Student Name…………………………………..

Teacher (circle one)   DKI   JOR   VNA

Homegroup ………….……………………

MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 1

Wednesday November 2nd 2016

Reading Time: 1:00 – 1:15pm (15 minutes)
Writing time: 1:15 – 2:15pm   (1 hour)

Instructions to students

This exam consists of **17** questions.
All questions should be answered in the spaces provided.
There are **65** marks available in this examination.
A decimal approximation will not be accepted if an exact answer is required.
Where more than one mark is allocated to a question working must be shown.
Students **may not** bring any notes or any calculators into this exam.
Diagrams in this exam are not to scale except where otherwise stated.

FORMULAS

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$\Pr(A \cap B) = \Pr(A).\Pr(B)$$
$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Newton's Iterative formula for approximating roots of a polynomial:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1) Given $A = \begin{bmatrix} -2 & 4 \\ -6 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ -2 & 7 \end{bmatrix}$, calculate the following:

a) $B - 2A$

b) AB

(2 + 2 = 4 marks)

2) Consider the set of simultaneous equations:

$$5x - 6y = 21$$
$$x - 2y = 5$$

a Write the set of equations as a matrix equation.

b Use a matrix method to solve the equations and hence determine the values of x and y.

(1 + 3 = 4 marks)

3) Using matrix methods:

a) <u>Calculate</u> the coordinates of the image of the point $(17,-5)$ under the translation defined by $T = \begin{bmatrix} -8 \\ 9 \end{bmatrix}$.	b) <u>Calculate</u> the coordinates of the image of the point $(6,-13)$ under the linear transformation defined by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

c) Describe the transformation defined by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. (See part b) above)

$(2 + 2 + 1 = 5$ marks$)$

4)   Find the exact values of

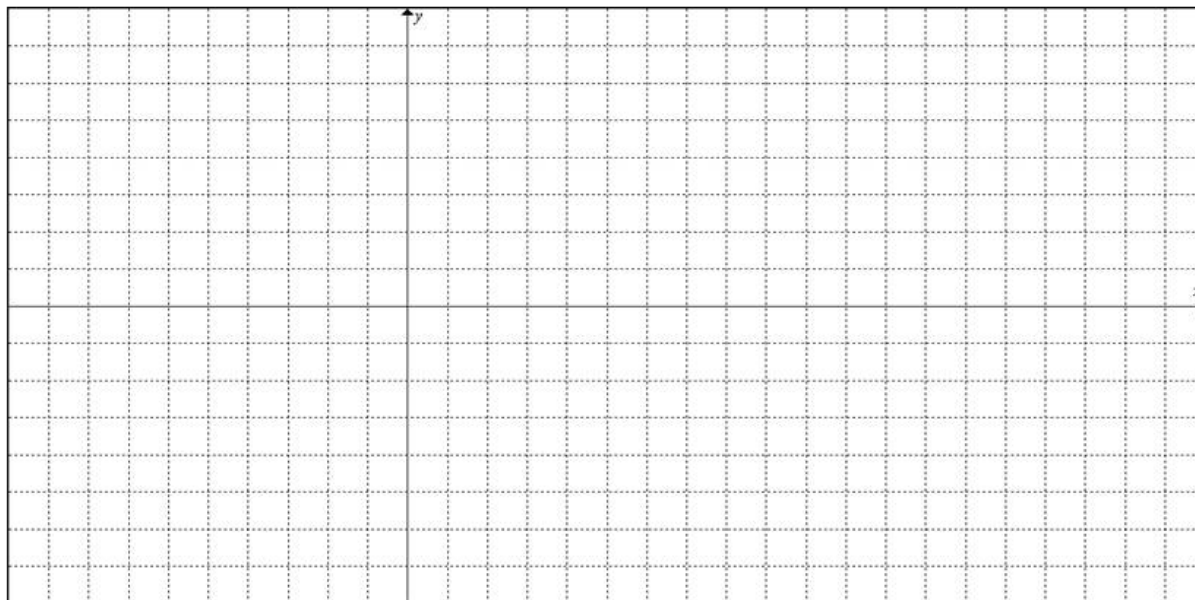| a) $\sin 60^0$	b) $\tan \dfrac{2\pi}{3}$	c) $\cos\left(-\dfrac{\pi}{6}\right)$

(3 marks)

5) Solve the following equation $2\sin x = \sqrt{3}, -2\pi \le x \le 2\pi$

(4 marks)

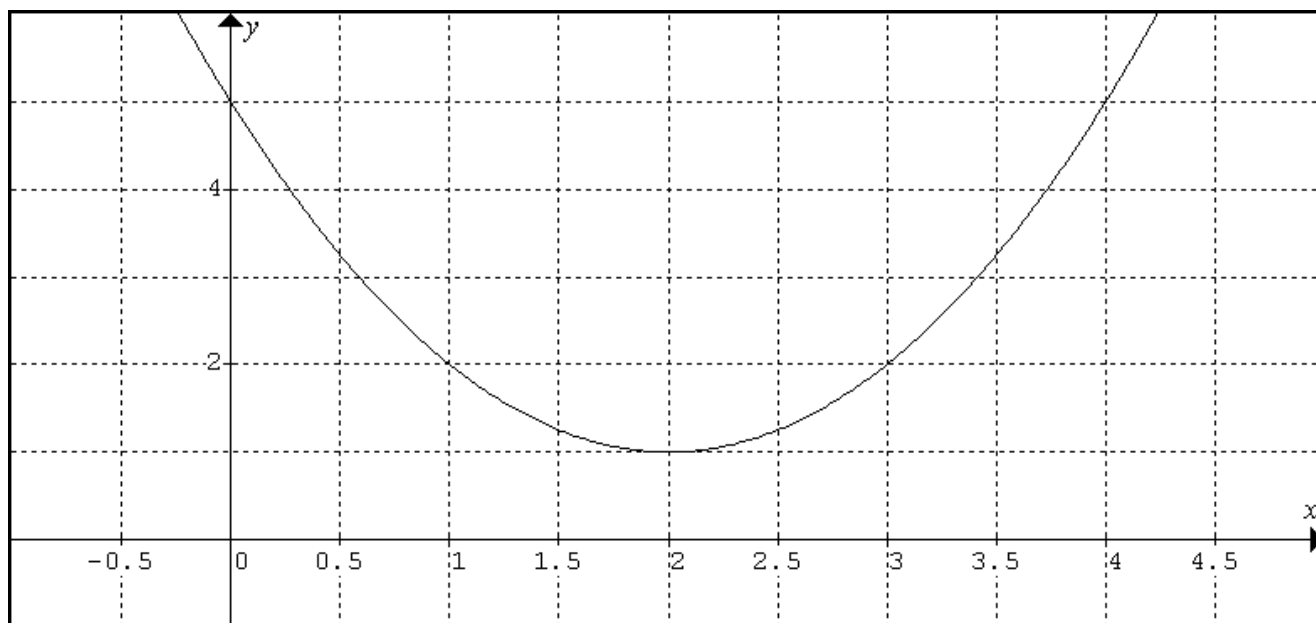6)   a) What is the period and the amplitude of the graph of $y = -4\cos 2x$?

       b) Sketch the graph, showing <u>one complete cycle</u>. Clearly label key points.

                                                                    (2 + 3 = 5 marks)

7)   Part of the graph of the function $f : R \rightarrow R, f(x) = (x-2)^2 + 1$ is shown below.

   a)   Find the average rate of change of $y = f(x)$ with respect to x, between x = 0 and x = 3.

b) Find the instantaneous rate of change of $y = f(x)$ with respect to x at the point where $x = 5$.

<div style="text-align: right">(2 + 2 = 4 marks)</div>

8)   Find, using first principles, the derivative of $y = x^2 + 5x + 1$

<div style="text-align: right">(3 marks)</div>

9)   If $f(x) = x^2(3x^2 - x) + 7$, find $f'(-1)$.

<div style="text-align: right">(3 marks)</div>

10) Find the derivatives of: (make sure all answers are expressed with positive indices)

a) $y = \dfrac{7x^2 - 2x}{x}$	b) $f(x) = \dfrac{4}{3x^4}$

(2 + 2 = 4 marks)

11) Simplify, making sure that all answers are expressed with positive indices

a) $\int (5x^3 + 3x^2 + 2)dx$	b) $\int \sqrt[3]{x^2}\,dx$

(2 + 2 = 4 marks)

12) Find $\lim\limits_{x \to 2} \dfrac{x^2 + 3x - 10}{x^2 - 4}$

(2 marks)

13) A particle moves in a straight line such that its displacement, x metres, from a fixed origin at time t seconds is modelled by $x = t^2 - 6t + 8, t \geq 0$.
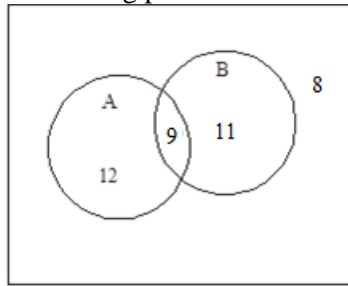
a Identify its initial position.

b Show, using calculus, that the particle is momentarily at rest at $t = 3$ seconds.

c What is the average speed of the particle over the first 4 seconds?

(1 + 2 + 3 = 6 marks)

14) Use this Venn Diagram to find the following probabilities:

a) Pr($B'\cap A$)	b) Pr($B \mid A$)	c) Pr($A' \cup B$)

(3 marks)

15) If Pr(B) = 0.42, Pr($A' \cap B$) = 0.16 and Pr(A') = 0.51,

a) complete this probability table

	B	B'	
A			
A'	0.16		0.51
	0.42		1

b) Find Pr($A \cap B'$)

c) Find Pr($A \cup B$)

d) Find Pr($A' \mid B$)

(2 +1 +1 + 2 = 6 marks)

16) Mr Oates needs two students to take some parents on a school tour. He chooses them randomly from a group of ten that were standing near his office. How many different groups of two could he choose?

2 marks

17)

a) Define an iterative formula using Newton's Method for the function $f(x) = 6x^3 + 4x - 3$.

b) Use this to calculate the value of x_1 when $x_0 = 1$. Give your answer as an exact value.

(2 + 1 = 3 marks)

END OF EXAMNATION