



# **MATHEMATICAL METHODS (CAS) UNITS 3 & 4**

**2014 Trial EXAMINATION 2**

**July 2014**

**Section A: CAS and reference book permitted**

There is a total of 47 marks available for this section.

Reading Time: 15 minutes

Writing time: 1 hour

### **Instructions to students**

Section A has 2 parts. Part 1 consists of 12 multiple-choice questions, which should be answered on the answer sheet supplied. Part 2 has 3 extended-answer questions.

All questions should be answered in the spaces provided.

The marks allocated to each of the questions are indicated throughout.

An exact answer is required for all questions unless specified otherwise.

Where more than one mark is allocated to a question, appropriate working must be shown.

Diagrams in this trial exam are not drawn to scale.

**Part 1: Multiple Choice. Choose the best answer and circle it on the answer sheet supplied.****Question 1**

The maximal domain of the function  $y = \log_e(2x+1)$  is

- A.  $x \in R$
- B.  $x \in R \setminus \left\{-\frac{1}{2}\right\}$
- C.  $x \in \left[-\frac{1}{2}, \infty\right)$
- D.  $x \in \left(-\frac{1}{2}, \infty\right)$
- E.  $x \in \left[\frac{1}{2}, \infty\right)$

**Question 2**

The simultaneous linear equations

$$2x + (k-2)y = 2$$

$$(k+1)x + 2y = -1$$

have no solutions for

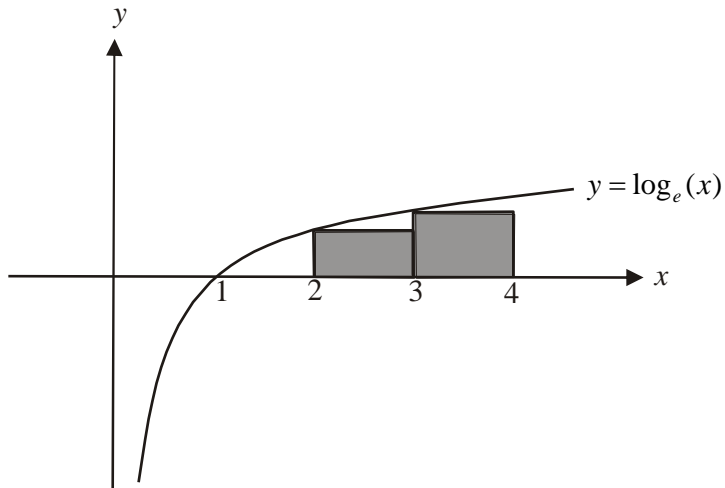
- A.  $k = -2$
- B.  $k = 3$
- C.  $k \in \{-2, 3\}$
- D.  $k \in R$
- E.  $k \in R \setminus \{-2, 3\}$

**Question 3**

The rate of change of the function  $y = \frac{e^x - e^{3x}}{x^2}$  with respect to  $x$ , at the point where  $x = 2$  is

- A.  $e^{-6}$
- B.  $\frac{-e^6}{2}$
- C.  $e^2 + e^{-4}$
- D.  $e^2 - \frac{e^6}{2}$
- E.  $\frac{-e^2(e^4 - 1)}{4}$

## Question 4



The area under the curve  $y = \log_e(x)$  between  $x = 1$  and  $x = 4$  is approximated by the two shaded rectangles shown above. This approximate area in square units is

- A.  $\log_e(1.5)$
- B.  $\log_e(5)$
- C.  $\log_e(6)$
- D.  $2\log_e(2)$
- E.  $3\log_e(2)$

## Question 5

The graph of  $y = x - 3$  and  $y = x^2 + kx - 1$  do **not** intersect for

- A.  $k = 1 \pm 2\sqrt{2}$
- B.  $-1 - 2\sqrt{2} < k < -1 + 2\sqrt{2}$
- C.  $1 - 2\sqrt{2} < k < 1 + 2\sqrt{2}$
- D.  $k < -1 - 2\sqrt{2}$  and  $k > -1 + 2\sqrt{2}$
- E.  $k < 1 - 2\sqrt{2}$  and  $k > 1 + 2\sqrt{2}$

## Question 6

The average rate of change of the function  $y = |(x-1)(x-5)|$  between  $x = 2$  and  $x = 3$  is

- A.  $-1$
- B.  $0$
- C.  $\frac{1}{6}$
- D.  $1$
- E.  $7$

**Question 7**

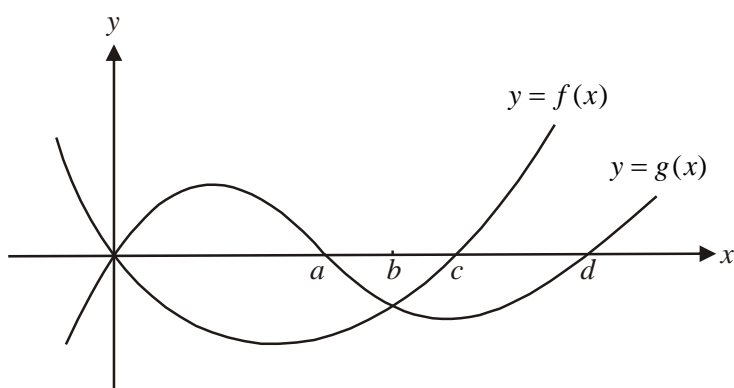
Inverses will not appear on the July exam.

Let  $f: [2, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = \sqrt{x-2}$ . The inverse function  $f^{-1}$  is given by

- A.  $f^{-1}: [0, \infty) \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = x^2 + 2$   
 B.  $f^{-1}: [0, \infty) \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = (x+2)^2$   
 C.  $f^{-1}: [2, \infty) \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = x^2 + 4$   
 D.  $f^{-1}: [2, \infty) \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = x^2 + 2$   
 E.  $f^{-1}: [0, 2] \rightarrow \mathbb{R}$ ,  $f^{-1}(x) = (x+2)^2$

**Question 8**

The graphs of  $y = f(x)$  and  $y = g(x)$  are shown below.



The area enclosed by these two graphs is given by

- A.  $\int_0^b (g(x) - f(x)) dx$   
 B.  $\int_0^b (g(x) + f(x)) dx$   
 C.  $\int_0^a (g(x)) dx - \int_0^b f(x) dx$   
 D.  $\int_0^a (g(x)) dx - \int_0^c f(x) dx$   
 E.  $\int_0^d (g(x)) dx - \int_0^c f(x) dx$

**Question 9**

The equation of the tangent to the curve with equation  $y = x^{\frac{1}{3}}$  at the point  $(8, 2)$  is given by

- A.  $y = \frac{x}{12} + \frac{4}{3}$   
 B.  $y = \frac{x}{12} + \frac{47}{6}$   
 C.  $y = \frac{x}{6} + \frac{2}{3}$   
 D.  $y = \frac{x}{6} + \frac{23}{3}$   
 E.  $y = \frac{4x}{3} - \frac{26}{3}$

**Question 10**

A transformation  $T: R^2 \rightarrow R^2$ , which maps the curve with equation  $y = e^x$  onto the curve with equation  $y = 4e^{x+1}$  could be given by

- A.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$   
 B.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix}$   
 C.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$   
 D.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$   
 E.  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

**Question 11**

The function  $g$  is continuous and differentiable for  $x \in R$ . The function  $g$  satisfies the following conditions.

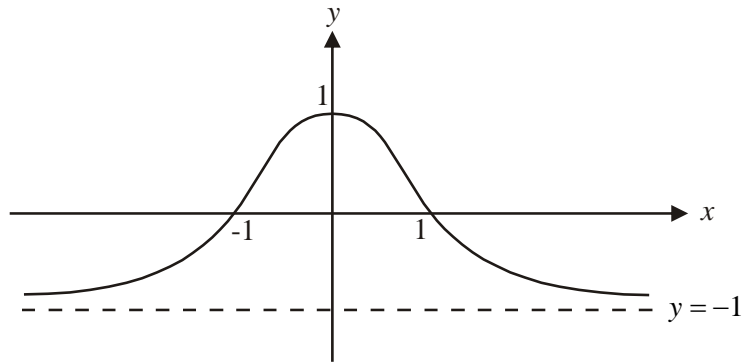
- $f'(x) = 0$  where  $x = -1, 0$  and  $4$
- $f'(x) > 0$  where  $x < -1$  and  $0 < x < 4$
- $f'(x) < 0$  where  $-1 < x < 0$  and  $x > 4$

It is true to say that the graph of  $y = g(x)$  has

- A. a stationary point of inflection where  $x = 0$   
 B. a point of inflection where  $x = 0$   
 C. a local maximum where  $x = 0$   
 D. a local minimum where  $x = 0$   
 E. two local minimum

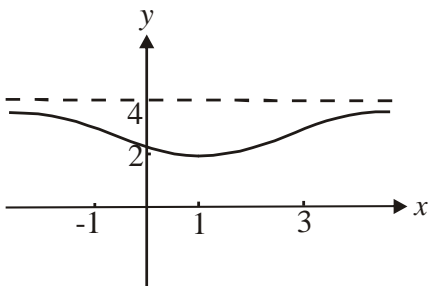
**Question 12**

The graph of  $y = f(x)$  is shown below.

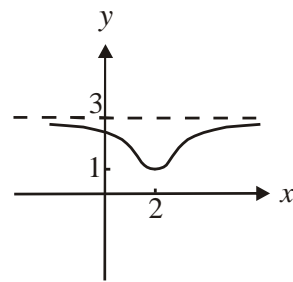


The graph of  $y = 3 - f(2x - 2)$  is given by

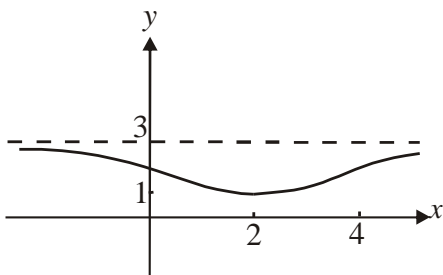
**A.**



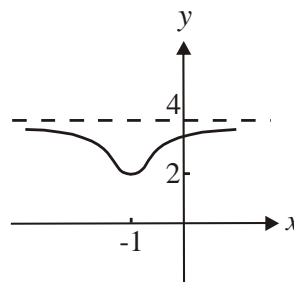
**B.**



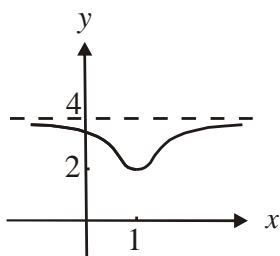
**C.**



**D.**



**E.**

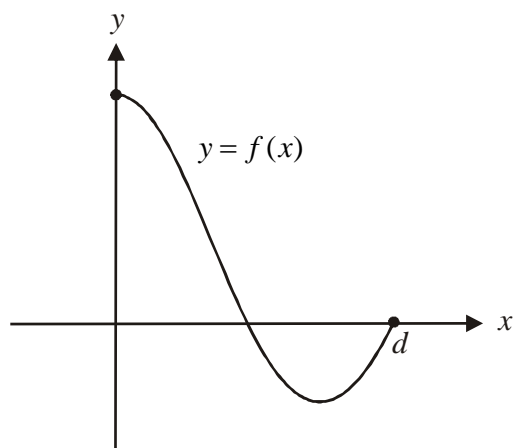


## Part 2

## Question 13

- a. Let  $f : [0, d] \rightarrow \mathbb{R}$ ,  $f(x) = 2 \cos\left(\frac{x}{4}\right) + 1$  where  $d$  is a constant.

The graph of  $y = f(x)$  is shown below.



- i. Find the value of  $d$ .

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- ii. Sketch the graph of  $y = |f(x)|$  on the set of axes shown above.

- iii. State the maximum value of  $|f(x)|$ .

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- iv. Find the value(s) of  $p$  for which the equation  $|f(x)| = p$  has three solutions.

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2+1+1+2=6 marks

**b.** Let  $g : [0, q] \rightarrow R$ ,  $g(x) = 2 \cos(nx) + 1$ , where  $n$  is a real constant and  $q$  is the largest possible value so that the inverse function  $g^{-1}$  exists.

**i.** Find  $q$  in terms of  $n$

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The function  $g$  passes through the point  $T(t, 1 - \sqrt{3})$

**ii.** Find the value of  $n$  in terms of  $t$ .

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2 + 2 = 4 marks

**Total Question 13 = 10 marks**



**Question 14**

Victoria James is a spy.

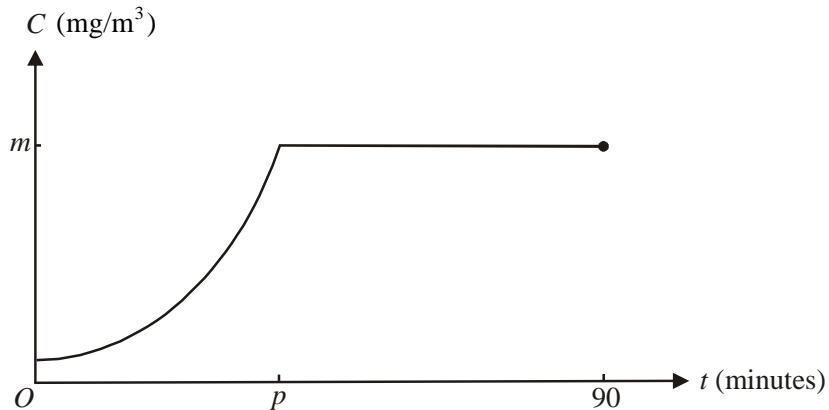
She is trapped in a space where poisonous gas is leaking.

The concentration  $C$ , in  $\text{mg/m}^3$ , of the gas  $t$  minutes after Victoria became trapped is given by the continuous function

$$C(t) = \begin{cases} \frac{500}{100-t}, & 0 \leq t \leq p \\ m, & p < t \leq 90 \end{cases}$$

where  $m$  and  $p$  are constants.

A graph of the function is shown below.



- a. What is the initial concentration of the gas in  $\text{mg/m}^3$ ?

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1 mark

- b. Find an expression for  $m$  in terms of  $p$ .

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1 mark

- c. Find the minimum and maximum values of  $m$ .

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2 marks

d. Find the function  $C'(t)$  for  $0 < t < p$ .

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1 mark

e. If the rate at which the concentration of the gas is increasing was  $1 \text{ mg/m}^3$  per minute, find the value of  $t$ . Express your answer in minutes correct to 2 decimal places.

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2 marks

f. If  $p=10$ , find the average concentration of the gas between  $t=0$  and  $t=p$ , correct to 2 decimal places.

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2 marks

If the concentration of the gas reaches  $6 \text{ mg/m}^3$  then a human cannot survive.

g. Given that Victoria is trapped for 90 minutes in this space, find the possible values of  $p$  in order for her to survive.

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2 marks

Total Question 14 = 11 marks

## Question 15

a. Consider the functions

$$f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = e^{-x} \quad \text{and} \quad g : [0, \infty) \rightarrow \mathbb{R}, g(x) = 2(x+1)$$

i. Write down the rule for  $f(g(x))$ .

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ii. What is the domain of  $f(g(x))$ ?

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iii. Sketch the graph of  $f(g(x))$ . Hence find range of  $f(g(x))$ .

iv. Find the area enclosed by the function  $y = f(g(x))$ , the  $x$  and  $y$  axes and the line with equation  $x = 1$ . Express your answer as an exact value.

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v. Describe the two transformations that the graph of  $y = f(x)$  undergoes to become the graph of  $y = f(g(x))$ .

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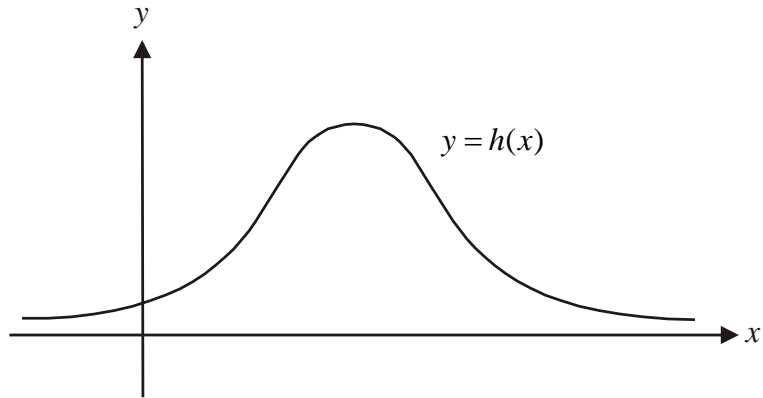
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- b.** The graph of the function  $h: \mathbb{R} \rightarrow \mathbb{R}$ ,  $h(x) = e^{-(x-a)^2}$  where  $a$  is a constant and  $a > 0$  is shown below.



- i.** Show that the stationary point of this graph occurs at  $(a, 1)$ .

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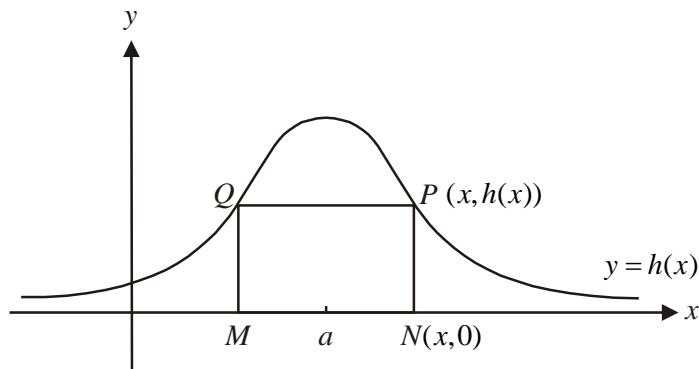
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- ii.** Find the values of  $x$  for which  $h(x)$  is strictly increasing.

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The diagram below shows the graph of  $y = h(x)$  where  $h(x) = e^{-(x-a)^2}$  and the rectangle  $MNPQ$ .



The side length  $MN$  lies on the  $x$ -axis, points  $P$  and  $Q$  lie on the graph of  $y = h(x)$  and the line  $x = a$  bisects the rectangle.

The coordinates of points  $N$  and  $P$  are  $(x, 0)$  and  $(x, h(x))$  respectively.

**iii.** Show that the area  $A$ , in square units, of the rectangle is given by

$$A = 2(x - a) \times e^{-(x-a)^2}.$$

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**iv.** Find the maximum area of rectangle  $MNPQ$ . You do not need to prove this is a maximum.

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2+1+1+3=7 marks

Total Question 15 =14 marks

**End of Section A**



**Mathematical Methods Multiple Choice Answer Sheet**  
**Choose and circle the best answer for each question**

|           |          |          |          |          |          |
|-----------|----------|----------|----------|----------|----------|
| <b>1</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>2</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>3</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>4</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>5</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>6</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>7</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>8</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>9</b>  | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>10</b> | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>11</b> | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |
| <b>12</b> | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> | <b>E</b> |

12 x 1 = 12 marks

Name: \_\_\_\_\_

Home Group: \_\_\_\_\_

**Mathematical Methods (CAS) Formulas****Mensuration**

|                                    |                        |                      |                        |
|------------------------------------|------------------------|----------------------|------------------------|
| area of a trapezium:               | $\frac{1}{2}(a+b)h$    | volume of a pyramid: | $\frac{1}{3}Ah$        |
| curved surface area of a cylinder: | $2\pi rh$              | volume of a sphere:  | $\frac{4}{3}\pi r^3$   |
| volume of a cylinder:              | $\pi r^2 h$            | area of a triangle:  | $\frac{1}{2}bc \sin A$ |
| volume of a cone:                  | $\frac{1}{3}\pi r^2 h$ |                      |                        |

**Calculus**

|  |   |
|--|---|
| $\frac{d}{dx}(x^n) = nx^{n-1}$                                       | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$   |
| $\frac{d}{dx}(e^{ax}) = ae^{ax}$                                     | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$  |
| $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$                              | $\int \frac{1}{x} dx = \log_e x  + c$   |
| $\frac{d}{dx}(\sin(ax)) = a \cos(ax)$                                | $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$  |
| $\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$                               | $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$   |
| $\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$       |   |
| product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ | quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ |
| chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$            | approximation: $f(x+h) \approx f(x) + hf'(x)$   |

**Probability**

|   |  |
|---|--|
| $\Pr(A) = 1 - \Pr(A')$                    | $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$                      |
| $\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ | transition matrices: $S_n = T^n \times S_0$                            |
| mean: $\mu = E(X)$                        | variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$ |

| probability distribution |                                     | mean                                      | variance   |
|--------------------------|-------------------------------------|---|--|
| discrete                 | $\Pr(X = x) = p(x)$                 | $\mu = \sum x p(x)$                       | $\sigma^2 = \sum (x - \mu)^2 p(x)$                       |
| continuous               | $\Pr(a < X < b) = \int_a^b f(x) dx$ | $\mu = \int_{-\infty}^{\infty} x f(x) dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$ |