Student Name A. Pls

Teacher (circle one) DKI JOR VNA

Homegroup Asgard

MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 1

Thursday June 9th 2016

Reading Time: 2:00 – 2:15pm (15 minutes)
Writing time: 2:15 – 3:15pm (1 hour)

Instructions to students

This exam consists of **13** questions.
All questions should be answered in the spaces provided.
There is a total of **65** marks available.
A decimal approximation will not be accepted if an exact answer is required.
Where more than one mark is allocated to a question working must be shown.
Students **may not** bring any notes or any calculators into this exam.
Diagrams in this exam are not to scale except where otherwise stated.

FORMULAS

Function and Graphs

Distance formula $d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint formula $x_M, y_M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Straight line graphs

General equation $y = mx + c$

Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation through point (x_1, y_1) given by $y - y_1 = m(x - x_1)$

Difference/sum of squares and cubes

$a^2 - b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Expansions

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

1. Solve these equations for x

(a) $6(x+2) = 4x-1$

$6x+12 = 4x-1$

$2x = -13$

$x = \frac{-13}{2}$

(b) $\frac{3}{x+2} = \frac{5}{x}$

$3x = 5(x+2)$

$3x = 5x+10$

$-2x = 10$

$x = -5$

2X2 =4 marks

2. Use the factor theorem and division to factorise $P(x) = 2x^3 - x^2 - 7x + 6$

$P(1) = 2-1-7+6 = 0 \Rightarrow (x-1)$ is a factor

$$x-1 \overline{)\, 2x^3 - x^2 - 7x + 6}$$

$2x^2 + x - 6$

$-(2x^3 - 2x^2)$

$x^2 - 7x$

$-(x^2 - x)$

$-6x + 6$

$-(-6x + 6)$

0

So $P(x) = (x-1)(2x^2+x-6)$

$= (x-1)(2x^2+4x-3x-6)$

$= (x-1)(2x(x+2)-3(x+2))$

$= (x-1)(x+2)(2x-3)$

5 marks

3. Expand

(a) $(2x+3)^3 = (2x)^3 + 3(2x)^2(3) + 3(2x)(3)^2 + 3^3$

$= 8x^3 + 36x^2 + 54x + 27$

(b) $(2x-1)(x+2)(x-3)$

$= (2x-1)(x^2-3x+2x-6)$

$= (2x-1)(x^2-x-6)$

$= 2x^3 - 2x^2 - 12x - x^2 + x + 6$

$= 2x^3 - 3x^2 - 11x + 6$

2+2=4 marks

4. Factorise the following completely

(a) $2x^3+54$

$= 2(x^3+27)$

$= 2(x^3+3^3)$

$= 2(x+3)(x^2-3x+4)$

(b) $12a^2+8a-15$

$= 12a^2+18a-10a-15$

$= 6a(2a+3)-5(2a+3)$

$= (6a-5)(2a+3)$

(c) $18a^2b-2b$

$= 2b(9a^2-1)$

$= 2b((3a)^2-1)$

$= 2b(3a+1)(3a-1)$

(d) Use completion of the square method
Leave your answer in exact form

w^2+2w-3

$= (w^2+2w+1)-1-3$

$= (w+1)^2-4$

$= (w+1+2)(w+1-2)$

$= (w+3)(w-1)$

4x2=8 marks

5. Use the discriminant to predict the number of solutions and the type for the following equations DO NOT SOLVE

$\Delta = b^2-4ac$

a) $x^2-6x+8=0$

$a=1 \quad b=-6 \quad c=8$

$\Delta = b^2-4ac$

$= 36-32$

$= 4$

2 solutions

b) $5v^2-2v+1=0$

$a=5 \quad b=-2 \quad c=1$

$\Delta = 4-20$

$= -16$

0 solutions

c) $3g^2-10=0$

$a=3 \quad b=0 \quad c=-10$

$\Delta = b^2-4ac$

$= 0+120$

$= 120$

2 solutions

3X2 = 6 marks

6. State the domain and range of $(x+1)^2+(y-4)^2=169$

circle centre $(-1,4)$, $r=13$

domain : $[-1-13, -1+13] = [14, 12]$

range : $[4-13, 4+13] = [-9, 17]$

2 marks

7. (a) Sketch the function $y = 1 - \frac{4}{(x-2)^2}$, where $-2 < x < 6$ Include any intercepts, endpoints and any other main feature(s)

asymptotes $y=1$, $x=2$

Intercepts

$y=0$

$\Rightarrow 1 - \frac{4}{(x-2)^2} = 0$

$1 = \frac{4}{(x-2)^2}$

$(x-2)^2 = 4$

$x-2 = \pm 2$

$\Rightarrow x = 0, 4$

$x=0$

$\Rightarrow y = 1 - \frac{4}{(-2)^2}$

$= 0$

Endpoints

at $x=-2$, $y = 1 - \frac{4}{16}$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

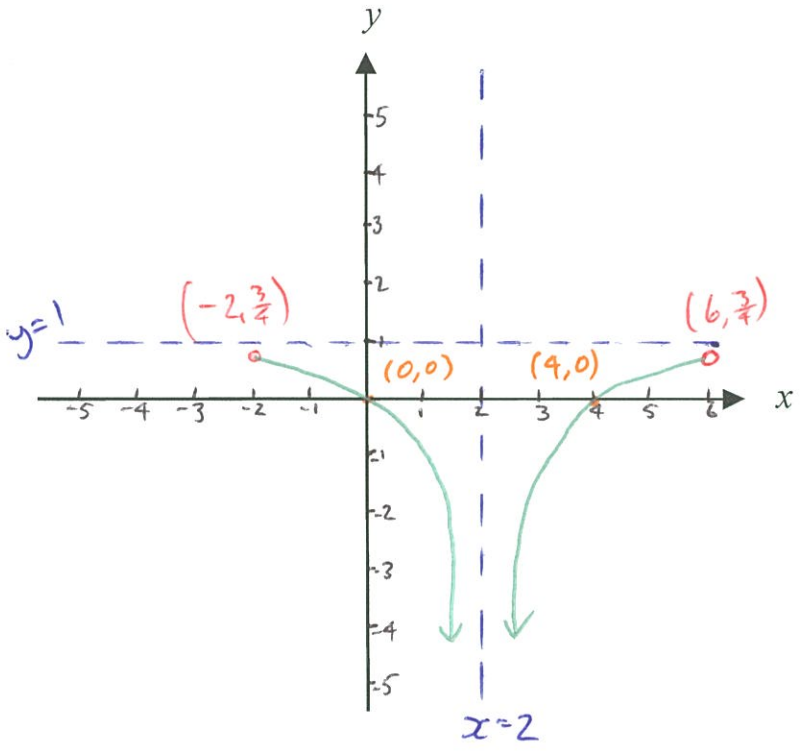
$(-2, \frac{3}{4})$

$x=6$

$y = 1 - \frac{4}{16}$

$= \frac{3}{4}$

$(6, \frac{3}{4})$



b) What is this shape called?

Truncus

c) What type of relation is this?

Function, many to one

d) What is the domain and range of this relation.

Domain: $(-2, 2) \cup (2, 6)$

Range: $(-\infty, \frac{3}{4})$

4+1+1+1=7 marks

8. State all the transformations required to transform the cubic $y = \sqrt{x}$ to $y = \sqrt{5x-2} + 3$

$y = \sqrt{5(x - \frac{2}{5})} + 3$

① Dilation of $\frac{1}{5}$ units from y-axis

② translation of $\frac{2}{5}$ units positive direction of x-axis

③ translation of 3 units positive direction of y-axis

3 marks

9. The perpendicular bisector of a line segment is the line passing through its mid point and perpendicular to the line segment. Find the equation of the perpendicular bisector of the line interval joining the points (–2, 5) and (4, –3).

$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{4-(-2)} = \frac{-8}{6} = -\frac{4}{3} \Rightarrow m_2 = \frac{3}{4}$

Mid point $= \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right) = \left(\frac{-2+4}{2}, \frac{5+(-3)}{2}\right) = (1, 1)$

Perpendicular Bisector : $y = mx + c$

sub in $m_2 = \frac{3}{4}$, $(1,1) \Rightarrow 1 = \frac{3}{4}(1) + c$

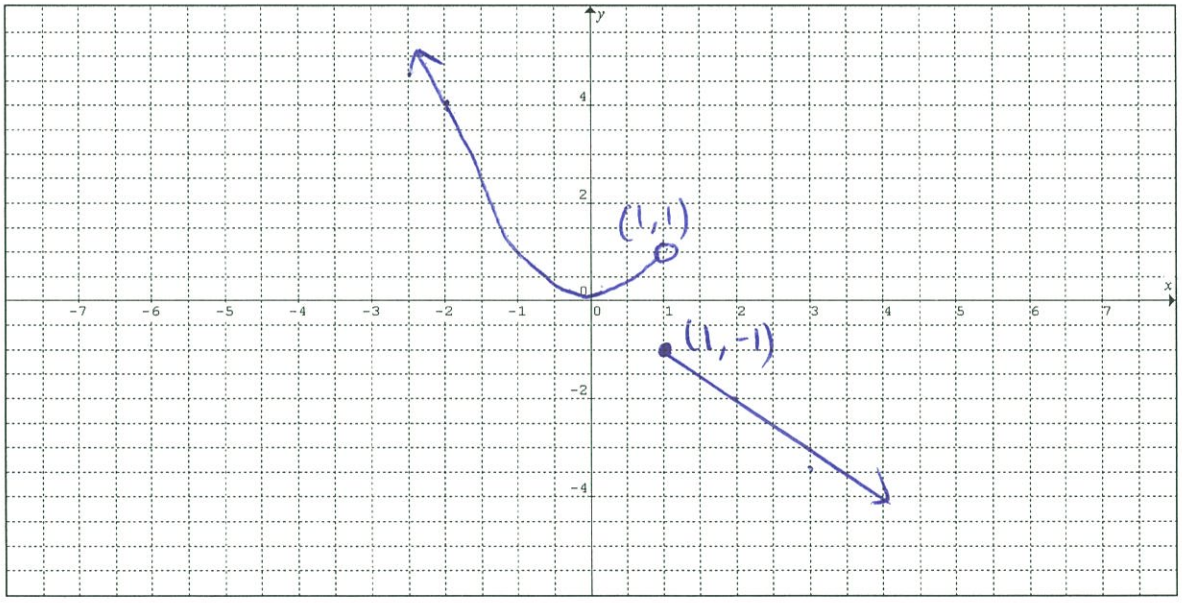
$c = 1 - \frac{3}{4}$

$= \frac{1}{4}$

so $y = \frac{3}{4}x + \frac{1}{4}$

5 marks

10. a) Sketch the graph of $h(x) = \begin{cases} x^2 & , \quad x < 1 \\ -x & , \quad x \geq 1 \end{cases}$



b) What is the domain and range of $h(x)$?

Domain : $\mathbb{R}$

Range : $(-\infty, -1] \cup [0, \infty)$

c) Find the value of $h(1)$.

$h(1) = -(1)$

$= -1$

d) State any value of x for which the function is not continuous.

$x = 1$

3+2+1+2=8 marks

11. Consider the equation for a circle: $x^2 - 4x + y^2 + 4y - 1 = 0$

a) State the coordinates of the centre of the circle and the size of the radius.

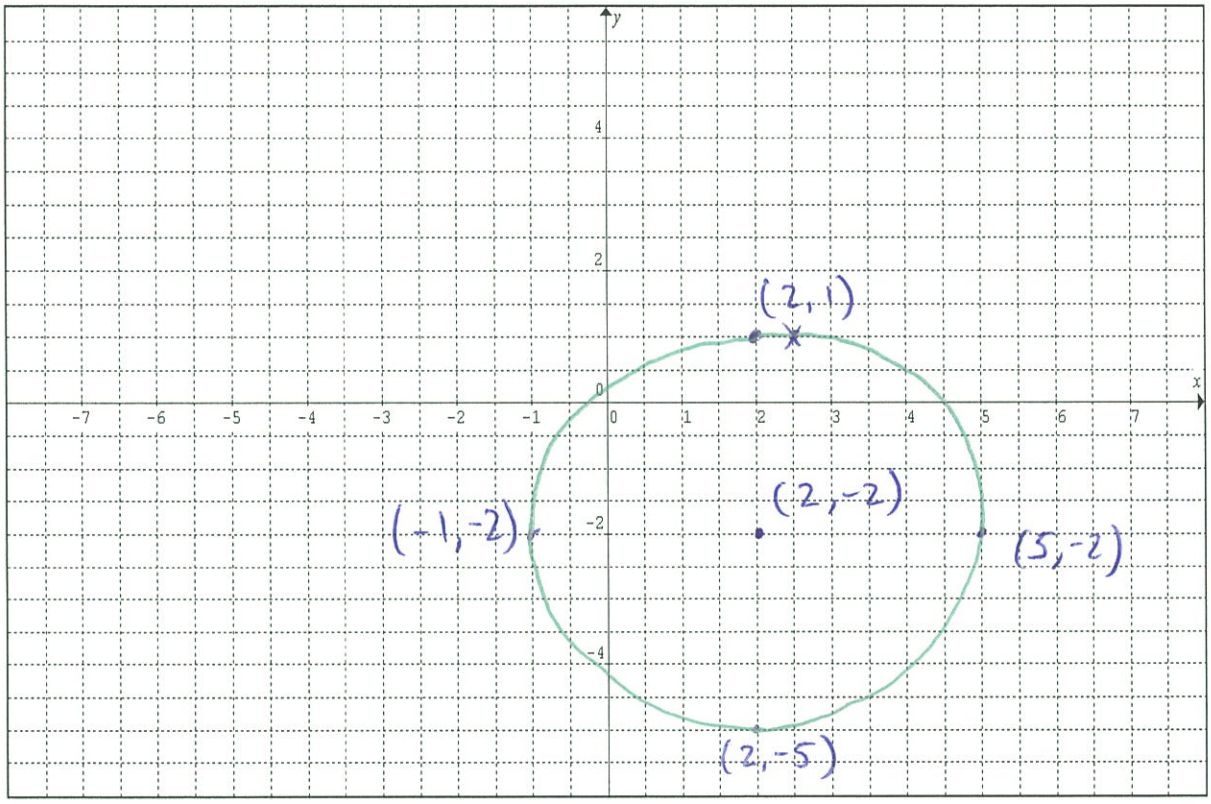
$x^2 - 4x + 4 + y^2 + 4y + 4 - 4 - 4 - 1 = 0$

$(x-2)^2 + (y+2)^2 - 9 = 0$

$(x-2)^2 + (y+2)^2 = 3^2$

centre: $(2, -2)$, radius $= 3$

b) Sketch the circle (do not calculate intercepts for this question)



3+2=5 marks

12. **a) Simplify** $\dfrac{9^{2n}\times 8^{n}\times 16^{n}}{6^{n}}$

$$= \frac{(3^2)^{2n}\times(2^3)^{n}\times(2^4)^{n}}{(2\times 3)^{n}}$$

$$= \frac{3^{4n}\times 2^{3n}\times 2^{4n}}{2^{n}\times 3^{n}}$$

$$= 3^{3n}\times 2^{6n}$$

b) Simplify $2\log_{10}3+\log_{10}4$

$$= \log_{10}(3)^2+\log_{10}(4)$$

$$= \log_{10}(9\times 4)$$

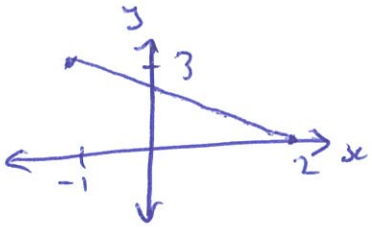
$$= \log_{10}(36)$$

4 marks

13. Consider the function $h:[-1,2]\to R, h(x)=2-x$

a) State the range of h(x)

$[0,3]$



b) In full notation, write the inverse of h(x)

$y=2-x$

So inverse:

$x=2-y$

$\Rightarrow y=2-x$

so $f^{-1}:[0,3]\to\mathbb{R}, f^{-1}(x)=2-x$

2+2=4 marks

END OF EXAMINATION 1