Student Name........SOLUTIONS..........

Teacher (circle one) JOR CWE

Homegroup ....

MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 2

Wednesday, 3th June, 2015

Reading Time: 9.00 – 9.15 (15 minutes)
Writing time: 9.15 – 10.45 (90 minutes)

Instructions to students

This exam consists of Section 1 and Section 2.

Section 1 consists of **12** multiple-choice questions, to be answered on the separate answer sheet. It is worth **12** marks.

Section 2 consists of **12** extended-answer questions that should be answered in the spaces provided. It is worth **77** marks

There is a total of **89** marks available.

All questions in Section 1 and Section 2 should be answered.

Unless otherwise stated, diagrams in this exam are not drawn to scale.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Students may bring one bound reference into the exam.
Students may bring an approved CAS calculator.

Section 1: Multiple Choice: Choose the best answer and write in the box shown. (12 marks)

1	B
2	B
3	D
4	A
5	D
6	E
7	B
8	C
9	E
10	C
11	D or A!
12	E

Student Name: SOLUTIONS **Home Group:** ____________

Teacher (circle): **Ms O'Rielly** **Ms Webb**

SECTION 1: MULTIPLE-CHOICE QUESTIONS

1 Which of the following relations are functions?

I $(x-2)^2+(y+1)^2=16$ X

II $y^2=\frac{2}{3}x-1$ X

III $y=-2x+4$

IV $y=4x^2$

A I and III
B III and IV
C I and II
D II and III
E I and IV

2 If $f(x)=2+\frac{3}{x}$ then the value of $f(3)-f(6)$ is

$f(3)=2+\frac{3}{3}$
$=3$

$f(6)=2+\frac{3}{6}$
$=2\frac{1}{2}$

$f(3)-f(6)$
$=3-2\frac{1}{2}$
$=0.5$

A. -3
B. $\frac{1}{2}$
C. $1\frac{1}{2}$
D. 3
E. $2\frac{1}{2}$

3 The expansion of $(x-3)^3(x+2)$ is given by

CAS

A x^2+x-12
B $x^3-2x^2-15x+36$
C $x^4+4x^3-27x-54$
D $x^4-7x^3+9x^2+27x-54$
E $x^4-5x^3-9x^2+81x-108$

4 The graph of the parabola with equation $y=-(x+3)^2-2$ has a turning point with coordinates

A $(-3,-2)$
B $(-3, 2)$
C $(3,-2)$
D $(9,-2)$
E $(3, 2)$

5 The equation $3x+2y-2=0$ has gradient and y-intercept respectively equal to:

$2y=-3x+2$
$y=-\frac{3}{2}x+1$

A $3, -2$
B $-2, 4$
C $\frac{2}{3}, \frac{4}{3}$
D $-\frac{3}{2}, 1$
E $-\frac{2}{3}, 4$

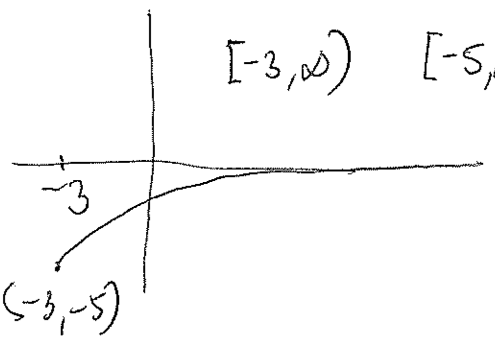
6 The variables a and b are related by the formula $a=\frac{4b}{b-1}$.
Rearrangement of the formula shows that b is equal to:

A $\frac{4a}{a+4}$
B $\frac{a}{a+4}$
C $\frac{a+4}{a}$
D $\frac{4a}{4+a}$
E $\frac{a}{a-4}$

7 A function has rule $f(x)=\sqrt{x+3}-5$.
The (implied) domain and range are:

A domain: $[3, \infty)$; range: $[-5, \infty)$
B domain: $[-3, \infty)$; range: $[-5, \infty)$
C domain: $(3, \infty)$; range: $(-5, \infty)$
D domain: $(-3, \infty)$; range: $(-5, \infty)$
E domain: $[-3, \infty)$; range: R

$[-3,\infty)$ $[-5,\infty)$



8 The maximal domain and range for $g(x)=\dfrac{2}{1+3x}$ are respectively:

A $R\setminus\left\{-\dfrac{3}{2}\right\}$ and $R\setminus\{0\}$

B $R\setminus\left\{-\dfrac{2}{3}\right\}$ and $R\setminus\{4\}$

C $R\setminus\left\{-\dfrac{1}{3}\right\}$ and $R\setminus\{0\}$

D $R\setminus\left\{-\dfrac{2}{3}\right\}$ and $R\setminus\{0\}$

E $R\setminus\left\{\dfrac{2}{3}\right\}$ and $R\setminus\{4\}$

9 The points (1, 4), (2, 0) and (4, *p*) lie on a straight line. The value of *p* is:

A +2

B – 2

C – 4

D – 6

E – 8

$m = \frac{-4}{1} = -4$

$4 = -4(1) + c$

$4 = -4 + c$

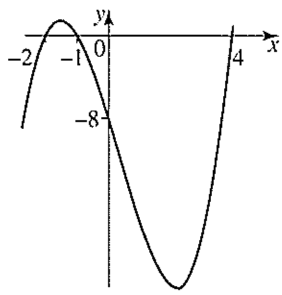
$\Rightarrow c = 8$

$y = -4x + 8$

$= -16 + 8$

$= -8$

10



$(x-2)(x-1)(x+4)$

The graph above represents the equation:

A $f(x)=(x-2)(x-1)(x+4)$

B $f(x)=-8(x+2)(x+1)(x-4)$

C $f(x)=(x+2)(x+1)(x-4)$

D $f(x)=(x-2)(x-1)(x+4)(x-8)$

E $f(x)=(x+2)(x+1)(x-4)(x+8)$

11 The expression $\dfrac{(m^2n)^4}{(2m^5n^2)^3}\div\dfrac{(m^5n^2)^2}{2mn^5}$ can be simplified to:

A $\dfrac{1}{4m^{16}n}$

B $\dfrac{2^2}{m^{16}n}$

C $\dfrac{1}{4m^8}$

D $\dfrac{1}{4m^{16}n}$

E $2^2m^{16}n$

$\dfrac{m^8n^4 \times \cancel{2}mn^5}{\underset{4}{8}m^{15}n^6 \times m^{10}n^4}$

$= \dfrac{m^9n^9}{4m^{25}n^{10}}$

$= \dfrac{1}{4m^{16}n}$

12 The expression $\log_n\left(\dfrac{1}{n^4}\right)$ equals:

A $\dfrac{n}{4}$

B $4n$

C 4

D n^{-4}

E -4

$= \log_n n^{-4}$

$= -4$

SECTION 2 EXTENDED-ANSWER QUESTIONS

13. A line joins the points with coordinates (–2, 5) and (6, 9).
Find:

a) The equation of the line that joins the 2 points.

$$m = \frac{9-5}{6--2} = \frac{4}{8} = \frac{1}{2}$$

$y = mx + c$

$5 = \frac{1}{2}x - 2 + c$

$5 = -1 + c$

$c = 6.$

$\underline{y = \frac{1}{2}x + 6}$

or

$\underline{2y - x - 12 = 0}$

b) The exact value (in simplest form) of the direct distance between the 2 points.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 - -2)^2 + (9-5)^2}$

$= \sqrt{8^2 + 4^2}$

$= \sqrt{64 + 16}$

$= \sqrt{80} = \sqrt{16 \times 5}$

$= 4\sqrt{5}$

c) The midpoint of the line.

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2+6}{2}, \frac{5+9}{2}\right)$

$= (2, 7)$

d) The equation of the perpendicular bisector of the line.

$m_1 = \frac{1}{2} \Rightarrow m_2 = -2$

$y = mx + c$

$7 = -2(2) + c$

$7 = -4 + c$

$11 = c$

$\underline{y = -2x + 11}$

or $\underline{2x + y - 11 = 0}$

e) State the domain and range of the line segment joining the points (-2, 5) and (6,9)

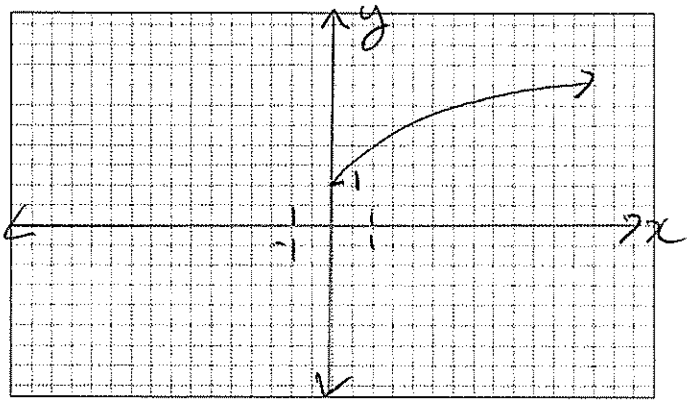
domain $= [-2, 6]$

range $= [5, 9]$

2 + 2 + 2 + 3 + 2 =11 marks

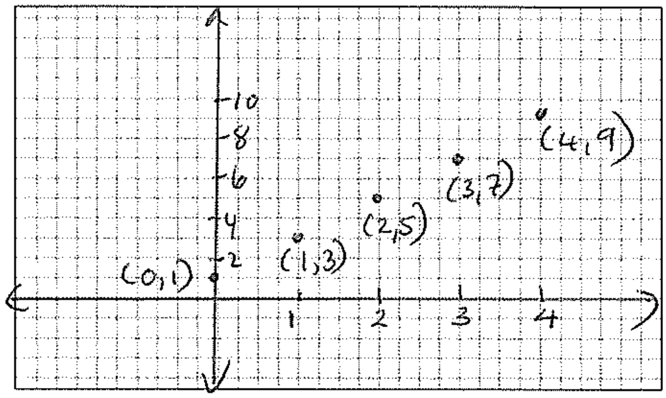
14. Sketch the graphs of each of the following equations. State the domain and range of each.

a) $y = \sqrt{x} + 1$, where $x \in R$



Domain: $[0, \infty)$
Range: $[1, \infty)$

b) $y = 2x + 1$, where $x \in \{0, 1, 2, 3, 4\}$



Domain: $\{0, 1, 2, 3, 4\}$
Range: $\{1, 3, 5, 7, 9\}$

3 + 3 = 6 marks

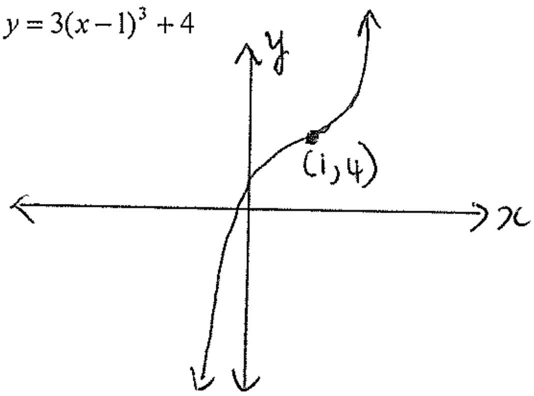
15. . (i) Sketch each of the following graphs, labelling the point of inflection or equation of any asymptotes where appropriate.

NOTE: You do **not** need to calculate any x or y intercept.

(ii) State the equation of the relevant basic shape graph.

(iii) State all dilations and translations required to draw them from the basic shape.

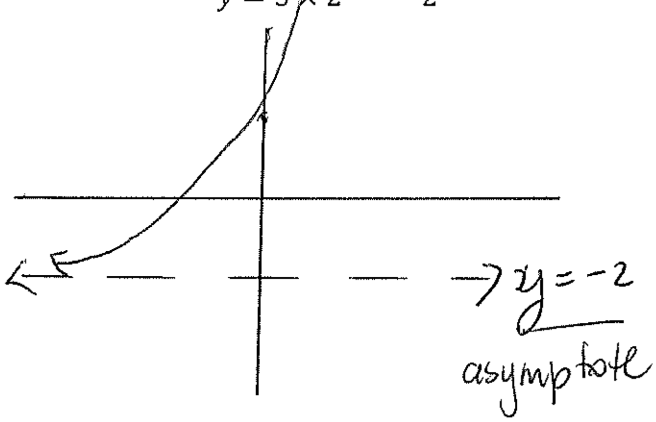
$y = 3(x-1)^3 + 4$



(ii) $y = x^3$

(iii) - Dilated 3 units from the x-axis
- translated 1 unit in the positive direction of the x-axis and 4 units in the positive direction of the y-axis

$y = 3 \times 2^{x+1} - 2$



(ii) $y = 2^x$

(iii) Dilated 3 units from the y-axis
translated 1 unit in the negative direction of the x-axis & 2 units in the negative direction of the y-axis

3+ 3 = 6 marks

16. a) Convert the following quadratic into turning point form: $y = x^2 + 4x - 7$

$y = x^2 + 4x - 7$

$= (x^2 + 4x + (2)^2) - (2)^2 - 7$

$= (x+2)^2 - 11$

a) Hence, state the co-ordinates of the turning point.

$(-2, -11)$

b) State the domain and range

domain : R range $[-11, \infty)$

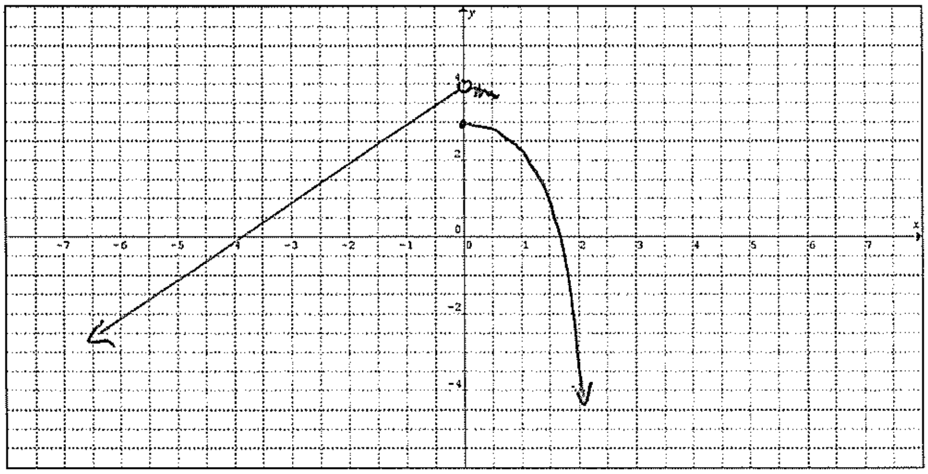
c) What translation would map the parabola $y = x^2$ onto $y = x^2 + 4x - 7$?

translation of 2 units in the negative direction of the x-axis and 11 units in the negative direction of the y-axis.

2 + 1 + 2 + 2 = 7 marks

17. If $f(x) = \begin{cases} 3 - x^2 & , \quad x \geq 0 \\ x + 4 & , \quad x < 0 \end{cases}$

Sketch this graph.



Find:

a) the range of $f(x)$ and

$(-\infty, 4)$

b) the value for $f(-2)$.

$f(-2) = -2 + 4$

$= 2$

3+2 = 5 marks

18. (a) Factorise $x^3 - 5x^2 - 4x + 20$

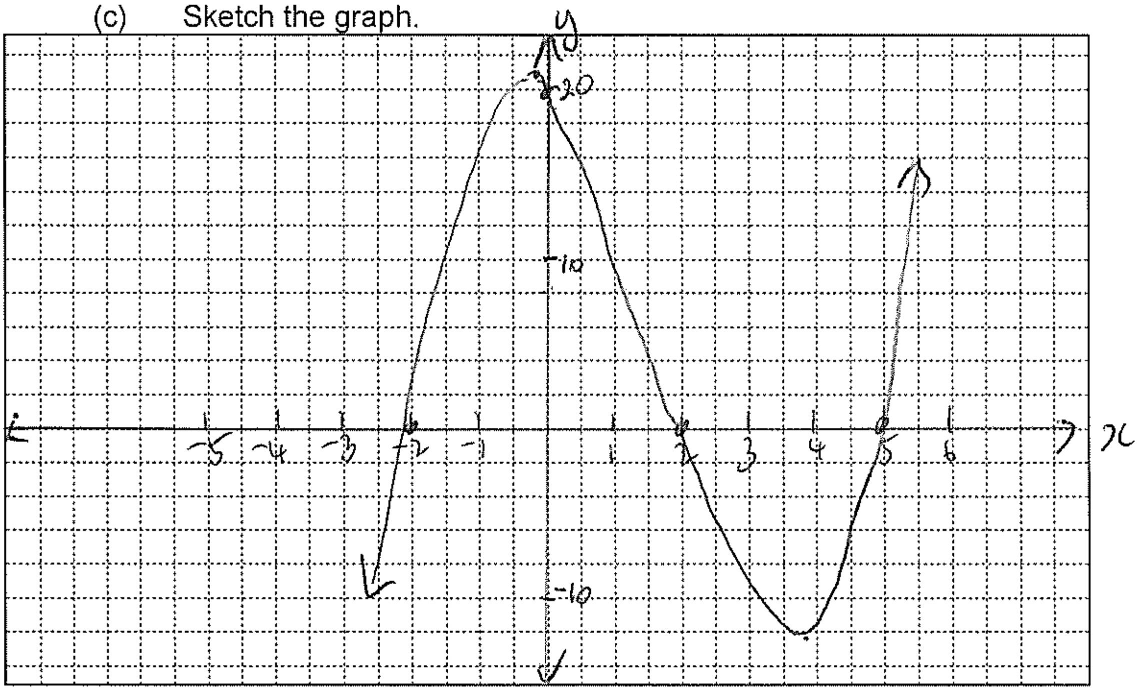
$(x+2)(x-2)(x-5)$

(b) What are the co-ordinates of the axes intercepts for the graph of $= x^3 - 5x^2 - 4x + 20$?

x-intercepts $(-2,0)$ $(2,0)$ $(5,0)$

y-intercept $(0,20)$

(c) Sketch the graph.



(d) What are the co-ordinates of the turning points, correct to correct to 2 decimal places.

$(-0.36, 20.75)$ $(3.69, -12.60)$

1 + 2 +2 + 2 = 7 marks

19. Rewrite these in interval notation:

(a) $R^+ \backslash \{5\}$ $(0,5) \cup (5,\infty)$

(b) $R^+ \backslash \{1 \le x < 4\}$ $(0,1) \cup [4,\infty)$

(c) $R^+ \cup \{-5 < x < -3\}$ $(-5,-3) \cup (0,\infty)$

3 marks

20. If $f(x) = 3 - x^2$, find:

a) f(-2)

$f(-2) = 3-(-2)^2$
$= 3-4$
$= -1$

b) $f(m-3)$ $= 3-(m-3)^2$
$= 3-(m^2-6m+9)$
$= 3-m^2+6m-9$
$= -m^2+6m-6$

1 + 2 = 3 marks

21. Write in simplest index notation:

a) $3^{n+1} \times 9^{2n+3} \div 27^{1-3n}$ $= 3^{n+1} \times (3^2)^{2n+3} \div (3^3)^{1-3n}$ $= 3^{n+1} \times 3^{4n+6} \div 3^{3-9n}$ $= 3^{n+1+4n+6-(3-9n)}$ $= 3^{5n+7-3+9n}$ $= 3^{14n+4}$	b) $\dfrac{(a^{-3}\sqrt{b^3})^4 \times (\sqrt{2}a^4b^{-3})^3}{\sqrt{2}\left(ab^{-2}\right)^{-2}}$ $= \dfrac{\cancel{a^{-12}}\, b^{\frac{3}{2}\times 4} \times 2^{\frac{3}{2}} \cancel{a^{12}}\, b^{-9}}{2^{\frac{1}{2}} a^{-2} b^4}$ $= \dfrac{2^{\frac{3}{2}} b^{-3}}{2^{\frac{1}{2}} a^{-2} b^4}$ $= \dfrac{2a^2}{b^7}$

2 + 3 = 5 marks

22. Solve for x in the following equations:

a) $3^{4x+1} = 243$ $3^{4x+1} = 3^5$ $4x+1 = 5$ $4x = 4$ $x = 1$	b) $5^{2x} - 6(5^x) + 5 = 0.$ $(5^x)^2 - 6(5x) + 5 = 0$ Let $a = 5^x$ $a^2 - 6a + 5 = 0$ $(a-5)(a-1) = 0$ $a = 5$ or $a = 1$ $\therefore 5^x = 5$ or $5^x = 1$ $5^x = 5^1$ or $5^x = 5^0$ $x = 1$ or $x = 0$

2 + 3 = 5 marks

23.

a) Evaluate $\log_2(256)$, showing all working

$= \log_2 2^8$

$= 8\log_2 2$

$= 8 \times 1 = 8$

b) Simplify $4\log_{10} 2 - 2\log_{10} 8$

$\cancel{4}\log_{10} 2^4 - \log_{10} 8^2$

$= \log_{10} \frac{16}{64}$

$= \log_{10}\left(\frac{1}{4}\right)$

c) Solve for *x* where $\log_5(2x-3) = 2$

$5^2 = 2x-3$

$2x-3 = 25$

$2x = 28$

$x = 14$

2 + 2 + 2 = 6 marks

24. The number of rabbits that are left on a farm t weeks after a virus is released is given by the function

$$N(t) = 15 + \frac{96}{t+3} \text{ rabbits per hectare.}$$

(a) How many rabbits per hectare were on the farm when the virus was released?

$$N(0) = 15 + \frac{96}{3} = \underline{47} \text{ rabbits/hectare}$$

(b) How many rabbits per hectare are there 13 weeks after the virus was released?

$$N(13) = 15 + \frac{96}{13+3}$$

$$= 21 \text{ rabbits/hectare.}$$

(c) How long after the virus is released are there 23 rabbits per hectare?

$$23 = 15 + \frac{96}{t+3}$$

$$8 = \frac{96}{t+3}$$

* or solve on CAS

$$t+3 = \frac{96}{8}$$

$$t+3 = 12 \Rightarrow t = 9 \text{ weeks}$$

(d) Will the virus kill all the rabbits? Explain your answer.

No, as t increases, the function approaches the asymptote of $N(t)=15$. The no. rabbits will not go below 15 rabbits/hectare.

1 + 1 + 2 + 2 = 6 marks

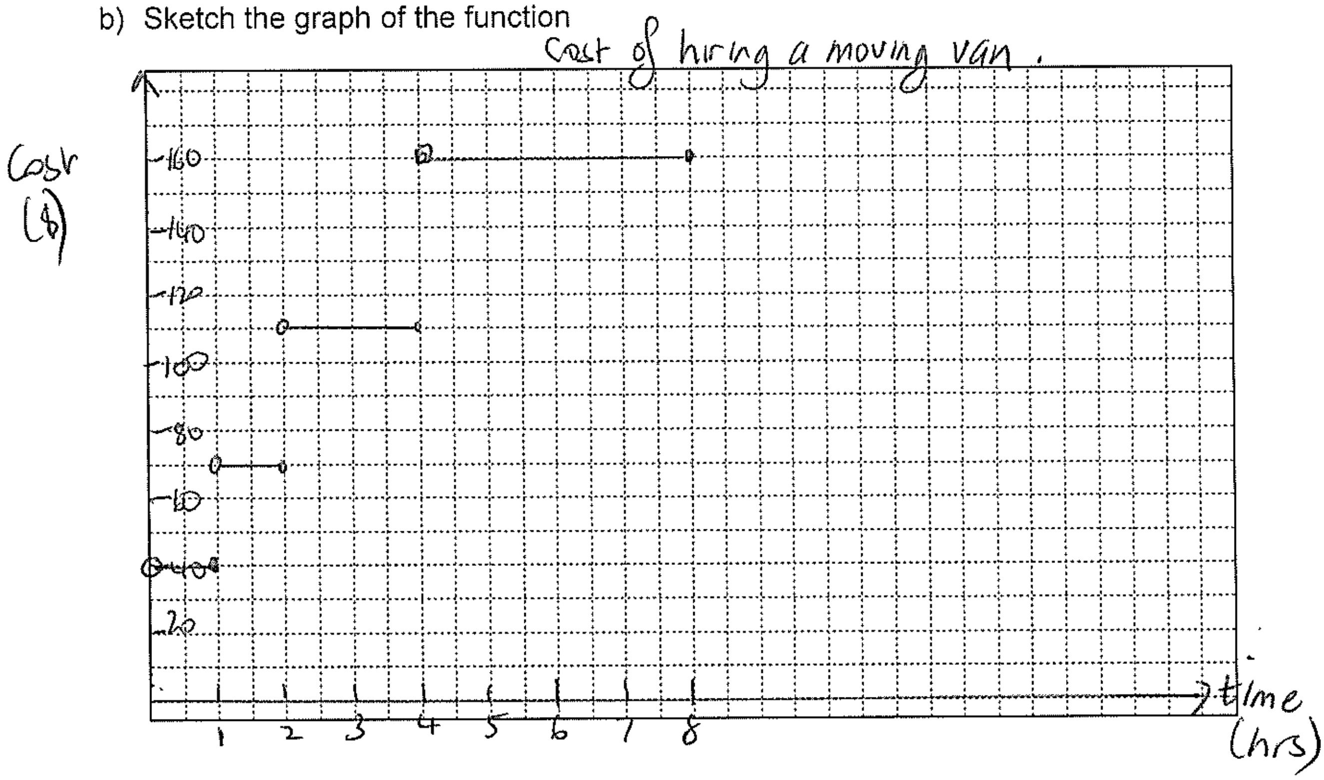
25. The cost of hiring a moving van is described in the table below:

Hours of Hire	Cost
Up to 1	$40
Over 1 up to 2	$70
Over 2 up to 4	$110
Over 4 up to 8	$160

a) State the cost function, C(t), for hiring up to 8 hours

$$C(t) = \begin{cases} 40 & 0 < t \leq 1 \\ 70 & 1 < t \leq 2 \\ 110 & 2 < t \leq 4 \\ 160 & 4 < t \leq 8 \end{cases}$$

b) Sketch the graph of the function



c) State the domain and range of the function.

domain : $(0, 8]$

range $\{40, 70, 110, 60\}$

2 + 3 + 2 = 7 marks

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