Student Name....SOLUTIONS....

Teacher (circle one) DKI JOR VNA

Homegroup ....

MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 2

Section	# of questions	# of questions to be answered	Number of marks
1	16	16	16
2	9	9	62
		Total	78

Monday November 7th 2016

Reading Time: 11:45 – 12:00pm (15 minutes)
Writing time: 12:00 – 1:30pm (90 minutes)

Instructions to students

This exam consists of **17** questions. All questions should be answered in the spaces provided. There are **65** marks available in this examination. A decimal approximation will not be accepted if an exact answer is required. Where more than one mark is allocated to a question working must be shown. Students **may not** bring any notes or any calculators into this exam. Students may bring one bound reference book into the exam. Students may bring a CAS and a Scientific calculator into the exam. No paper or electronic dictionaries may be used. Diagrams in this exam are not to scale except where otherwise stated.

FORMULAS

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \cap B) = \Pr(A).\Pr(B)$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Newton's Iterative formula for approximating roots of a polynomial:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Section A: Multiple Choice. Choose the best answer and RECORD ON ANSWER SHEET.

1 If $\mathbf{A} = \begin{bmatrix} 2 & 7 \\ -5 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 11 & 3 \\ -2 & 0 \end{bmatrix}$ then $2\mathbf{A} + 3\mathbf{B}$ is equal to

A $\begin{bmatrix} -29 & 5 \\ -4 & 8 \end{bmatrix}$

B $\begin{bmatrix} 13 & 10 \\ -7 & 4 \end{bmatrix}$

C $\begin{bmatrix} 28 & 27 \\ -19 & 12 \end{bmatrix}$

D $\begin{bmatrix} 37 & 23 \\ -4 & 8 \end{bmatrix}$

E $\begin{bmatrix} 37 & 23 \\ -16 & 8 \end{bmatrix}$

2 The determinant of the matrix $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$ is

A -7

B -1

C 1

D 7

E 13

$10-3$

3 The matrix which determines the transformation, a dilation from the x-axis of factor 2 followed by reflection in the line $y = x$ is

A $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

C $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

D $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

E $\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$

4 The matrix which describes the composition of mappings

- dilation of factor 3 from the x-axis
- reflection in the line $y = x$
- reflection in the x-axis

is

A $\begin{bmatrix} 3 & 0 \\ -1 & 3 \end{bmatrix}$

B $\begin{bmatrix} 0 & 0 \\ -1 & 3 \end{bmatrix}$

C $\begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix}$

D $\begin{bmatrix} 0 & 0 \\ -3 & 1 \end{bmatrix}$

E $\begin{bmatrix} 0 & 3 \\ -1 & 0 \end{bmatrix}$

5 The exact value of $\cos(-330°)$ is:

A $\frac{1}{2}$

B $-\sqrt{3}$

C $-\frac{\sqrt{3}}{2}$

D $\frac{\sqrt{3}}{2}$

E $\sqrt{3}$

6 If $\sin a = 0.2$, then $\cos a$ must equal:

A 0.80

B 0.96

C 0.98

D $\sqrt{0.96}$

E 0.2

$\cos(a) = \sqrt{1-\sin^2(a)}$

7 The following limits $\lim_{x\to -3}(3x^2-4)$ and $\lim_{x\to -3}(4-3x^2)$ evaluate respectively to:

A 23 and 9
B −23 and 31
C −31 and 31
D −23 and 23
E 23 and −23

8 If $y=-2x^3+6x^2-4x+7$, then $\frac{dy}{dx}$ is equal to:

A $-2x^2+6x-4$
B $-3x^2+2x-4$
C $-5x^2+8x-4$
D $4x^2+12x-4$
E $-6x^2+12x-4$

9 If $y = x^3 - 2x^2 + 3x - 4$, then the gradient of the *perpendicular* to this curve at $x = {}^-2$ is:

A $3x^2 - 4x + 3$
B $\frac{-1}{23}$
C $\frac{-1}{3x^2-4x+3}$
D $\frac{1}{23}$
E $^-1$

$3x^2-4x+3$

$3(4)+8+3$

23

10 If a curve passes through the point (2, −3) and at any point has a gradient given by $3x^2+2x-1$, then the y-intercept of the curve must be:

A −17
B −13
C −9
D −5
E −1

x^3+x^2-x+c

$2^3+2^2-2+c=-3$

$10+c=-3$

$c=-13$

11 A colony of organisms multiplies at a rate according to $N(t)=4t^2+1$. The average rate of increase between $t = 3$ and $t = 4$ is:

A 28
B 37
C 65
D 18
E 38

$t=3$ $N(3)=37$

$N(4)=65$

12 For the missile fired with displacement equation $s(t)=800-16t-3t^2$, the instantaneous rate of change (velocity) at $t = 3$ is:

A −16
B −6
C 725
D −34
E 34

$s'(t)=-16-6t$

13 The height, h, of a netball is given by the equation $h(x)=-0.1x^2+0.9x+1$ where x is the horizontal displacement of the ball. The horizontal displacement when the ball reaches its maximum height is:

A 9
B 4
C 5
D 4.7
E 4.5

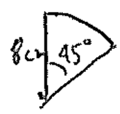
$-0.2x+0.9=0$

$-0.2x=-0.9$

$x=\frac{9}{2}$

14 An arc subtends an angle of 45^0 at the centre of a circle of radius 8 cm. The length of the arc (in cm) is:

A 360
B 6.70
C 0.68
D 5.34
E 53.61



6.28

$\frac{\pi}{4}\times 8$

15 The function with the rule $f(x) = 4\tan\left(\frac{\pi x}{2}\right)$ has period

A 4

B $\frac{1}{2}$

C 2

D $\frac{\pi}{2}$

E $\frac{2}{\pi}$

$\frac{\pi}{\frac{\pi}{2}}$ $\quad \frac{\cancel{\pi}}{1} \times \frac{2}{\cancel{\pi}}$

16 A box contains six red blocks and four blue blocks. Three blocks are drawn from the box without replacement.

The probability that all of the blocks are red is

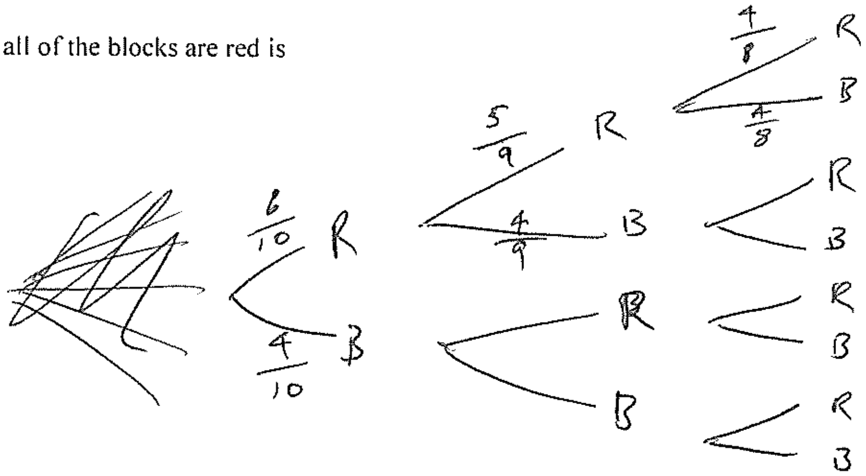
A $\frac{8}{125}$

B $\frac{3}{500}$

C $\frac{24}{29}$

D $\frac{27}{125}$

E $\frac{1}{6}$



$$\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{120}{720}$$

Section B: Short and Extended response questions. Answer in the space provided.

1. (4 marks)

Evaluate the root of the equation $x^3 = x + 8$ by carrying out two iterations of Newton's Method, starting at $x_0 = 2$. Give your answer to 4 decimal places.

$$x_1 = x_0 - \frac{f(x_0)}{f(x_0)}$$

$$f(x) = -x^3 + x + 8$$
$$f'(x) = -3x^2 + 1$$

$$x_1 = 2 - \frac{-2^3 + 2 + 8}{-3 \times 2^2 + 1}$$
$$= 2 - \frac{2}{-11}$$
$$= \frac{24}{11}$$
$$x_2 = \frac{24}{11} - \frac{-\left(\frac{24}{11}\right)^3 + 2 + 8}{-3\left(\frac{24}{11}\right)^2 + 1}$$
$$= 2.1664$$

2. (5 marks)

The curve with equation $y = \sin(x)$ is subject to the following transformations;

- a dilation in the y-direction (from the x axis) of 3 units
- a reflection in the x axis

a What is the equation of the transformed graph?

$$y = -3\sin(x)$$

b i) What is the maximum value of y on the transformed curve?

3

ii) When does this maximum occur (exact answer)?

$\frac{3\pi}{2}$

c What is the exact value of y when $x = \frac{\pi}{3}$? (Show working)

$$y = -3\sin\left(\frac{\pi}{3}\right)$$
$$= -3 \times \frac{\sqrt{3}}{2}$$
$$= \frac{-3\sqrt{3}}{2}$$

1 + 1 + 1 +2 = 5 marks

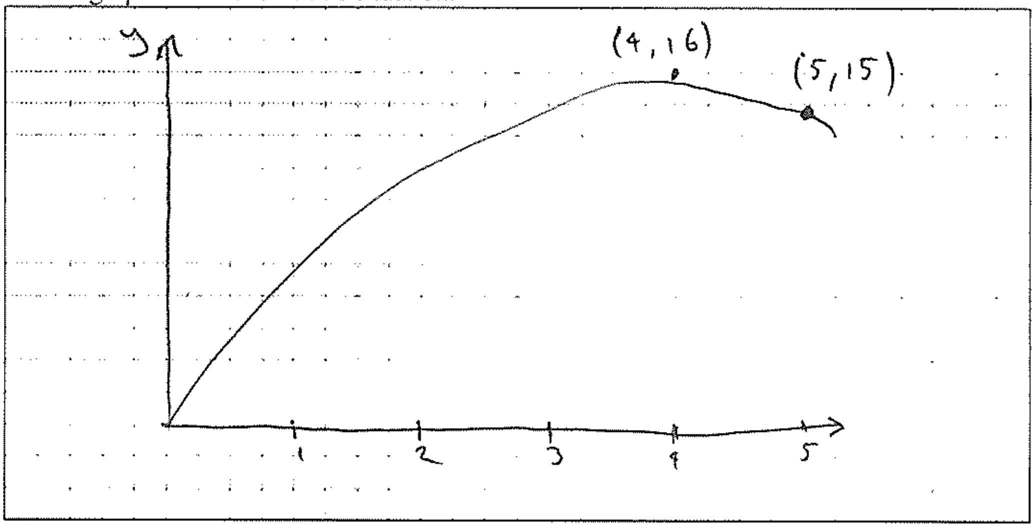
3. (8 marks)

Harry hits a ball; which was resting on flat ground, up into a nearby tree. The ball becomes stuck at a point 5 metres horizontally from where it was hit. The path of the ball is given by

$$y = -x^2 + 8x, \quad x \geq 0,\ y \geq 0$$

where x is the horizontal distance in metres from the point where the ball rested on the ground and y is the height in metres of the ball above the ground.

a. Sketch a graph which shows this situation.



2 marks

b. How high vertically above the ground is the ball when it becomes stuck? **(label this on your graph)**

15 m

1 mark

c. What is the maximum height that the ball reaches above the ground before it becomes stuck? **(label this on your graph)**

at $x = 4$, $\underline{y = 16}$

16 m

2 marks

d. Had the ball not become stuck in the tree, how far horizontally from its starting position would it have landed?

8 m

1 mark

e. During the ball's flight, over what horizontal distance was the ball 15 metres or higher above the ground?

2 m

at $x = 3$, $y = 15$

so from 3 to 5

2 marks

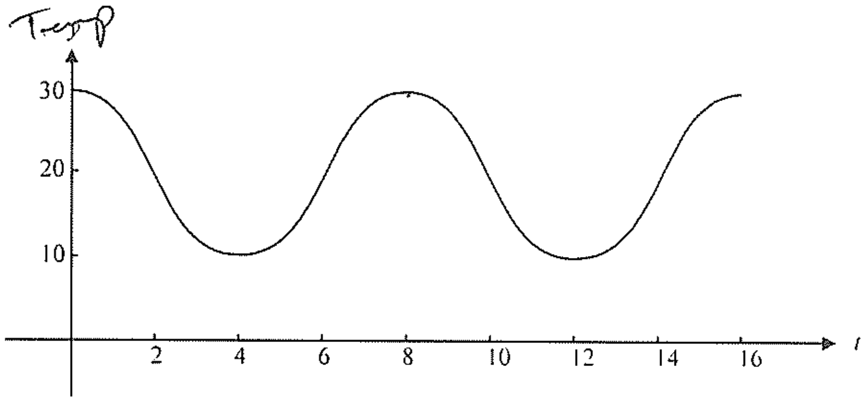
4. (13 marks)

In a laboratory, the temperature in a controlled environment is given by

$$y(t) = a\cos(bt) + c$$

where a, b and c are constants, y represents the temperature in degrees Celsius and t represents the time in hours where $t = 0$ corresponds to noon on Wednesday.

The graph of the function is shown below.



a. Explain why

a. $a = 10$ amplitude

b. $b = \frac{\pi}{4}$ $P = \frac{2\pi}{b} \Rightarrow P = 8$ so $8 = \frac{2\pi}{b}$, $b = \frac{2\pi}{8} = \frac{\pi}{4}$

c. $c = 20$ moved up 20

$1 + 1 + 1 = 3$ marks

b. Write down:

a. the minimum temperature in the controlled environment.

10°C

b. the mean temperature in the controlled environment.

20°C

$1 + 1 = 2$ marks

c. Write down the temperature in the controlled environment at 3pm on Wednesday. Express your answer in degrees Celsius correct to 2 decimal places.

$t = 3$, $y(3) = 10\cos\left(\frac{\pi}{4} \times 3\right) + 20$

$= 12.93$ °C

1 mark

d. At what time did the temperature in the controlled environment first reach 25°C?

$t = 1\frac{1}{3}$ hours $\Rightarrow$ 1:20 pm Wednesday

2 marks

e. Hence find for what period of time the temperature is greater than or equal to 25°C between noon and midnight on Wednesday?

first $1\frac{1}{3}$ hrs, at $6\frac{2}{3}$ till $9\frac{1}{3}$

$\Rightarrow 2\frac{2}{3}$

so for 4 hours

3 marks

In a second controlled environment the temperature is given by $p(t) = 8\cos\left(\frac{\pi t}{4}\right) + 20,\ t \geq 0$ where p represents the temperature in degrees Celsius and t represents the time in hours where $t = 0$ corresponds to noon on Wednesday.

f. When is the temperature in the original controlled environment first one degree higher than in the second controlled environment?
Explain how you found your answer.

$10\cos\left(\frac{\pi t}{4}\right) + 20 = 8\cos\left(\frac{\pi t}{4}\right) + 21$

intersection at

$1\frac{1}{3}$ so 1:20pm

2 marks

5. (3 marks)
If $\sin\theta = 0.3$, write down the value of:

a $\sin(\pi - \theta)$	b $\sin(\pi + \theta)$	c $\sin(-\theta)$
0.3	−0.3	−0.3

6. (9 marks)

A particle moves in a straight line and starts from a fixed point O on the line.

Its acceleration a m/s^2 is given by $a = 24t - 72$ for any time t seconds, $t \geq 0$.

a What is its acceleration at $t = 5$?

$$a|_{t=5} = 24 \times 5 - 72$$
$$= 48 \text{ ms}^{-2}$$

1 mark

b If its initial velocity is 96 m/s, what is its velocity for any time t seconds?

$$v(t) = 12t^2 - 72t + c$$
$$v(0) = 96 = c$$
$$\Rightarrow \quad v(t) = 12t^2 - 72t + 96$$

2 marks

c When is its velocity zero?

$$0 = 12t^2 - 72t + 96$$
$t = 2$ and $t = 4$

1 mark

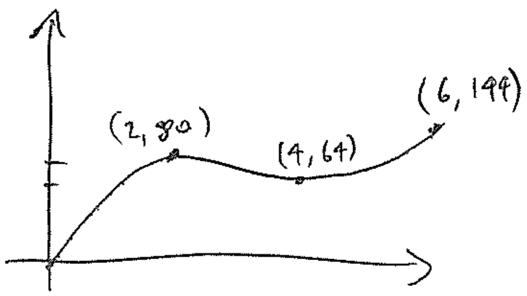
d What is its displacement from O at any time t seconds?

at $t = 0$, $x(0) = 0$

$$\Rightarrow \quad x(t) = 4t^3 - 36t^2 + 96t$$

1 mark

e Find the total distance covered by the particle in the first 6 seconds.



2 seconds = 80 m

next 2 seconds = 16 m

next 2 seconds = 80 m

total = 176 m

4 marks

7. (4 marks)

From a group of 12 female students, 2 female staff, 10 male students and 3 male staff, a committee of 6 is to be formed.

Find the number of different committees if:

a. there are no restrictions.

$^{27}C_6 = 296010$

b. All committee members must be students

$^{22}C_6 = 74613$

c. There is an equal number of males and females on the committee

$^{14}C_3 \times {}^{13}C_3 = 104104$

d. The committee must comprise of 1 male staff member, 1 female staff member, 2 female students and 2 male students.

$^{3}C_1 \times {}^{2}C_1 \times {}^{12}C_2 \times {}^{10}C_2 = 17820$

8. (14 marks)

In an animated computer game an adventurer can travel from his base to a treasure chest by three different routes. The route is chosen randomly.

The probability of the adventurer travelling on the desert route is $\frac{1}{2}$.

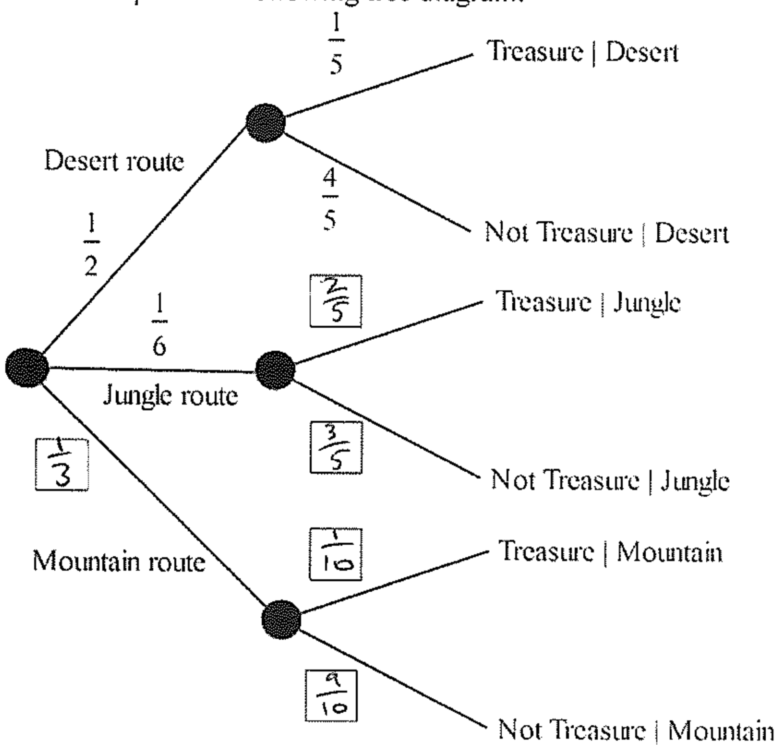
The probability of the adventurer travelling on the jungle route is $\frac{1}{6}$.

The probability of the adventurer travelling on the mountain route is $\frac{1}{3}$.

The probability that the adventurer obtains the treasure by a given route is given in the table below.

Treasure given desert route	Treasure given jungle route	Treasure given mountain route
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$

a Complete the following tree diagram.



3 marks

b Find the probability that the adventurer follows the desert route and obtains the treasure.

$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$

1 mark

c Find the following probabilities:

i the adventurer obtains the treasure

$$\frac{1}{10} + \frac{1}{6} \times \frac{2}{5} + \frac{1}{3} \times \frac{1}{10} = \frac{1}{5}$$

2 marks

ii the adventurer does not obtain the treasure

$$1 - \frac{1}{5} = \frac{4}{5}$$

2 marks

d Find the probability that, given the adventurer obtains the treasure, he has travelled:

i the desert route

$$Pr(\text{desert} \mid \text{treasure}) = \frac{Pr(\text{desert and treasure})}{Pr(\text{treasure})}$$

$$= \frac{1}{10} \times \frac{5}{1} = \frac{1}{2}$$

2 marks

ii the jungle route

$$Pr(\text{jungle} + \text{treasure}) = \frac{1}{6} \times \frac{2}{5} = \frac{1}{15}$$

$$Pr(\text{jungle} \mid \text{treasure}) = \frac{1}{15} \times \frac{5}{1} = \frac{1}{3}$$

2 marks

iii the mountain route.

$$Pr(\text{mountain} + \text{treasure}) = \frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$$

$$Pr(\text{mountain} \mid \text{treasure}) = \frac{1}{30} \times \frac{5}{1} = \frac{1}{6}$$

2 marks

9. (2 marks)

Find all possible rational x intercepts of $3x^3 + 4x^2 - 7x + 6$

factors of 3 are $\pm 1, \pm 3$

factors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$

Possible roots are $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 3, \pm \frac{3}{2},$

END OF EXAMINATION