

**STUDENT NUMBER**

Letter

Figures


Words

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# MATHEMATICAL METHODS (CAS)

## Written examination 1

Created by James Hancock, 2012

### QUESTION AND ANSWER BOOK

#### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	38

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

#### Materials supplied

- Question and answer book of 13 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

#### Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

### Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

### Question 1

a. Differentiate  $x^3 e^{2x}$  with respect to  $x$ .

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1 mark

b. Let  $f(x) = \log_e(\tan(2x))$ . Find  $f'(\frac{\pi}{8})$ .

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2 marks

**Question 2**

a. Find an antiderivative for  $12xe^{3x^2}$ .

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1 mark

b. Solve  $2\log_e \sqrt{x-2} + \log_e(x-4) = 1$  giving exact answers.

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3 marks



#### Question 4

**a.**

Given that  $f(x) = \frac{1}{2x^3} - 2$ , find  $g(x) = f\left(\frac{1}{x}\right)$ .

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2 marks

**b.**

Find where  $g(x) = 0$ .

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1 mark

### Question 5

a.

Thomas drives to school each day and has to suffer through red lights. The probability that he will be stopped at a certain number of red lights is given in the discrete table below.

<b>Pr(X=x)</b>	m	0.2	0.4	0.15	k
<b>x</b>	0	1	2	3	4

Find k and m given that the expected number of lights he stops at is 2.05.

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2 marks

b.

Find the probability that he stops at the number of lights he has to stop at in 2 consecutive days adds to 3.

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2 marks

**Question 6**

**a.**

Brad has accidentally dropped his ice cream on the ground in a spherical blob. Unfortunately, it's a hot day and the ice cream begins to melt at a rate of  $\frac{2T}{5} \text{ mm}^3/\text{s}$ , where T is the outside temperature.

Assuming that the blob of ice cream remains spherical while melting, find the rate at which the radius changes in terms of T and r.

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2 marks

**b.** Given that it is 25°C on this day, find the rate at which the radius is decreasing when the volume of the ice cream is  $32\pi/3 \text{ mm}^3$ .

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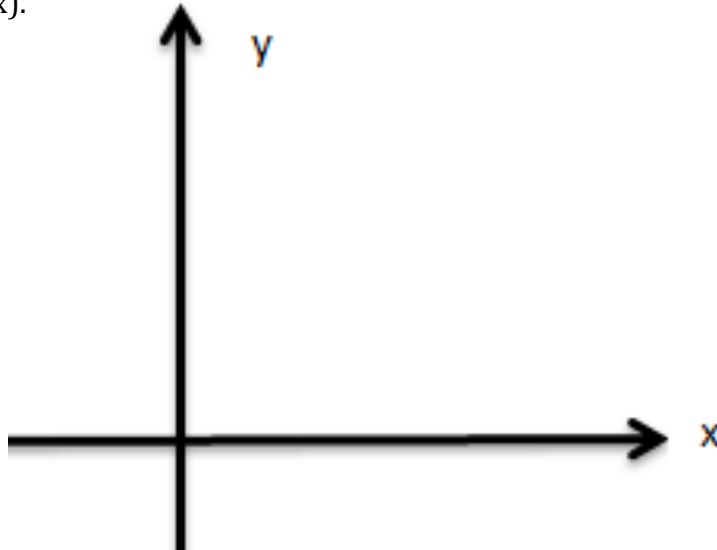
2 marks

**Question 7**

$$f(x) = \begin{cases} k \cos(x), & 0 \leq x \leq \frac{\pi}{2} \cup \frac{3\pi}{2} \leq x \leq 2\pi \\ 0, & \text{otherwise} \end{cases}$$

$f(x)$  describes a probability distribution function.

a. Sketch  $f(x)$ .



1 mark

b. Prove that  $k = \frac{1}{2}$ .

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2 marks

c. Using information from previous steps or otherwise, find  $\Pr(X < \frac{\pi}{2} \mid X < \frac{7\pi}{4})$

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3 marks



### Question 8

a.

Given that  $y = -8x + a$  is normal to the equation  $f(x) = \sqrt{x} + b$  at the point  $(c,9)$ , find  $a$ ,  $b$  and  $c$ .

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4 marks

**Question 9 -**

An unfair coin has a probability  $p$  of getting landing on heads.

**a.**

Find the probability that out of 3 attempts, 3 tails show up.

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1 mark

**b.**

Find the probability that out of 3 attempts, exactly 2 are heads.

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1 mark

**c.**

Find the value of  $p$  such that the probability that makes the probability of 2 heads out of 3 coin flips a maximum.

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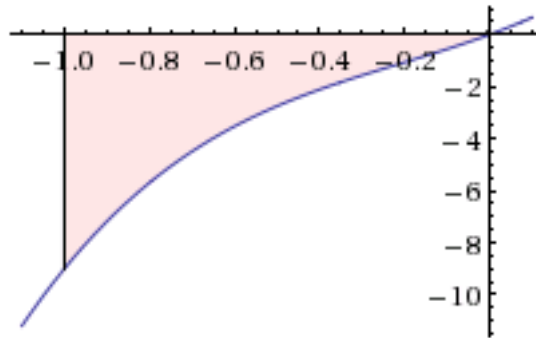
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2 marks

### Question 10

If the area shaded below is equal to 19, and the graph of  $f(x)$  passes through the point  $(0,0)$  find  $a$ ,  $b$ ,  $c$  and  $d$  for  $f(x) = a(x^3 + 5x^2 + 6x + d)$  where  $b$  and  $c$  are  $x$ -intercepts of the function  $f(x)$ .



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4 marks