Student Name……………………………………

Teacher (circle one)   AMA   VNA

Homegroup ……………………………………

MATHEMATICAL METHODS (CAS) UNIT 1

EXAMINATION 2

Friday June 9th 2017

Reading Time: 8.45-9:00 (15 minutes)
Writing time: 9.00-10.30 (90 minutes)

Instructions to students

This exam consists of Section 1 and Section 2.

Section 1 consists of **12** multiple-choice questions, to be answered on the separate answer sheet. It is worth **12** marks.

Section 2 consists of **7** extended-answer questions that should be answered in the spaces provided. It is worth **62** marks

There is a total of **74** marks available.

All questions in Section 1 and Section 2 should be answered.

Unless otherwise stated, diagrams in this exam are not drawn to scale.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Students may bring one bound reference into the exam.
Students may bring an approved CAS calculator.

MATHEMATICAL METHODS (CAS) UNIT 1

EXAM 2

SECTION 1

ANSWER SHEET

Student Name…………………………………….

Teacher (circle one) AMA VNA

Homegroup……………………………………

Question	**Answer (A – E)**
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11	
12	

MULTIPLE-CHOICE QUESTIONS

Question 1
The equation of the line passing through the points (–2, 3) and (4, 6) is:
A $2y - x - 8 = 0$
B $2y + x + 8 = 0$
C $y - 2x - 8 = 0$
D $2y + 2x + 8 = 0$
E $y + 2x - 8 = 0$

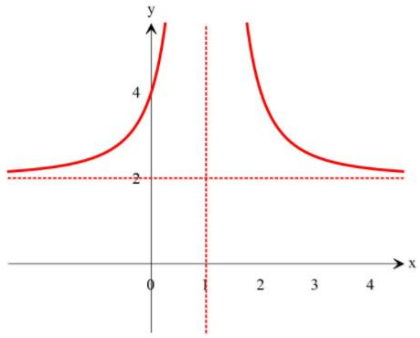
Question 2
M is the midpoint of XY. The coordinates of M and Y are (7,–3) and (5,4) respectively. The coordinates of X are
A $(6, \frac{1}{2})$
B (4, -14)
C (2, -7)
D (9, -10)
E (6, -5)

Question 3
The maximum value of the function $h(x) = -2x^2 + 4x - 6$ is
A -4
B -2
C -1
D 2
E 4

Question 4
The value of the discriminant in $3x^2 + 8x - 2$ is:
A $2\sqrt{11}$
B 44
C $2\sqrt{22}$
D 88
E 73

Question 5
The values of x for which $x^2 + x - 20 < 0$ are given by
A $x \in (-5,4)$
B $x \in [-4,5]$
C $x \in [-5,4]$
D $x \in (-\infty,-5) \cup (4,\infty)$
E $x \in (-\infty,-5] \cup [4,\infty)$

Question 6
The rule for the graph shown is of the form $y = \frac{a}{(x-b)^2} + c$. The values of a, b and c are:



A $a = 1, b = 2 \text{ and } c = 3$
B $a = 1, b = 2 \text{ and } c = 4$
C $a = -1, b = 2 \text{ and } c = 6$
D $a = 2, b = 1 \text{ and } c = 2$
E $a = 4, b = 1 \text{ and } c = 2$

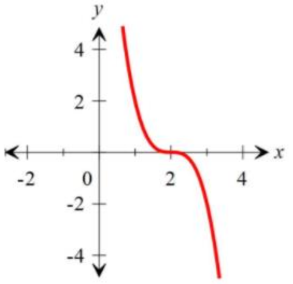
Question 7
If $f(x) = 2x^3 - 3x$, then $f(2x - 1)$ equals:
A $16x^3 + 6x$
B $16x^3 - 6x$
C $16x^3 + 24x^2 + 6x - 1$
D $16x^3 - 6x^2 + 24x + 1$
E $16x^3 - 24x^2 + 6x + 1$

Question 8
The polynomial $P(x) = x^3 + 2x^2 - 10x + d$ is divided by $x - 1$. If the remainder is 6, the value of d is:

A 10
B 14
C -10
D 2
E 13

Question 9

Choose a function that best represents the graph below:



A $f(x) = -2(x-2)^2$

B $f(x) = 2x^3$

C $f(x) = -2x^3 + 2$

D $f(x) = -2(x-2)^3$

E $f(x) = -2(x+2)^3$

Question 10

The x-axis intercepts of the graph with the equation $y = (x+a)^2(x^2 - b^2)$, where a and b are positive constants, are

A $a, -a, b, -b$

B $a, b, -b$

C $a, -a, b$

D $-a, b, -b$

E $a, -a, -b$

Question 11

The curve with equation $y = x^3$ is transformed under a dilation of factor 2 from the y-axis followed by a translation of 3 unit in the positive direction of the x-axis. The equation of the image is:

A $y = 2(x-3)^3$

B $y = 2x^3 + 3$

C $y = \frac{(x-3)^3}{8}$

D $y = \frac{(x-3)^3}{2}$

E $y = \frac{(x+3)^3}{8}$

Question 12

Two events A and B are independent. Given that $\Pr(A) = 0.2$ and $\Pr(B) = 0.6$ then $Pr(A \cup B)$ is equal to

A 0.12

B 0.52

C 0.68

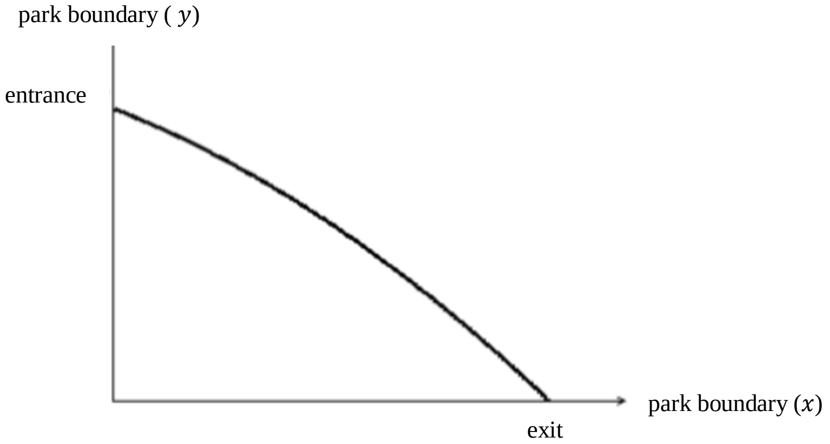
D 0.8

E 0.92

SECTION 2 EXTENDED-ANSWER QUESTIONS

Question 1 (7 marks)

A parabolic path passes through a local park, as shown in the diagram below. The equation of the path is given by $y = -\frac{1}{5}x^2 - 4x + 100$.



1.a

Find the co-ordinates of the point of entry

1 mark

1.b

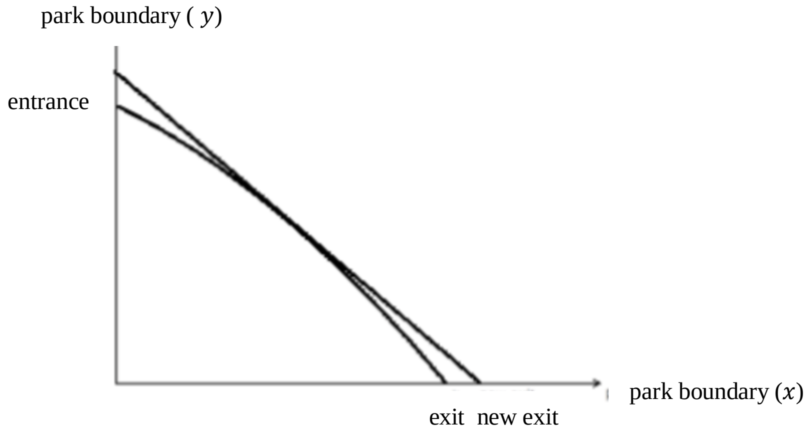
The point of exit has co-ordinates $(10\sqrt{b} - a, 0)$. Find the values of a and b.

2 marks

1.c

The council decides to build another path through the park. The equation of the new path is

$y = -6x + 105$



Show that the new path intersects the old path at 5km east and 75km north.

2 marks

1.d

Find, to 2 decimal places, the length of the new path

2 marks

Question 2 (Probability) 6 marks

When Ted gets to play tennis, he plays just once for the week. There is 30% chance that he plays on a Saturday (S), otherwise he plays on a Wednesday (W).
He plays mixed tennis (M) on Saturdays 80% of the time and on Wednesdays 60% of the time.

a) Represent this information on a tree diagram showing clearly the probabilities on each of the branches.

2 marks

b) What is the probability that Ted plays mixed tennis on a Wednesday?

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1 mark

c) What is the probability that Ted plays mixed tennis?

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1 mark

d) Given that Ted played mixed tennis this week, what was the probability that he played on a Saturday?

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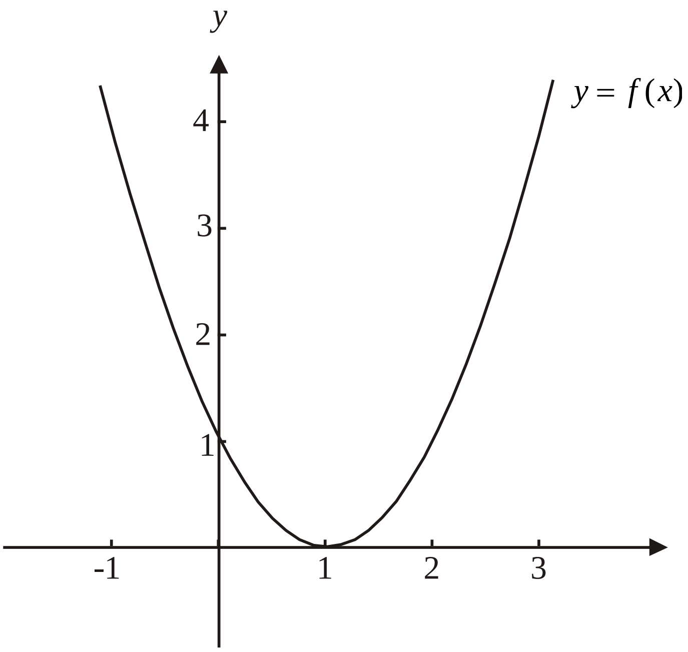
2 marks

Question 3 (11 marks)

The graph of the function $f: R \to R,\ f(x) = (x-1)^2$ is shown below.



a. Write down

i. the domain of f

____

ii. the range of f

____

iii. the value of $f(2)$

____

$1+1+1=3$ marks

b. The inverse function; f^{-1}, does not exist. Explain why.

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1 mark

Suppose that the domain of f is restricted to $x \in [-3, a)$ so that an inverse function f^{-1} exists.

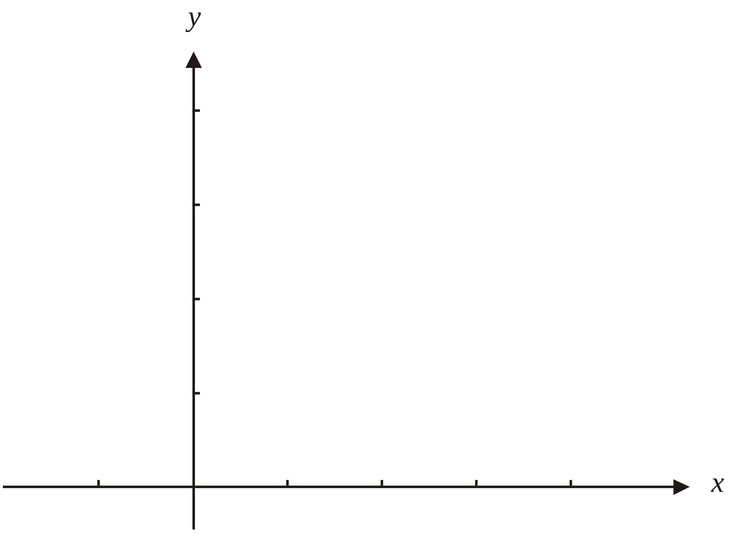
c. Write down a possible value of a.

____

1 mark

Consider the function $g:[1,\infty) \to R,\ g(x) = (x-1)^2$

d. On the set of axes below sketch the graph of $y = g(x)$. Clearly label on your graph any endpoints and/or intercepts.



2 marks

e. On the same set of axes above, sketch the graph of the inverse function of g; that is, sketch the graph of $y = g^{-1}(x)$.
Clearly label any endpoints and/or intercepts.

2 marks

f. State the rule using function notation for $g^{-1}(x)$.

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____

2 marks

Question 4 (10 marks)

A man stands on a balcony and hits a tennis ball into a yard for his dog to fetch.
The path of the ball is given by $y = -x^2 + 10x + 4,\ x \geq 0, y \geq 0$ where *y* represents the distance in metres above the ground and *x* represents the horizontal distance in metres from the point where the ball was hit.

(a) How high above the ground was the ball when it was hit?

1 mark

(b) Express the equation of the path of the ball in turning point form.

1 mark

(c) What is the maximum height reached by the ball?

1 mark

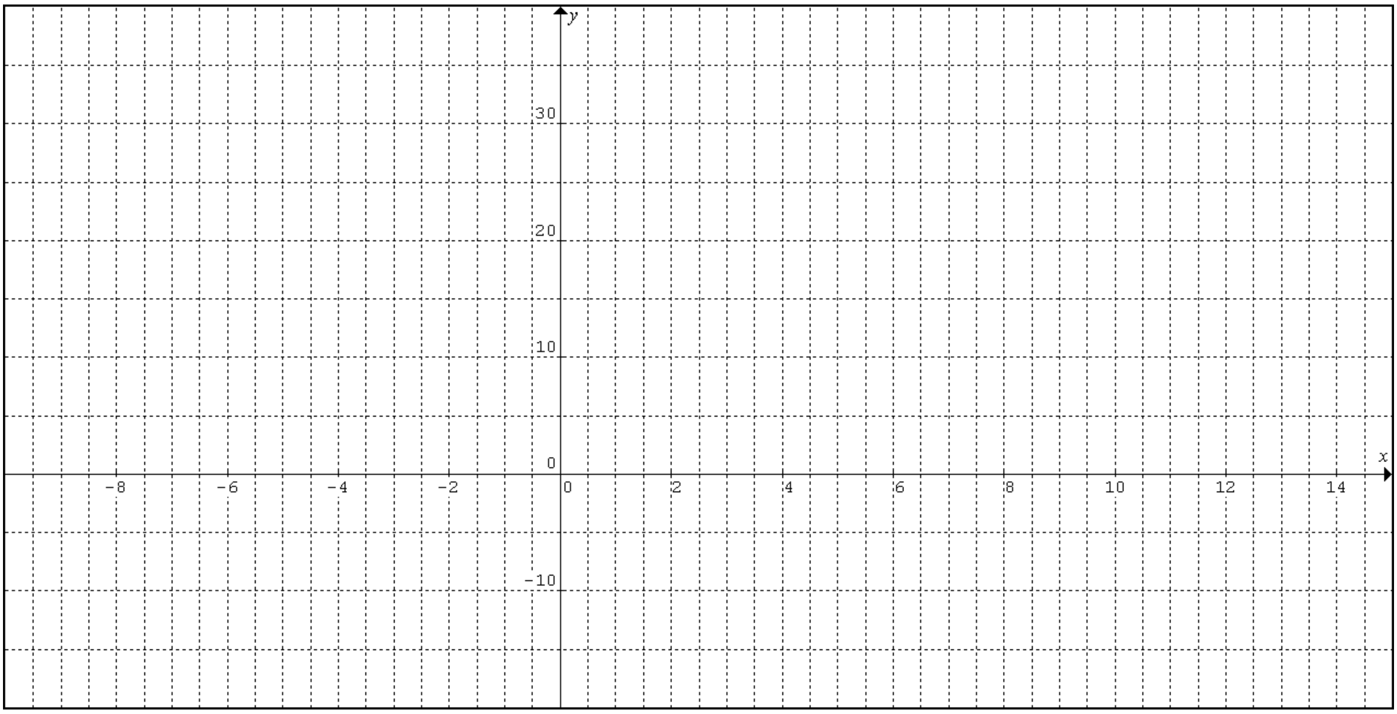
(d) What is the horizontal distance of the ball from the owner when this maximum height is reached?

1 mark

(e) Find the exact horizontal distance the ball is from the owner when it first hits the ground.

3 marks

(f) Sketch a graph of path of the tennis ball, labelling all key features.



3 marks

Question 5 (8 marks)

Let $A = \begin{bmatrix} 38 & 28 \\ 30 & 25 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

a) Find AB.

1 marks

b) Explain why the matrix product BA does not exist.

1 mark

c) Find the determinant of matrix A.

1 mark

d) Find A^{-1}, the inverse of A.

1 mark

e) i) Write using matrices the transformation for a translation of 2 units to the right and 4 down, reflection in the x axis and vertical dilation by a factor of 3, in that order

2 marks

ii) Apply the transformation to find what the curve with equation $y = \frac{1}{x^2}$ is mapped to under the transformation.

2 marks

Question 6 (11 marks)

A droving track winds its way through farmland criss-crossing a straight road.

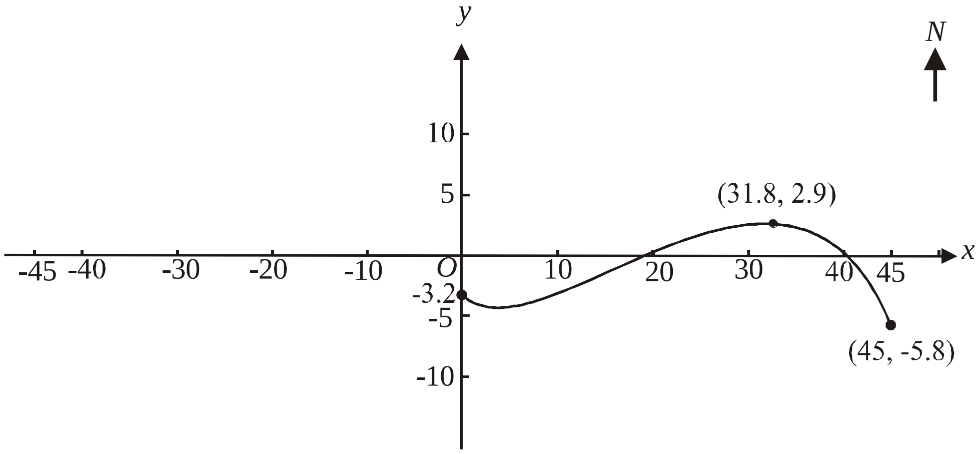
The track can be represented by the function

$$f : [-45, 45] \rightarrow R,\ \ f(x) = -0 \cdot 00001(x-40)(x-20)(x+10)(x+40)$$

where the unit of measurement is the kilometre and the origin O is located at $(0,0)$.

The road; which runs in an east-west direction, is represented by the x-axis.

Part of the graph of $y = f(x)$ for $x \in [0,45]$ is shown below.
The y - coordinate of the endpoint, the coordinates of the local maximum and the y-intercept are expressed correct to one decimal place.



a. On the set of axes above, complete the graph of $y = f(x)$ for $x \in [-45,0]$. Label clearly the coordinates (to 1 decimal place where appropriate) of any endpoints, intercepts and local maximum or minimum.

3 marks

b. How far south of the road is the track at the point 10 km east of the origin at O?

1 mark

c. What is the furthest distance south of the road that the track reaches? Express your answer in km correct to 1 decimal place.

1 mark

d. What is the straight line distance from the start of the track to the end of the track? Express your answer in kilometres correct to 2 decimal places.

2 marks

A controversial water pipe is planned. The proposed path of the pipeline can be described by the function

$$p:[-45,45] \to R,\ \ p(x) = \frac{-x}{40} + 2.$$

e. In how many places will the proposed pipeline cross the track?

1 mark

f. **i.** By moving the pipeline north 4.5 km, write down the rule of the new function that describes this new position of the pipeline.

____

1 mark

A radical plan to change the shape of the **original** track is considered where each point on the track has its perpendicular distance from the road halved, thus "flattening out" the shape of the track. For example the point at (31.8, 2.9) would be moved to (31.8, 1.45).

g. **i.** Describe the type of transformation that this represents.

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____

ii. Write down the rule of a function that describes this newly shaped track.

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1+1=2 marks

Question 7 (9 marks)

a) Consider the cubic polynomial $P(x) = x^3 - 3x^2 + 7x - 4$

i) Show the equation $x^3 - 3x^2 + 7x - 4 = 0$ has a root which lies between $x = 0$ and $x = 1$.

2 marks

ii) $P(1) = 1$ and $P(0) = -4$. Carry out two iterations of the method of bisection by hand to obtain two further estimates of this root.

2 marks

b) Find all possible rational x intercepts of $y = 2x^2 + 3x - 5$.

2 marks

c) By using completing the square, find the centre and radius of the following circle.

$$x^2 + y^2 - 11x - 10y + 24 = 0$$

3 marks

END OF EXAMINATION 2