Student Name: _____

Home Group: _____

Teacher's name: (please circle):   Ms Nation   Ms O'Rielly

Mathematical Methods

Unit 2

Wednesday 8th November 2017

Part I   **Total 57 marks**

Topics covered: -   **Combinatorics**
-   **Circular Functions**
-   **Rates of Change**
-   **Differential Calculus**
-   **Integral Calculus**
-   **Exponential Functions and Logarithms**

Complete working must be shown and simplified wherever possible in order to gain full marks.

Reading Time: 15 minutes

Writing Time: 60 minutes

Students are NOT permitted to use any calculators or reference books for this section.

No paper or electronic dictionaries may be used.

Useful formulae:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$^nC_r = \frac{n!}{(n-r)!\,r!} \qquad\qquad ^nP_r = \frac{n!}{(n-r)!}$$

Newton's Iterative formula for approximating roots of a polynomial:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Any question worth more than 1 mark <u>must have</u> the appropriate working shown to justify the extra marks.

1) A menu offers a choice of five entrees, four mains and three desserts.

   Find the number of meal choices possible

   a) if one of each must be chosen for a 3 course meal.

   b) if you have the choice of not having the dessert if you prefer.

   (2 marks)

2) The digits 0, 1, 2, 3 and 4 are used to make a 3-digit number. No digit is repeated.

   a) How many different 3-digit numbers are possible, if 0 cannot be the first digit?

   b) If any of the 3-digit numbers in part a is equally likely to have been made, find the probability that number made is greater than or equal to 230.

   (1 + 2 = 3 marks)

3) Evaluate $^{100}C_2$

   (1 mark)

4) In how many ways can four girls be selected for a table tennis team, if seven girls try out?
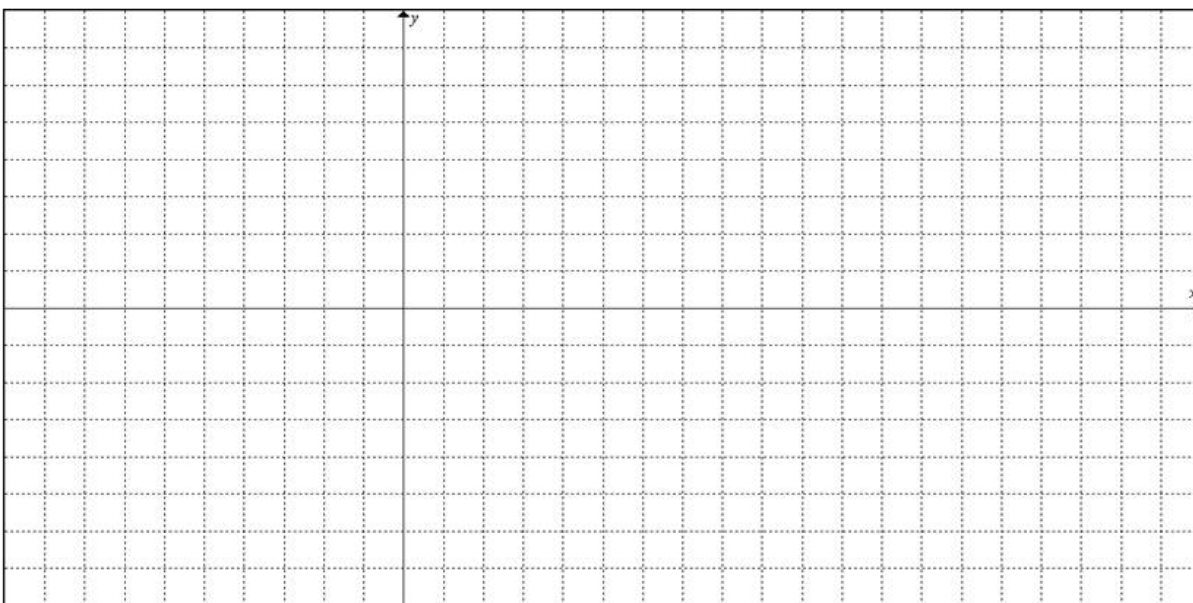
   (2 mark)

5) Find the exact values of:

a) $\sin 120^0$	b) $\tan \dfrac{4\pi}{3}$
c) $\sin\left(-\dfrac{\pi}{6}\right)$	d) $\cos \dfrac{9\pi}{4}$

(4 marks)

6) a) What is the period and the amplitude of the graph of

$$y = -3\sin 2x ?$$

b) Sketch the graph, showing <u>one complete cycle</u>. Clearly label key points.

(2 + 3 = 5 marks)

3

7) Solve the following equation, giving your answer(s) as exact values:

$$\sqrt{2}\sin x + 1 = 0, \quad 0 \le x \le 4\pi$$

(3 marks)

8) If sin θ = 0.66, cos θ= 0.75 and tan θ= 0.87, write down the value of:

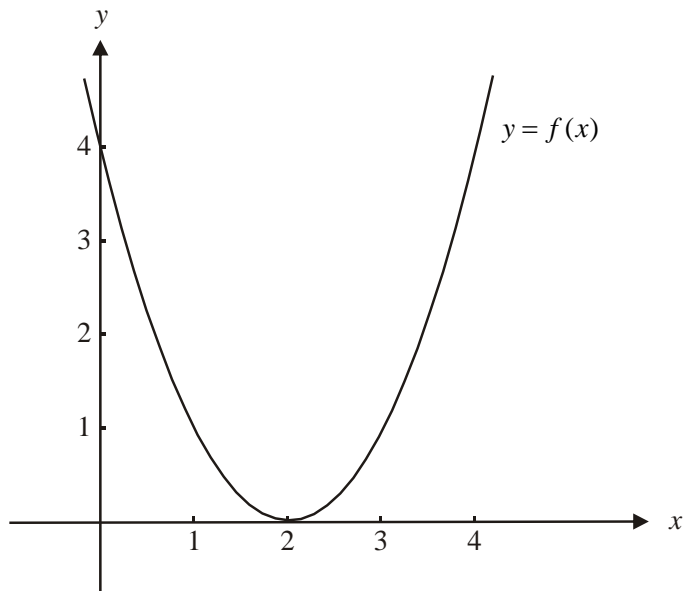| sin (2π - θ)	tan (π + θ)
cos (-θ)	cos (π - θ)

(4 marks)

9) If Evie drives at 60 km/h for 2 hours and 110 km/h for 3 hours, what is her average speed for the entire journey?

(1 mark)

10) The graph of the function $f : R \rightarrow R$, $f(x) = (x-2)^2$ is shown below.

a) Find the average rate of change of $y = f(x)$ with respect to x, between x = 1 and x = 4.

b) Find the instantaneous rate of change of $y = f(x)$ with respect to x at the point where

i) $x = 2$	ii) $x = 4$

(1 + 1 + 1 = 3 marks)

11) If $f(x) = (x + 2)(x + 3)$, find $f'(3)$.

(2 marks)

5

12) Find, using first principles, the derivative of
$$f(x) = 3x^2 + x - 2$$

(3 marks)

13) Find $\lim_{x \to 3} \dfrac{x^2 - x - 12}{x^2 - 16}$

(2 marks)

14) Evaluate:

a) $\int \left(4x^3 - x^2 + 9\right) dx$	b) $\int \dfrac{3x^4 + 5x^3}{2x} dx$

(1 + 2 = 3 marks)

6

15) A particle moves in a straight line with velocity of $v(t) = 6t^2 - 4t$ (m/s) at time t seconds (t ≥ 0). The particle has an initial position $x(t)$ of 3m left of the origin, O.

    a) Find the equation of the position of the particle, $x(t)$

    b) Find the acceleration of the particle at $t = 2$ seconds

<div align="right">(2 + 2 = 4 marks)</div>

16) Simplify these expressions using appropriate index or logarithm laws:

a) $\dfrac{25^{x+3} \times 5^{6x}}{125^{2x-1}}$	b) $\dfrac{(2x^4 y^{-3})^3}{2(x^{-3} y^2)^2}$	c) $2\log_{10} 5 + \log_{10} 4$

<div align="right">(3 x 2 = 6 marks)</div>

17) Solve the following equations for x:

a) $9^{2x} = 27^{2x-4}$	b) $\log_2(3x - 5) = 4$

(2 x 2 = 4 marks)

18)   a) Sketch the graph of the function $y = 10^{-x} + 1$, $x \in R$ on the set of axes below. <u>Indicate clearly on the graph any intercepts or asymptotes.</u>
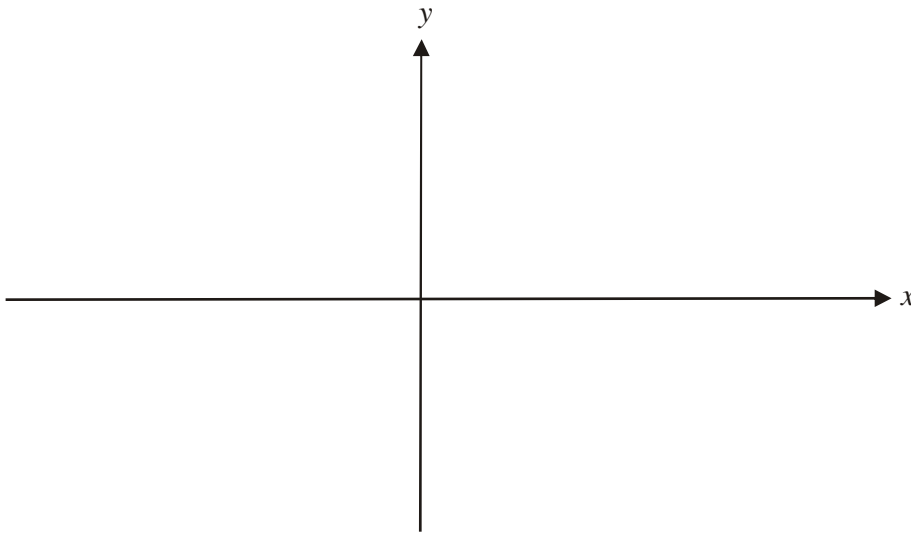
b) What is the range of this function?

(4 + 1 = 5 marks)

END OF EXAM