Student Name Answers McAnswerface

Teacher (circle one) DKI JOR VNA

Homegroup Trenzalore

MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 2

Monday June 6th 2016

Reading Time: 11.30-11.45 (15 minutes)
Writing time: 11.45-1.15 (90 minutes)

Instructions to students

This exam consists of Section 1 and Section 2.

Section 1 consists of **12** multiple-choice questions, to be answered on the separate answer sheet. It is worth **12** marks.

Section 2 consists of **13** extended-answer questions that should be answered in the spaces provided. It is worth **73** marks

There is a total of **85** marks available.

All questions in Section 1 and Section 2 should be answered.

Unless otherwise stated, diagrams in this exam are not drawn to scale.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Students may bring one bound reference into the exam.
Students may bring an approved CAS calculator.

MATHEMATICAL METHODS (CAS) UNIT 1

EXAM 2

SECTION 1

ANSWER SHEET

Student Name..... Topof DaClass .....

Teacher (circle one) DKI JOR. VNA

Homegroup..... Siluria .....

Question	Answer (A – E)
1.	E
2.	B
3.	C
4.	B
5.	A
6.	B
7.	D
8.	B
9.	C
10.	B
11	A
12	E

MULTIPLE-CHOICE QUESTIONS

Question 1
The equation of the line passing through the points (–1, –3) and (3, 4) is:

A $4x + 7y + 25 = 0$
B $4x - 7y + 5 = 0$
C $4x - 7y - 17 = 0$
D $7x - 4y + 17 = 0$
E $7x - 4y - 5 = 0$

Question 2
The exact distance between the points (–1, 5) and (5, –1) is:

A $2\sqrt{6}$
B $6\sqrt{2}$
C $4\sqrt{2}$
D 12
E $2\sqrt{3}$

Question 3
The expanded form of $(3x+2)^3$ is:

A $3x^3+8$
B $3x^3+12x+8$
C $27x^3+54x^2+36x+8$
D $27x^3+27x^2+3x+8$
E $3x^3+6x^2+12x+8$

Question 4
Solutions to the quadratic equation $2x^2+4x-3=0$ are:

A $x=\left\{-1\pm\sqrt{40}\right\}$
B $x=\left\{\dfrac{-2\pm\sqrt{10}}{2}\right\}$
C $x=\left\{-1\pm\sqrt{10}\right\}$
D $x=\left\{\dfrac{4\pm\sqrt{40}}{4}\right\}$
E $x=\left\{1\pm\sqrt{40}\right\}$

Question 5
The graph with equation $y=6x^2+x-2$ has x-intercepts:

A $x=\left\{-\dfrac{2}{3},\dfrac{1}{2}\right\}$
B $x=\left\{-\dfrac{2}{3},-\dfrac{1}{2}\right\}$
C $x=\left\{-\dfrac{1}{2},\dfrac{2}{3}\right\}$
D $x=\left\{\dfrac{1}{2},\dfrac{2}{3}\right\}$
E $x=\{1,-2\}$

Question 6
The radius measure of the circle $9(x-5)^2+9(y+2)^2=81$ is:

A 9
B 3
C $3\sqrt{2}$
D $2\sqrt{3}$
E 81

Question 7
The maximal domain and range for $f(x)=\sqrt{5-2x}$ respectively are:

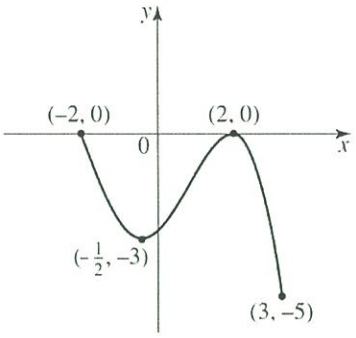
A $(-\infty, 2.5)$ and $(0, \infty)$
B $(-\infty, 2.5)$ and $[0, \infty)$
C $(-\infty, 2.5]$ and $(0, \infty)$
D $(-\infty, 2.5]$ and $[0, \infty)$
E $(-\infty, 2.5]$ and $[2.5, \infty)$

Question 8
When $6x^3-5x^2+x+3$ is divided by $2x+3$, the quotient and remainder respectively are:

A $3x^2-7x-11,\ -30$
B $3x^2-7x+11,\ -30$
C $3x^2+7x+11,\ -30$
D $3x^2+7x-11,\ -30$
E $3x^2-7x-11,\ -32$

Question 9

The range of the function shown below is:



A [−2, 3]
B [−2, 2]
C [−5, 0]
D [−3,0]
E [−5, −3]

Question 10

The expression $\frac{3}{x^2-5x-6}$ is undefined when:

A $x=0$
B $x=-1, x=6$
C $x=-3, x=-2$
D $x=-6, x=1$
E $x=2, x=3$

Question 11

The logarithmic form of the statement $729=9^3$ is:

A $\log_9(729)=3$
B $\log_3(729)=9$
C $\log_3(9)=729$
D $\log_{729}(3)=9$
E $\log_{729}(9)=3$

Question 12

$\frac{\left(3m^{-2}n^{\frac{1}{2}}\right)^2}{4\left(m^6n^3\right)^{\frac{1}{2}}}$ is equal to:

A $\frac{9m}{2n^{\frac{1}{2}}}$
B $\frac{9n^{\frac{1}{2}}}{4m}$
C $\frac{3m^7}{2n^{\frac{1}{2}}}$
D $\frac{9n^{\frac{1}{2}}}{4m^7}$
E $\frac{9}{4m^7n^{\frac{1}{2}}}$

SECTION 2 EXTENDED-ANSWER QUESTIONS

Question 13

Find the equation of the line which which passes through the point (–1, 2).and is perpendicular to the line with equation $2x - 4y + 3 = 0$.

$4y = 2x + 3$

$y = \frac{1}{2}x + \frac{3}{4} \Rightarrow m = \frac{1}{2}, \; m_{\perp} = -2$

so $y - 2 = -2(x+1)$

$y - 2 = -2x - 2$

$y = -2x$

4 marks

Question 14

Write down an expression to represent the midpoint of the line joining the point $(3k, 3h + 1)$ with $(4 - 5k, 2h - 5)$. Then simplify if possible.

midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\Rightarrow \left(\frac{(3k) + (4-5k)}{2}, \frac{(3h+1) + (2h-5)}{2}\right)$

$\Rightarrow \left(\frac{4-2k}{2}, \frac{5h-4}{2}\right)$

$\Rightarrow \left(2-k, \frac{5h-4}{2}\right)$

3 marks

Question 15

(a) For the parabola with equation $y = x^2 + 5x - 1$, find the vertex by completing the square.

$$y = \left(x^2 + 5x + \left(\frac{5}{2}\right)^2\right) - \left(\frac{5}{2}\right)^2 - 1$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{4}{4}$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{29}{4}$$

so vertex at $\left(\frac{-5}{2}, \frac{-29}{4}\right)$

(b) Hence find the domain and range.

Domain: $\mathbb{R}$

Range: $\left[\frac{-29}{4}, \infty\right)$

(c) What translation would map the parabola $y = x^2$ onto $y = x^2 + 5x - 1$?

Translation of $\frac{5}{2}$ units negatively along the x-axis

Translation of $\frac{29}{4}$ units vertically down along the y-axis

2+2+2=6 marks

Question 16

a) Write down the numbers in row 4 of Pascal's triangle.

1 4 6 4 1

a) Hence, expand $(2x - 3)^4$, using the binomial coefficients from Pascal's triangle. You must show at least 1 line of algebraic working.

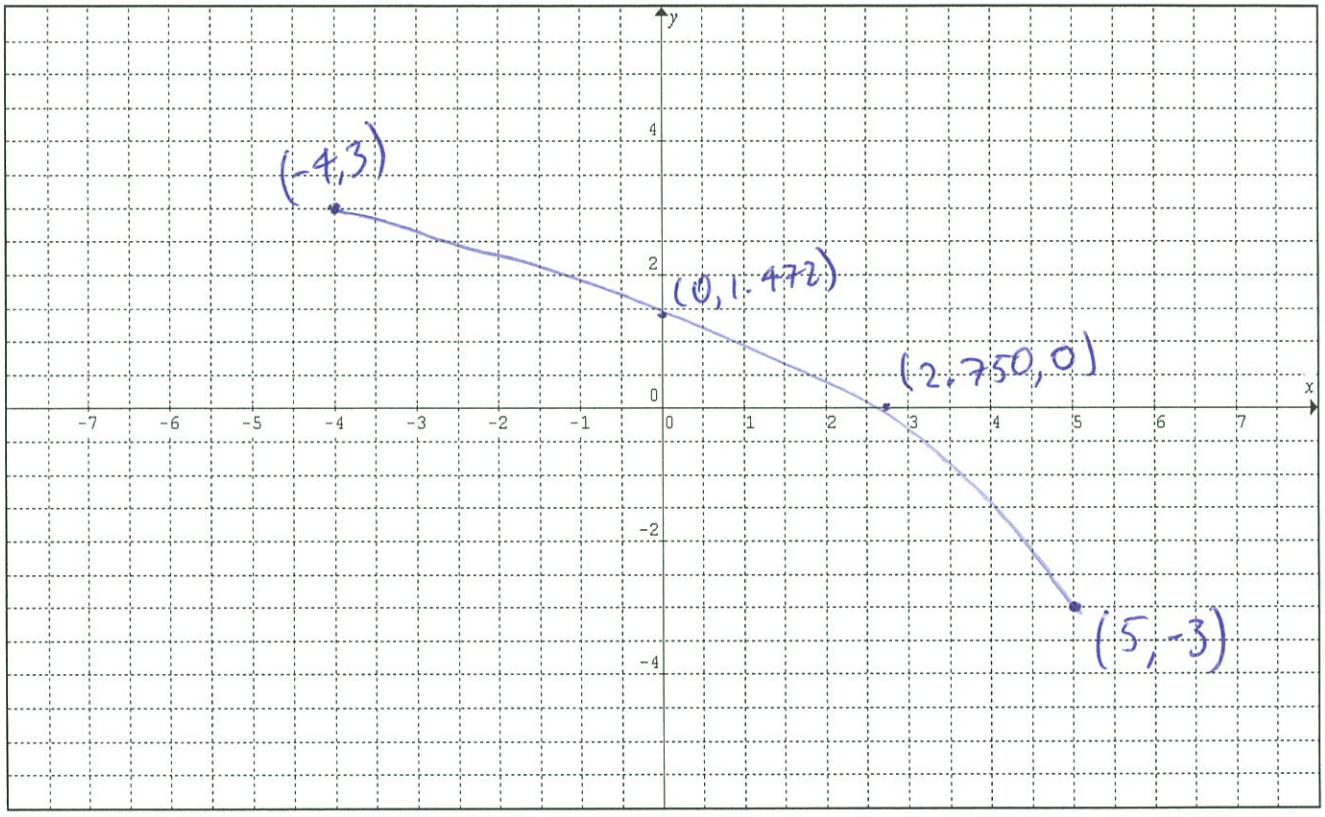
$$(2x-3)^4 = 1(2x)^4 + \left(4(2x)^3(-3)^1\right) + 6(2x)^2(-3)^2 + 4(2x)^1(-3)^3 + (-3)^4$$

$$= 16x^4 - 96x^3 + 216x^2 - 216x + 81$$

(1 + 2 = 3 marks)

Question 17

a. **i.** On the set of axes below sketch the graph of the relation $f:[-4,5] \to R$ *where* $f(x) = 2(\sqrt{5-x}) - 3$.
Indicate clearly on the graph any intercepts or endpoints to **3 decimal places**.



ii. Write down the domain and range of this relation.

Domain : $[-4, 5]$

Range : $[-3, 3]$

iii. The basic shape of this graph was $y = \sqrt{x}$. Clearly state the transformations (dilations, reflections and translations that would be needed to give $y = 2(\sqrt{5-x}) - 3$

$y = 2(\sqrt{-(x-5)}) - 3$

1. Dilation of a factor of 2 away from x-axis
2. Reflection in y axis
3. Translation of 5 units in positive direction along x-axis
4. Translation of 3 units down along y axis

3+2+4=9 marks

Question 18

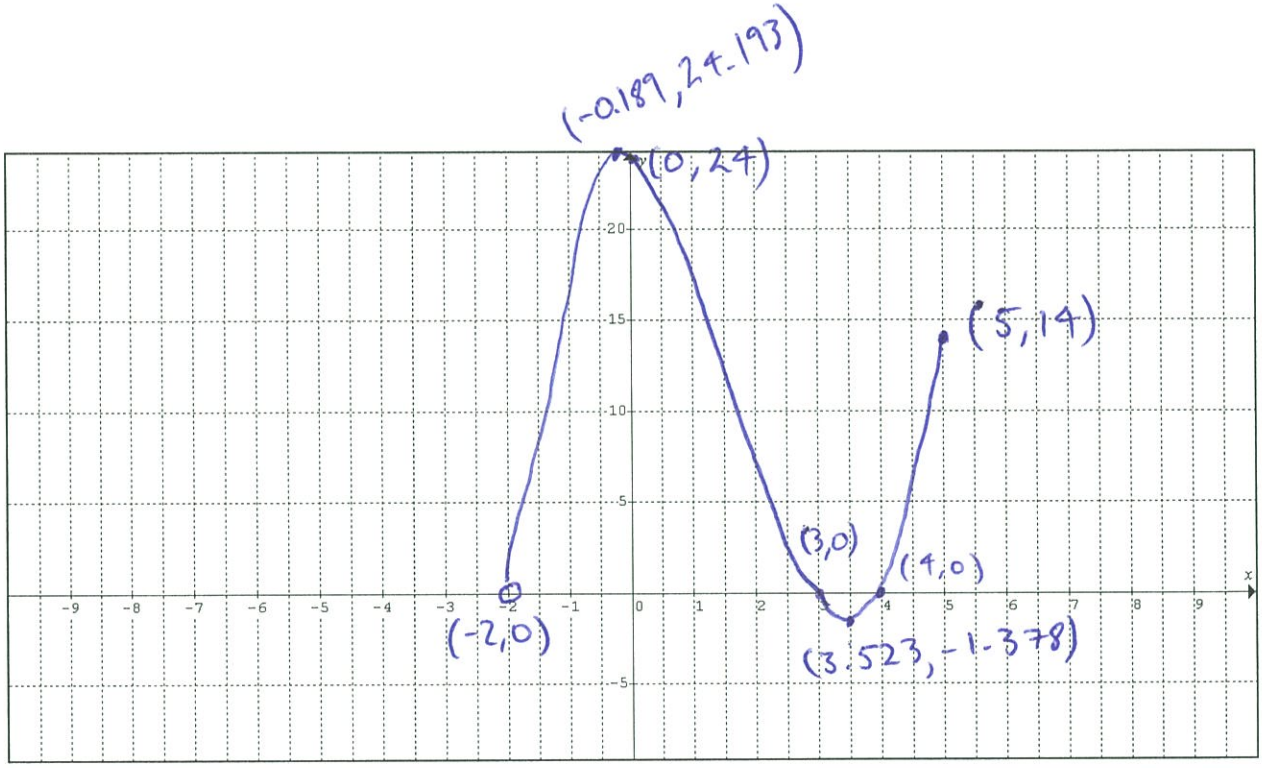
a) Factorise $x^3 - 5x^2 - 2x + 24$, then sketch the graph of

$g:(-2,5] \to R$ *where* $g(x) = x^3 - 3x^2 - 4x + 12$.

Clearly label the **coordinates** of the intercepts and endpoints.

$g(x) = (x+2)(x-3)(x-4)$

Endpoints: $(-2, 0)$ open , $(5, 14)$ closed



b) Using CAS find the **coordinates** of the turning points to **3 decimal places**. Label these on your graph

T.P. $(-0.189, 24.193)$

T.P. $(3.523, -1.378)$

c) Hence find the domain and range. (**3 decimal places**.)

Domain $(-2, 5]$

Range $[-1.378, 24.193]$

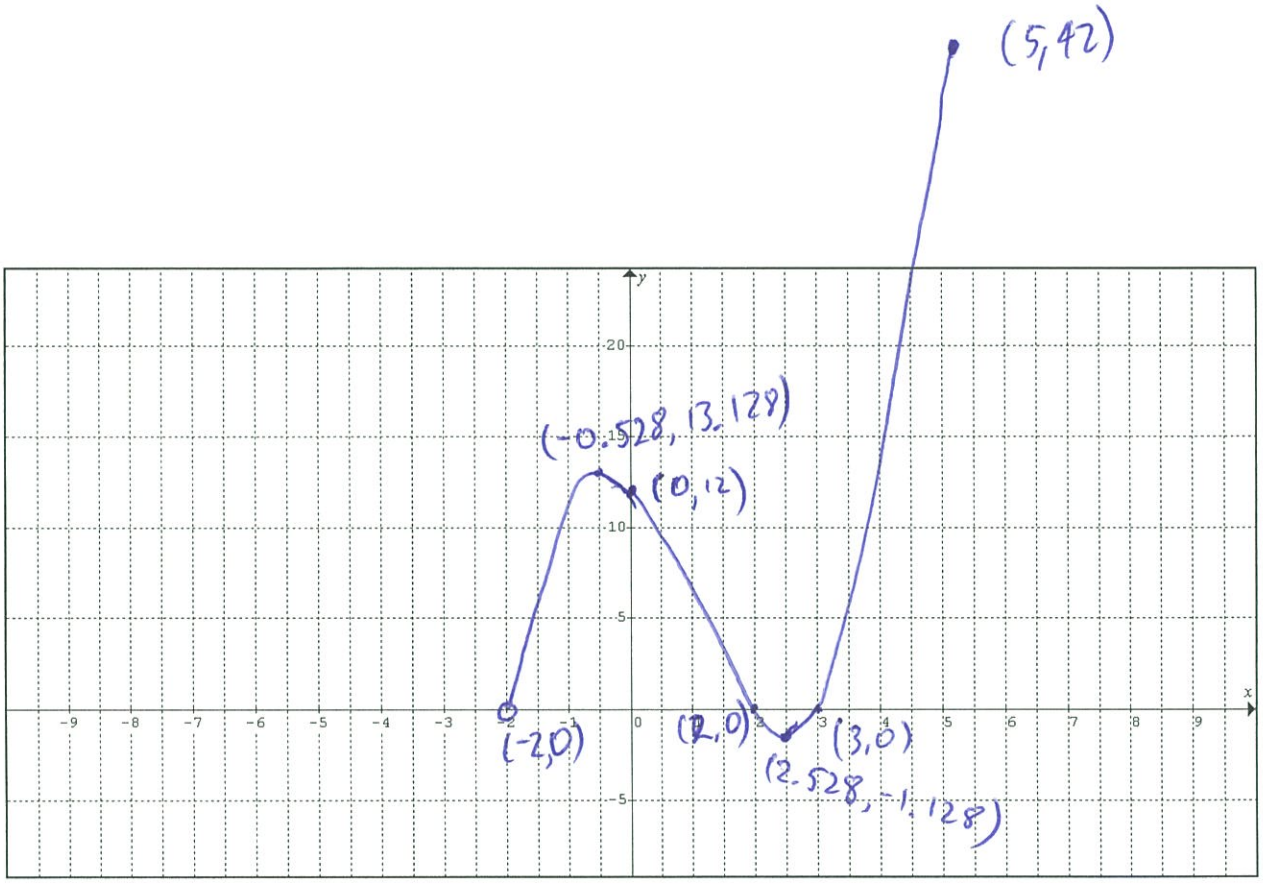
5+2+2=9 marks

Question 18

a) Factorise ~~$x^3 - 5x^2 - 2x + 24$~~, then sketch the graph of
$g:(-2,5] \to R$ *where* $g(x) = x^3 - 3x^2 - 4x + 12$.
Clearly label the **coordinates** of the intercepts and endpoints.

$g(x) = (x+2)(x-2)(x-3)$

Endpoints : $(-2,0)$, $(5,42)$



b) Using CAS find the **coordinates** of the turning points to **3 decimal places**. Label these on your graph

T.P. $(-0.528, 13.128)$

T.P. $(2.528, -1.128)$

c) Hence find the domain and range. (**3 decimal places**.)

Domain $(-2,5]$

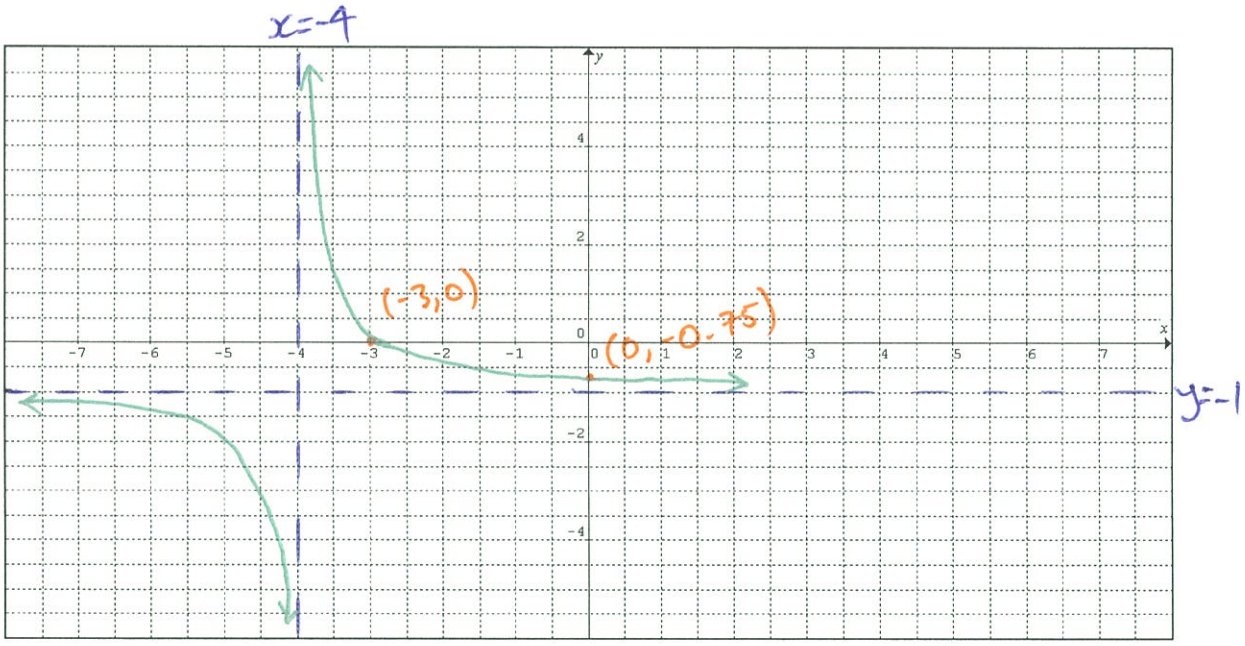
Range $[-1.128, 42]$

5+2+2=9 marks

Question 19

a) i. On the set of axes below sketch the graph of the relation $y = \dfrac{1}{(x+4)} - 1$.

Clearly label the **coordinates** of the intercepts and the equations of any asymptotes.



ii. State whether or not this relation is a function. Give the reason for your answer.

Function

Passes vertical line test

Each x-value has one unique y-value

iii. State the domain and Range of y

Domain: $\mathbb{R} \setminus \{-4\}$

Range: $\mathbb{R} \setminus \{-1\}$

3+2+2=7 marks

Question 20

a) Give the equation for the upper half of a circle with centre (0,0) and radius 6.

circle: $x^2+y^2=36$ upper half: $y = \sqrt{36-x^2}$

b) Give the equation for the lower half of a circle with centre (4,-3) and radius 2.

circle: $(x-4)^2+(y+3)^2=4$

$(y+3)^2 = 4-(x-4)^2$

lower half: $y = -\sqrt{4-(x-4)^2} - 3$

2 marks

Question 21

If $f(x) = \begin{cases} x+1, & x \in [-5,0) \\ 5-x^2, & x \in [0,3) \\ x, & x \in [3,6) \end{cases}$

Find

a) the value for $f(3)$.

$f(3) = 3$

b) The range of, $f(x)$

Range: $[-4,6)$

c) Sketch this graph. Place the **coordinates** on each end point and intercept.

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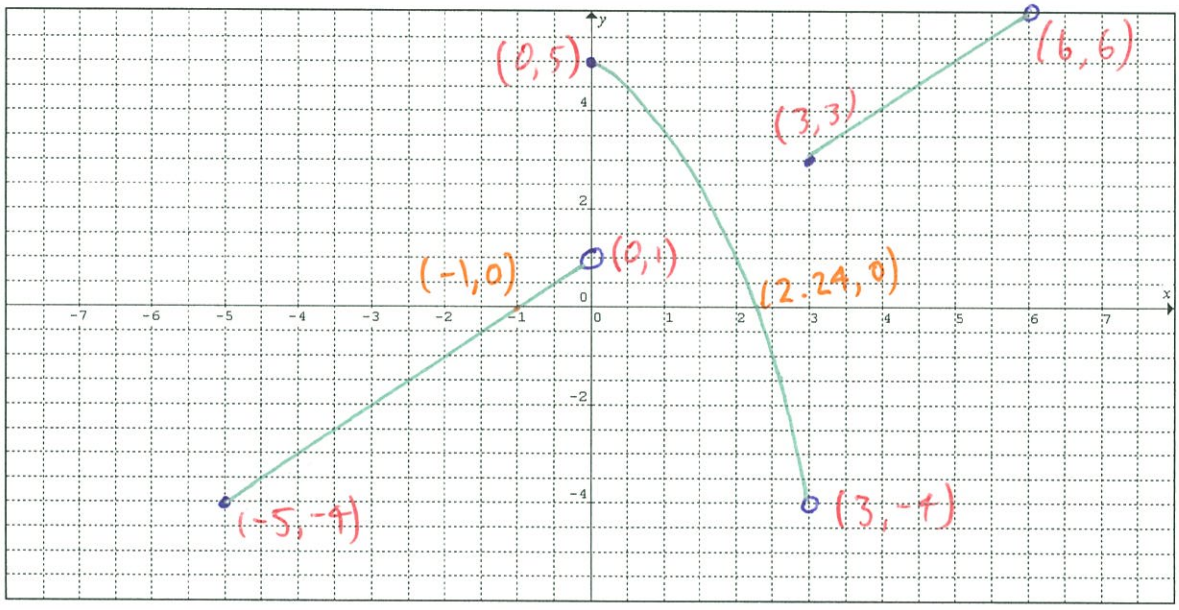


1+1+3 = 5 marks

Question 22

Write in simplest index notation:

a) $\dfrac{(a^2b^3)^2 \times (4ab^{-2})^3}{16(a^{-3}b^5)^2}$

$= \dfrac{a^4 b^6 \times 4^3 a^3 b^{-6}}{16 a^{-6} b^{10}}$

$= \dfrac{4^3 a^7}{16 a^{-6} b^{10}}$

$= \dfrac{4a^{13}}{b^{10}}$

b) $\dfrac{(^-3a^n b^{2n-2})^3 \times 4(ab^{1-n})^2}{(2a^{n+2}b^3)^6}$

$= \dfrac{(-3)^3 a^{3n} b^{6n-6} \times 4 a^2 b^{2-2n}}{2^6 a^{6n+12} b^{18}}$

$= \dfrac{-27 a^{3n+2} b^{4n-4} \times 4}{1664 a^{6n+12} b^{18}}$

$= \dfrac{-27 a^{-3n-10} b^{4n-22}}{16}$

$= \dfrac{-27 b^{4n-22}}{a^{3n+10}}$

3 + 3 = 6 marks

Question 23

22. Solve for x in the following equations:

a) $2^{2x-3} - 64^x = 0.$

$2^{2x-3} = 64^x$

$2^{2x-3} = (2^6)^x$

$2^{2x-3} = 2^{6x}$

$2x - 3 = 6x$

$-3 = 4x$

$x = \dfrac{-3}{4}$

b) $\log_x(2) - \log_x(128) = {}^-3$

$\log_x\left(\dfrac{2}{128}\right) = -3$

$x^{-3} = \dfrac{1}{64}$

$\dfrac{1}{x^3} = \dfrac{1}{64}$

$x^3 = 64$

$x^3 = 4^3$

$x = 4$

2+2=4 marks

Question 24

(a) What are the values k can take for which the equation $(k+2)x^2+(3k+3)x+2k+2=0$ has only one rational solution?

$\Delta = 0$

$\Delta = b^2 - 4ac$

$(3k+3)^2 - 4(k+2)(2k+2) = 0$

$k = -1$ or 7

(b) For the values of k calculated in part (a), solve the equation.

$k=7 \Rightarrow 9x^2+24x+16=0 \Rightarrow x = -\frac{4}{3}$

$k=-1 \Rightarrow x^2=0 \Rightarrow x=0$

(c) Solve the equation when $k=-3$. Give your answer in EXACT Values.

$(-3+2)x^2 + (3(-3)+3)x + 2(-3)+2 = 0$

$-x^2 - 6x - 4 = 0 \Rightarrow x^2+6x+4=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow x = \frac{-6 \pm \sqrt{36-16}}{2}$

$= \frac{-6 \pm \sqrt{20}}{2}$

$= \frac{-6 \pm 2\sqrt{5}}{2}$

$= -3 \pm \sqrt{5}$

3+2+2=7 marks

Question 25

A cup of coffee is left to cool on a kitchen table. The temperature of the coffee T $(°C)$ after t minutes is measured and recorded in a table:

T (mins)	1	2	10	15
T (^{0}C)	80.5	76.4	50.9	39.3

a. Use your calculator to generate the equation of the line of best fit, $y = a \times b^x$, giving the coefficients to 2 d.p.

$T = 84.70 \times 0.95^t$

b. According to your equation, what is the initial temperature of the coffee, to the nearest degree?

85°

c. By how many degrees does the coffee cool in 5 minutes? Give your answer to the nearest degree

when $t = 5 \Rightarrow (5, 65.54)$

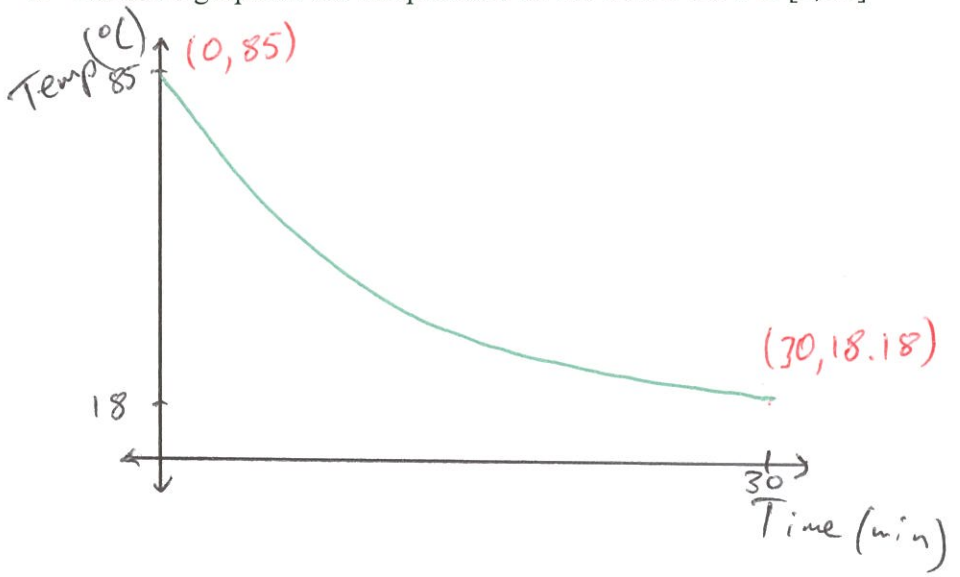
Cools down by $20^\circ C$

d. How long (in minutes, accurate to 2 decimal places) does it take the coffee to cool down to 48^{0}C?

$x = 11.07$

$\therefore$ It takes 11 min to cool to get to $48^\circ C$

e. Sketch a graph of the temperature of the coffee for $t \in [0,30]$



2 + 1 + 2 + 1 + 2 = 8 marks

END OF EXAMINATION