Student Name.....

Teacher (circle one) JOR CWE

Homegroup .....

SOLUTIONS

MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 1

Thursday, 4th June, 2015

Reading Time: 5 minutes
Writing time: 1 hour

Instructions to students

This exam consists of **13** questions.
All questions should be answered in the spaces provided.
There is a total of **61** marks available.
A decimal approximation will not be accepted if an exact answer is required.
Where more than one mark is allocated to a question working must be shown.
Students **may not** bring any notes or any calculators into this exam.
Diagrams in this exam are not to scale except where otherwise stated.

FORMULAS

Function and Graphs

Distance formula $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Midpoint formula $x_M, y_M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Straight line graphs

General equation $y = mx + c$ Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation through point (x_1, y_1) given by $y - y_1 = m(x - x_1)$

Difference/sum of squares and cubes

$a^2 - b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Expansions

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

1 Solve these equations for x:

(a) $3x^3 + 81 = 0$

$x^3 + 27 = 0$

$x^3 = -27$

$x^3 = (-3)^3$

$x = -3$

(b) $\frac{1}{x+2} = \frac{2}{6x-5}$

$6x - 5 = 2(x+2)$

$6x - 5 = 2x + 4$

$4x = 9$

$x = \frac{9}{4}$

2 + 3 = 5 marks

2 Use the factor theorem and division to factorise $W(x) = x^3 - 5x^2 - 2x + 24$

$P(2) = 8 - 20 - 4 + 24 \neq 0$

$P(-2) = -8 - 20 + 4 + 24 = 0 \Rightarrow (x+2)$ is a factor

$$\begin{array}{r|l} & x^2 - 7x + 12 \\ \hline x+2 & x^3 - 5x^2 - 2x + 24 \\ & -x^3 + 2x^2 \\ & \quad -7x^2 - 2x \\ & \quad -7x^2 - 14x \\ & \quad\quad 12x + 24 \\ & \quad\quad 12x + 24 \\ & \quad\quad 0 \end{array}$$

$P(x) = (x+2)(x^2 - 7x + 12)$

$= (x+2)(x-4)(x-3)$

5 marks

3 Let $f(x) = -3x^4 + 2x^2 - 3$

Evaluate:

i) $f(-1) = -3(-1)^4 + 2(-1)^2 - 3$

$= -3 \times 1 + 2 \times 1 - 3$

$= -3 + 2 - 3 = -4$

ii) $f(\sqrt{2})$

$= -3(2^{\frac{1}{2}})^4 + 2(2^{\frac{1}{2}})^2 - 3$

$= -3 \times 2^2 + 2 - 3$

$= -12 + 2 - 3 = -13$

2 marks

4 Expand

(a) $(2x-1)(x+1)(1-x)$

$= (2x-1)(1-x^2)$

$= 2x - 2x^3 - 1 + x^2$

$= -2x^3 + x^2 + 2x - 1$

(b) $(3x+2)^3$

$= (3x)^3 + 3(3x)^2(2) + 3(3x)(2)^2 + (2)^3$

$= 27x^3 + 54x^2 + 36x + 8$

2 + 2 = 4 marks

5 Factorise the following completely

$4x^2 + 2x - 2$

$= 2(2x^2 + x - 1)$

$= 2(2x^2 + 2x - x - 1)$

$= 2(2x(x+1) - (x+1))$

$= 2(x+1)(2x-1)$

$3x^3y - 12xy^3$

$= 3xy(x^2 - 4y^2)$

$= 3xy(x+2y)(x-2y)$

$x^3 - 8$

$= (x-2)(x^2 + 2x + 4)$

3 x 2 = 6 marks

6 Simplify this expression using appropriate logarithm or index laws

a) $\dfrac{25^{x+3} \times 5^{6x}}{125^{2x-1}} = \dfrac{(5^2)^{x+3} \times 5^{6x}}{(5^3)^{2x-1}} = \dfrac{5^{2x+6} \times 5^{6x}}{5^{6x-5}}$

$= \dfrac{5^{8x+6}}{5^{6x-3}}$

$= 5^{8x+6-(6x-3)}$

$= 5^{14x+9}$

b) $3\log_3 18 + \log_3 2 - 2\log_3 12$

$= \log_3 18^3 + \log_3 2 - \log_3 12^2$

$= \log_3 \dfrac{18^3 \times 18^3 \times 18^9 \times 2}{12 \times 12}$

$= \log_3 (3 \times 3 \times 9)$

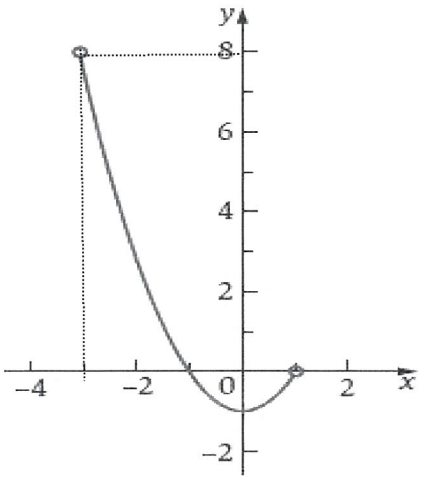
$= \log_3 81$

$= \log_3 3^4$

$= 4$

3+3=6 marks

7 State the domain and range of this graph



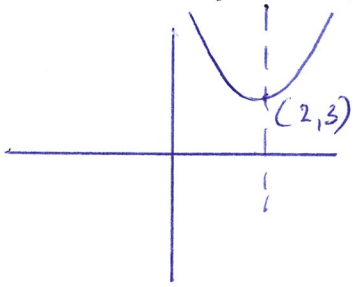
Domain $(-3, 1)$

Range $[-1, 8)$

2 marks

8 For the function $f : D \to R, f(x) = 2(x-2)^2 + 3$ find *D*, the largest domain for which the function is 1:

1



2 answers possible:

either $[-\infty, 2]$ or $[2, \infty)$

2 marks

9 List all the transformations that have been applied to the graph of $y = x^2$ to transform it into the graph of $y = \frac{1}{2}(x+4)^2 + 1$

1. Dilation $\frac{1}{2}$ unit from x-axis
2. Translation 4 units in the negative direction of the x-axis
3. Translation 1 unit in the positive direction of the y-axis

3 marks

10 Consider the points: A (5,-1) and B (1,3)

(a) Find the distance from *A* to *B*.
Express your answer in simplest surd form.

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(1-5)^2+(3--1)^2}$

$= \sqrt{4^2+4^2} = \sqrt{32} = \sqrt{16\times 2} = 4\sqrt{2}$

(b) Find the midpoint of the line segment *AB*

$M = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$= \left(\frac{1+5}{2}, \frac{3+-1}{2}\right)$

$= (3, 1)$

(c) Show that the point $A(5,-1)$ lies on the line with equation $y = 2x-11$.

if $x=5$, $y = 2(5)-11$

$= -1$

$=$ RHS

$\therefore$ $(5,-1)$ does lie on the equation

(d) Find the equation of the line that passes through the point $A(5,-1)$ and is perpendicular to the line $y = 2x-11$. Leave your answer in the form $ay+bx+c=0$

for 2 lines perpendicular: $m_1 m_2 = -1$

Line 1 : $m_1 = 2$

$\therefore$ $m_2 = -\frac{1}{2}$

$y = mx+c$ Sub in $(5,-1)$

$-1 = -\frac{1}{2}(5)+c$

$-1+\frac{5}{2} = c$

$\frac{3}{2} = c$

$y = -\frac{1}{2}x + \frac{3}{2}$

$2y = -x+3$

$x+2y-3=0$

2 + 1 +1 + 3 = 7 marks

$2y+x-4=0$

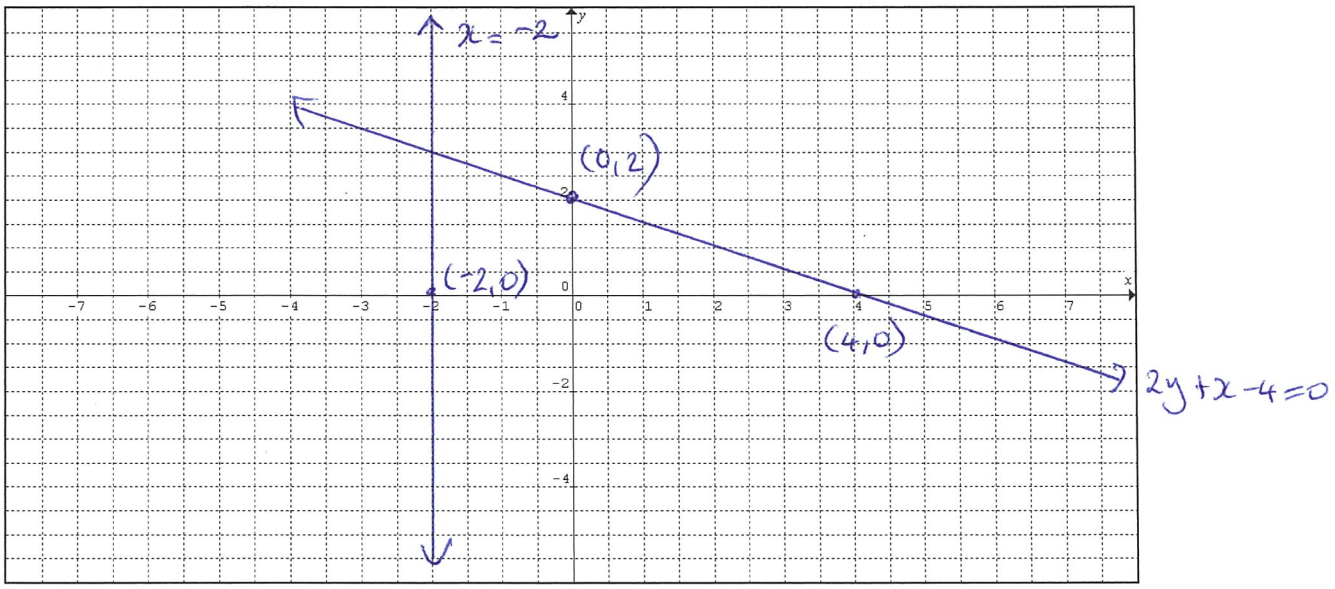
x-int $y=0$ | y int $x=0$

$x-4=0$ | $2y-4=0$

$x=4$ $(4,0)$ | $2y=4 \Rightarrow y=2$ $(0,2)$

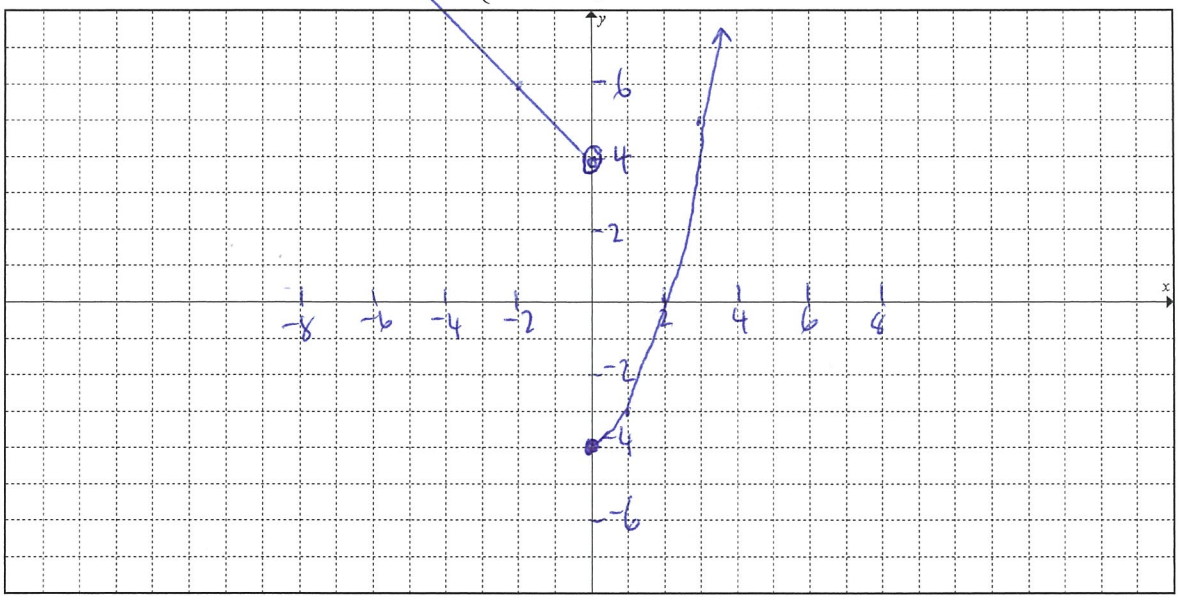
11 Sketch the graphs of the following, labelling axes intercepts with their coordinates.

(a) $2y+x-4=0$ (b) $x=-2$



2 marks

12 (a) Sketch the graph of $f(x)=\begin{cases} 4-x & , \quad x<0 \\ x^2-4 & , \quad x\geq 0 \end{cases}$



(b) What is the domain and range of $f(x)$?

domain: $\mathbb{R}$ range $[-4,\infty)$

(c) Find the value of $f(3)$. $f(3)=(3)^2-4=5$

(d) Is $f(x)$ a function or a relation? Give reasons

function - each x value has only 1 unique y value.

3 + 2 + 1 +2 = 8 marks

13 Consider the curve with equation. $y = \dfrac{1}{x-3} - 4$

(a) State the equations of the asymptotes.

$x = 3, \quad y = -4$

(b) What are the coordinates of any axes intercepts?

x int $y = 0$

$0 = \dfrac{1}{x-3} - 4$

$4 = \dfrac{1}{x-3}$

$x - 3 = \dfrac{1}{4}$ $\quad (3.75, 0)$

$\Rightarrow x = 3\frac{3}{4}$

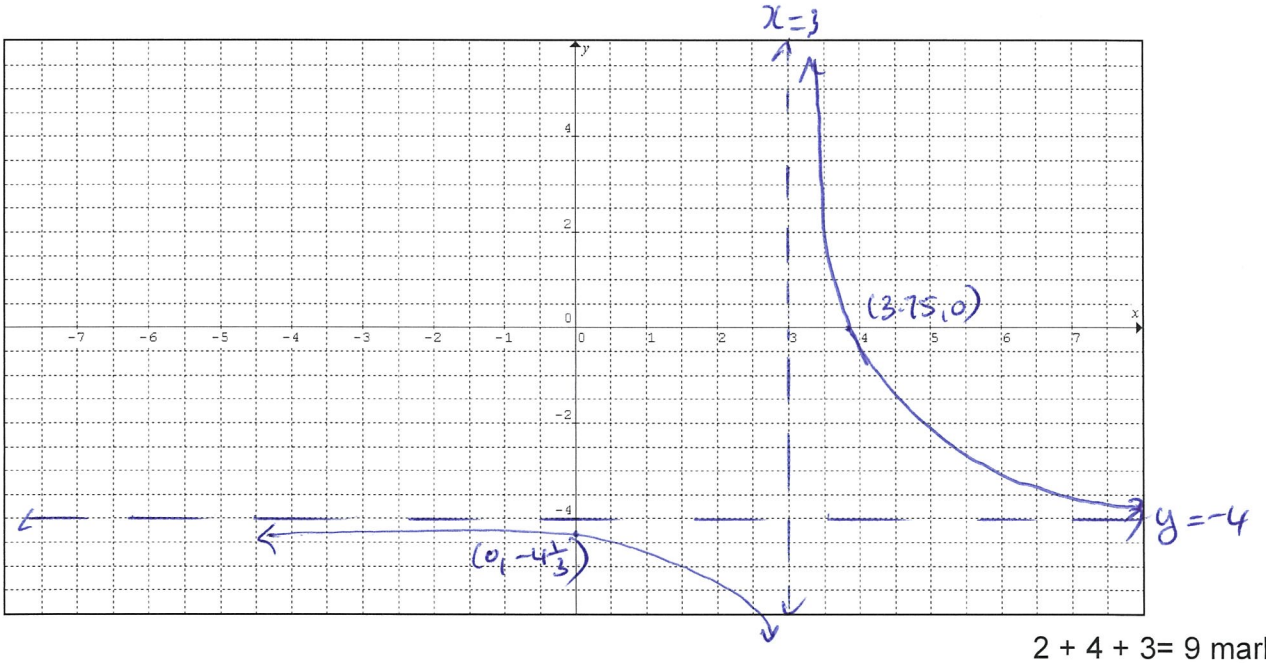
y-int $x = 0$

$y = \dfrac{1}{-3} - 4$

$= -4\frac{1}{3}$

$(0, -4\frac{1}{3})$

(c) Sketch the curve. Label all axes intercepts with their co-ordinates and asymptotes with their equations.



2 + 4 + 3= 9 marks

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