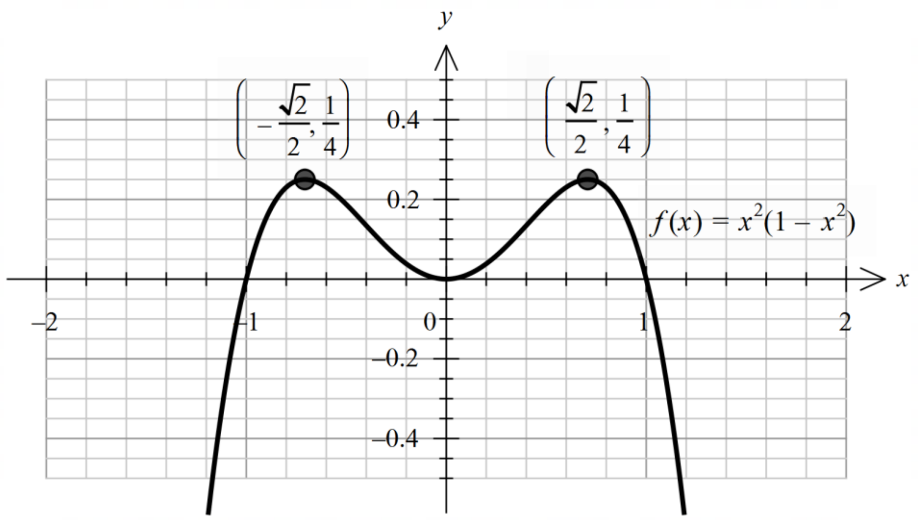
This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Section A – Multiple-choice questions

Question	Correct answer	Comments
1	C	
2	A	
3	D	
4	A	
5	B	
6	C	
7	B	
8	B	
9	D	
10	A	
11	E	$h(t)=12-3\cos\left(\frac{2\pi}{365}(t-175)\right)$ where $t \in Z^+$. The number of days with at least 11 hours of daylight during one year, 365 days, is $103+(365-246)=222$ days. The graphs of $y=h(t)=12-3\cos\left(\frac{2\pi}{365}(t-175)\right)$ where $t \in R$ and $y=11$ are shown below.

Question	Correct answer	Comments
		
12	C	
13	D	$A = -\int_{-1}^{2} g(x)dx$, as $g(x) < 0$ $= -\left[f(x)\right]_{-1}^{2}$, as $f'(x) = g(x)$ $= -f(2) + f(-1)$ $= 1 + 2$ $= 3$
14	D	The algorithm stops when $-0.01 < \cos(x) - x < 0.01$. $x = 0.6$, $\cos(0.6) - 0.6 = 0.2253... > 0.01$ $x = 0.61$, $\cos(0.61) - 0.61 = 0.2096... > 0.01$ Check alternatives $x = 0.73$, $\cos(0.73) - 0.73 = 0.0151... > 0.01$ $x = 0.74$, $\cos(0.74) - 0.74 = -0.0015... > -0.01$ Since $\cos(0.74) - 0.74 > -0.01$, $x = 0.74$
15	D	$C(n) = 0.5n^2 - 37.5n + 1900 + \frac{1250}{n}$, $n \in Z^+$ Solve $C'(n) = 0$, $n = 38.34...$, but n is discrete. $C(38) = 1229.89...$, $C(39) = 1230.05...$ The minimum cost is closest to $\$1229.89$.
16	A	$\frac{d}{dx} g(f(x))$ $= g'(f(x)) \times f'(x)$ $= g'(f(0)) \times f'(0)$ $= g'(2) \times f'(0)$ $= 3 \times -1$ $= -3$

Question	Correct answer	Comments
17	D	$\Pr(A\|B)=1$ $\frac{\Pr(A\cap B)}{\Pr(B)}=1$ $\Pr(A\cap B)=\Pr(B)$, $B\subseteq A$ $\Pr(C\|B)=\Pr(C)$ $\frac{\Pr(C\cap B)}{\Pr(B)}=\Pr(C)$ $\Pr(C\cap B)=\Pr(B)\times\Pr(C)$, B and C are independent.
18	D	$f:[0,2\pi]\to R,\ f(x)=\cos\left(ax+\frac{\pi}{6}\right)$, $f(0)=\frac{\sqrt{3}}{2}$ To get the smallest value of a such that there is a unique solution to $f(x)=1$ and a unique solution to $f(x)=-1$, $f(x)$ must equal 1 when $x=2\pi$ and only at $x=2\pi$. Solve $\cos\left(2\pi a+\frac{\pi}{6}\right)=1$, $a=\frac{12n-1}{12}$, $n\in Z^+$, $a=\frac{11}{12}$ is the smallest value. This value can also be found using the slider functionality on CAS. The graph of $y=f$ when $a=\frac{11}{12}$ is shown below. 
19	C	$(x^2-x-2)f'(x)>0$ $f'(x)>0$ when $\{x:-1<x<3\}$, $x^2-x-2>0$ when $\{x:x<-1\}\cup\{x:x>2\}$ So $(x^2-x-2)f'(x)>0$, when $\{x:2<x<3\}$. $f'(x)<0$ when $\{x:x<-1\}\cup\{x:x>3\}$, $x^2-x-2<0$ when $\{x:-1<x<2\}$ So $(x^2-x-2)f'(x)>0$, when $\{x:2<x<3\}$ only.
20	E	$f:R\to R,\ f(x)=x^2(1-x^2)$ and $g(x):[0,1]\to R, g(x)=f(x-a)$ a translates the graph of f horizontally.

Question	Correct answer	Comments
		For $g(0)$ to be an absolute maximum, the graph of f has to be translated at least $\frac{\sqrt{2}}{2}$ units to the left. So $a \in \left(-\infty, -\frac{\sqrt{2}}{2}\right]$. The maximum value the graph of f can be translated to the right is $\frac{\sqrt{2}}{2}$ units. The minimum value occurs when $g(0) = g(1)$ which is when $a = \frac{1}{2}$. Hence, $a \in \left(-\infty, -\frac{\sqrt{2}}{2}\right] \cup \left[\frac{1}{2}, \frac{\sqrt{2}}{2}\right]$. These values can also be found using the slider functionality on CAS. Part of the graph of $y = f$ is shown below. 

Section B

Question 1a.

$x = 0,\ x = \pm\sqrt{3}$

Question 1b.

Solving $f'(x) = 0$ or $3x^2 - 3 = 0$

$(-1, 2)$ and $(1, -2)$

Question 1ci.

$p = 0$

Question 1cii.

$p < 0$

Question 1di.

$f'(x) = 3x^2 - p$ gives $f'(0) = -p$

The tangent line is given by $y = -px + c$, where $c = 0$ as it goes through the origin

Therefore $y = -px$

Question 1dii.

$$A = \int_0^p \left(f(x) + px \right) dx$$

$$= \frac{p^4}{4}$$

Question 1diii.

$k = \pm 1$

Question 2a.

$\frac{5}{4}, 1$

Question 2b.

$h(\theta) < 1.1$

$-0.64 < \theta < 0.64$ or $(-0.64, 0.64)$

Question 2c.

$\theta(0)=\frac{\pi}{3}\cos(0)=\frac{\pi}{3}$ (Point A)

Question 2d.

$\frac{4\pi}{9}$

Question 2e.

22 times

Question 2f.

$\frac{\pi}{3}$

Question 2gi.

$h(\theta(1))$

$=h(-0.2207...)$

$=1.01$

Question 2gii.

$h(\theta(t))=\frac{3}{2}-\frac{1}{2}\cos\left(\frac{\pi}{3}\cos\left(\frac{9t}{2}\right)\right)$

Question 2giii.

$\frac{1}{30}\int_{0}^{30} h(\theta(t))\,dt$

$=1.13$

Question 3a.

2.1

Question 3b.

$\frac{1}{7}$

Question 3c.

$X \sim \text{Bi}(10, 0.1)$

$\Pr(X \geq 3) = 0.0702$

Question 3di.

10

Question 3dii.

4

Question 3diii.

$\sigma = 2$

$\Pr(0 < C < 8) = \frac{608}{625}$ or 0.9728

Question 3ei.

$\frac{3}{20}$ or 0.15

Question 3eii.

$(0.108, 0.192)$

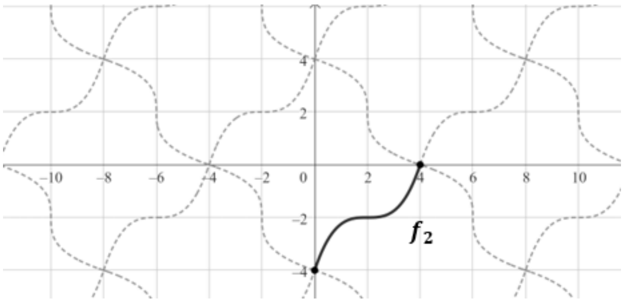
Question 3f.

$\hat{p} = 0.1375 = \frac{11}{80}$

$n = 80$

Question 4a.

The graph must be drawn and labelled.



Question 4b.

$b=-2$, $c=2$

$a=\frac{1}{4}$

Question 4c.

$f(x-4)+4=\frac{1}{4}(x-2)^3+6,\ 0<x\le 4$

OR

$f(x+4)-4=\frac{1}{4}(x+6)^3-2,\ -8\le x<-4$

Question 4d.

$f^{-1}(x)=\sqrt[3]{4(x-2)}-2=2^{\frac{2}{3}}(x-2)^{\frac{1}{3}}-2$

Domain $[0,4]$

Question 4e.

$m=1,\ n=-1$

Question 4f.

$A=4\int_{-4}^{0} f(x)dx$ or $A=4\sqrt{2}\times 4\sqrt{2}$ (area of square)

$=32$

Alternative methods were possible.

Question 4g.

$f'(-4)=3,\ g'(-4)=-\frac{1}{3}$

$m_1\times m_2=-1$, angle is $90°$

Question 4h.

Some of the equations are shown below. There are other possibilities.

$y = f\left(x \pm 8k\right) = \frac{1}{4}\left(x + 2 \pm 8k\right)^3 + 2$ LHS upper

$y = f\left(x \pm 4 \pm 8k\right) - 4 = \frac{1}{4}\left(x + 6 \pm 8k\right)^3 - 2$ RHS lower

$y = -g\left(-x \pm 8k\right) = g\left(x \pm 4 \pm 8k\right) + 4$ RHS upper

$= \sqrt[3]{-4(x - 2 \pm 8k)} + 2$

Question 5a.

Inverse: $x = e^{-ey}$

$f^{-1}(x) = -\frac{1}{e}\log_e(x)$

Domain of f^{-1} is range of f which is $\left(0, \infty\right)$.

Question 5b.

$$A = \int_0^{\frac{1}{e}} f(x)dx + \int_{\frac{1}{e}}^{1} f^{-1}(x)dx$$

$$= \frac{2e - 3}{e^2}$$

Question 5ci.

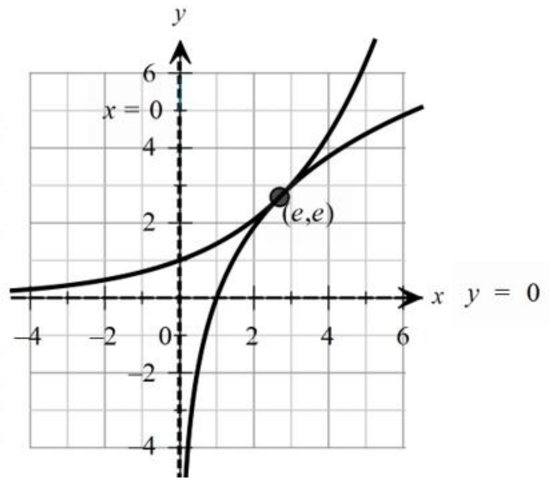
$y = x$

Question 5cii.

The graphs must be drawn correctly.

Asymptotes must be labelled with their equations.

The coordinates of the point of intersection must be shown with exact values.



Question 5di.

$g_2^{-1}(x) = -\frac{1}{4}\log_e(x)$

Question 5dii.

$x = 0.028,\ x = 0.301,\ x = 0.894$

Question 5e.

Number of points	Value(s) of a
0	$a > \frac{1}{e}$
1	$[-e, 0) \cup \{e^{-1}\}$
2	$0 < a < \frac{1}{e}$ given
3	$a < -e$