Student Name: Solutions.

Home Group: ____________

Teacher's name: (please circle): **Mrs O'Rielly** **Mrs Nation**

Mathematical Methods

Unit 2 EXAM 2

17th November 2017

Total 80 marks

Topics covered:

- **Combinatorics**
- **Circular Functions**
- **Rates of Change**
- **Differential Calculus**
- **Integral Calculus**
- **Exponential Functions and Logarithms**

Complete working must be shown and simplified wherever possible in order to gain full marks.

Reading Time: **15 minutes**

Writing Time: **90 minutes**

Students may bring one bound reference book into the exam.
Students may bring a CAS calculator and a Scientific calculator into the exam.

No paper or electronic dictionaries may be used.

Teacher's name: (please circle): Mrs O'Rielly Mrs Nation

SECTION A: MULTIPLE CHOICE SECTION: CHOSE THE CORRECT ANSWER AND RECORD THE LETTER ON THE COLOURED SHEET.

1 For his holiday reading Geoff has selected eight detective novels, three biographies and four science fiction books, but he only has room in his case for three books. If he selects one book from each group, how many combinations of books are possible?

A 15
B 28
C 56
D 20
E 96

2 $^{21}C_3$ is equal to

A 21!
B $\frac{21!}{3!}$
C $\frac{21!}{18!3!}$
D $\frac{21!}{18!}$
E $\frac{18!3!}{21!}$

3 If $\sin\theta = -\frac{2\sqrt{2}}{3}$, then a possible value of $\cos\theta$ is:

A $-\frac{1}{3}$
B $\frac{2\sqrt{2}}{3}$
C $-\frac{\sqrt{5}}{9}$
D $-\frac{1}{\sqrt{8}}$
E $\frac{1}{9}$

4 Over the interval $[-\pi, \pi]$, the graphs of y= − cos θ and y= sin θ intersect at:

A $-\frac{\pi}{4}, -\frac{\pi}{4}$
B $\frac{3\pi}{4}, \frac{5\pi}{4}$
C $-\frac{5\pi}{4}, -\frac{13\pi}{4},$
D $\frac{\pi}{4}, \frac{5\pi}{4}$
E $-\frac{\pi}{4}, \frac{3\pi}{4}$

5 The function $f: R^+ \to R\ where\ f(x) = log_2(3x)$, has an inverse function f^{-1}. The rule for f^{-1} is given by

A $f^{-1}(x) = 2^x$
B $f^{-1}(x) = 3^x$
C $f^{-1}(x) = \frac{2^x}{3}$
D $f^{-1}(x) = 2^{\frac{x}{3}}$
E $f^{-1}(x) = log_2(\frac{x}{3})$

6 Let $y = x^2 - 7x + 10$. The average rate of change of y with respect to x over the interval $4 \le x \le 6$ is:

A − 3
B − 2
C 1
D 3
E 4

7 Which one of the following functions has a graph with a vertical asymptote with equation $x = b$?

A $y = log_2(x - b)$

B $y = \frac{1}{x+b}$

C $y = \frac{1}{x+b} - b$

D $y = 2^x + b$

E $y = 2^{x-b}$

8 If $y = 3x(x-6)$, then $\frac{dy}{dx}$ is equal to:

A $6x^2-3$
B $6x+8$
C $6x-6$
D $6x-18$
E $6x^2$

9 For the curve with equation $y=x^2-8x$, the gradient of the curve at $x=5$ is:

A – 30
B –15
C – 8
D 2
E 17

10 The coordinates of the point on the curve with equation $y=\frac{1}{2}x^2-2x+5$ at which the tangent is parallel to the line $y=-4x+2$ are:

A (– 6, 35)
B (– 2, 10)
C (– 2, 11)
D (2, 3)
E (2, 6)

11 Let $\frac{dy}{dx} = 3 - 4x$ and $y = 1$ when $x = 0$. The value of y when $x = 2$ is:

A – 9
B – 7
C – 2
D –1
E 1

12 $\lim\limits_{h\to 0}\frac{9-(3+h)^2}{h}$ is equal to:

A – 6
B – 4
C – 2
D 0
E 8

13 A hardware shop stocks five colours of paint. New colours can be made by taking two of the existing colours and mixing them. Assuming each such mixing gives a new colour, the number of new colours possible is:

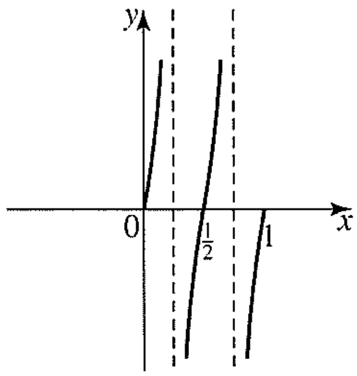
A 5
B 10
C 20
D 25
E 5! × 4!

14 The population of trout in a trout pond is growing. If the population, P, after t weeks is given by $P = 10 \times 1.1^t$, the average rate of growth of the population during the 5th week is closest to

A 16 trout per week
B 15 trout per week
C 1.5 trout per week
D 4 trout per week
E 15.35 trout per week

15 The graph depicted could have the equation:

A $y=2\tan \pi x$
B $y=\tan \pi x$
C $y=\tan\frac{\pi x}{2}$
D $y=\frac{1}{2}\tan \pi x$
E $y=\tan 2\pi x$



16 If $f(x)=\begin{cases} x-1, & x\in(0,1] \\ x^3-1, & x\in[-1,0)\end{cases}$ then $\lim\limits_{x\to 0} f(x)$ equals:

A 0
B 1
C -1
D does not exist
E x

SECTION B- SHORT ANSWER SECTION:

1 In how many ways can a team of four be selected from five Year 11 students and five Year 12 students

a) If there are no restrictions?

$^{10}C_4 = 210$

b) If there must be exactly three Year 12 students on the team?

$^5C_3 \times {}^5C_1 = 50$

c) If there must be exactly two Year 11 students on the team?

$^5C_2 \times {}^5C_2 = 100$

d) If the whole team is to be from Year 12?

$^5C_4 = 5$

(1 + 1 + 1 + 1 = 4 marks)

2 From a group of five boys and 4 girls, three school captains must be selected.

a) How many different combinations of three school captains, without any restrictions, are possible?

$^9C_3 = 84$

b) How many combinations contain a majority of female captains?

$$\left.\begin{array}{l} ^5C_1 \times {}^4C_2 = 30 \\ ^5C_0 \times {}^4C_3 = 4 \end{array}\right\} + \quad = 34$$

c) How many different combinations of three captains are possible if both genders must be represented?

$$\left.\begin{array}{l} ^5C_1 \times {}^4C_2 = 30 \\ ^5C_2 \times {}^4C_1 = 40 \end{array}\right\} + \quad 70.$$

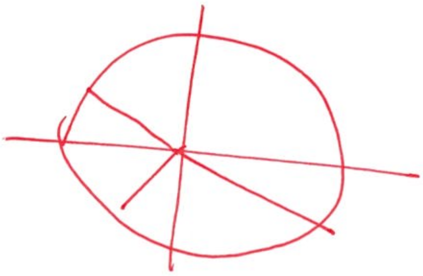
(1+2+2 = 5 marks)

3 If $\cos\theta = 0.6$, write down the value of:

a $\cos(2\pi - \theta)$ $= 0.6$

b $\cos(\pi - \theta)$ $= -0.6$

c $\cos(\pi + \theta)$ $= -0.6.$



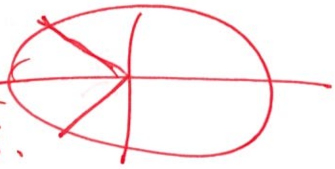
(3 marks)

4 Solve the equation $1 - 2\cos 2x = 2$, where $0 < x < 2\pi$, giving answers as exact values.

$-2\cos 2x = 1$ $\quad 0 < 2x < 4\pi$

$\cos 2x = \frac{-1}{2}$ $\quad$ Ref $\theta = \pi/3$

$2x = \pi - \frac{\pi}{3}, \ \pi + \frac{\pi}{3}, \ 3\pi - \frac{\pi}{3}, \ 3\pi + \frac{\pi}{3}.$



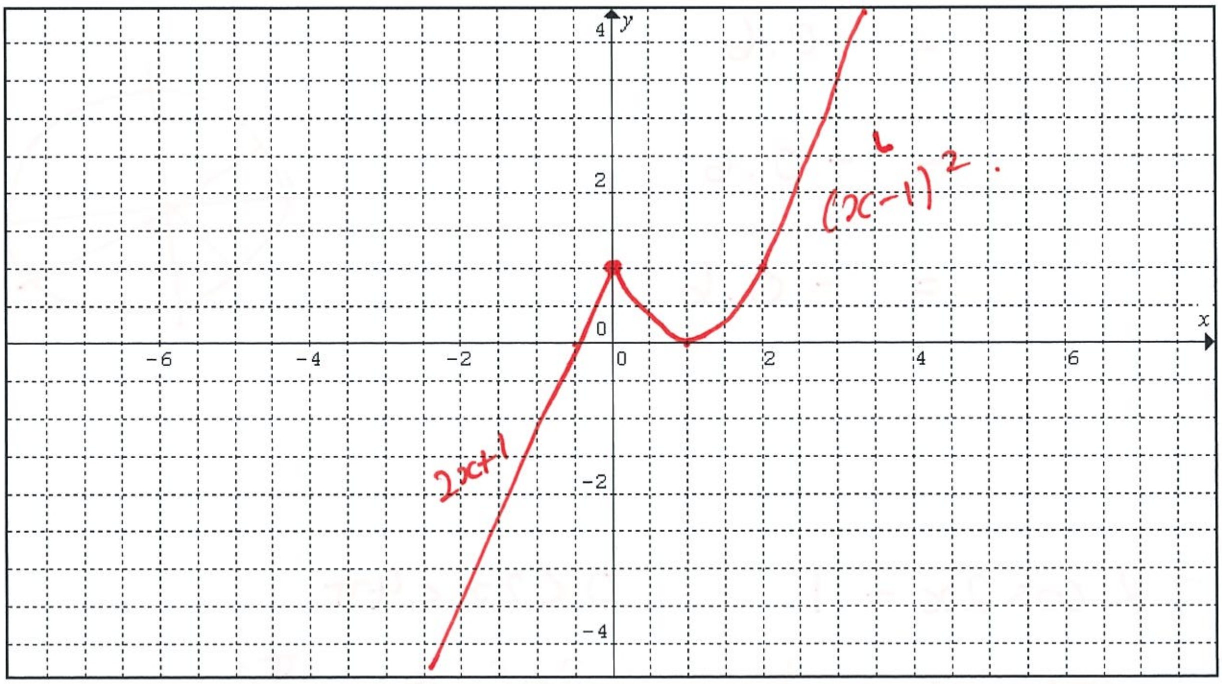
$2x = 2\pi/3, \ 4\pi/3, \ 8\pi/3, \ 10\pi/3$

$x = 2\pi/6, \ 4\pi/6, \ 8\pi/6, \ 10\pi/6$

$= \pi/3, \ 2\pi/3, \ 4\pi/3, \ 5\pi/3.$

(3 marks)

5 **(a)** Sketch the graph of $f(x) = \begin{cases} 2x+1 & , \quad x \le 0 \\ (x-1)^2 & , \quad x > 0 \end{cases}$



(b) For what values of *x* is the graph a continuous function?

$\mathbb{R}$

(c) Evaluate $\lim_{x \to 3} f(x)$

$\lim_{x \to 3} f(x) = 4.$

(3 + 1 + 1 = 5 marks)

6 Find the derivatives of:

a) $y = \dfrac{4}{x^2}$	b) $y = x^{\frac{5}{4}}$
$y = 4x^{-2}$ $\dfrac{dy}{dx} = -8x^{-3}$ $= \dfrac{-8}{x^3}$	$\dfrac{dy}{dx} = \dfrac{5}{4} x^{\frac{1}{4}}$ $= \dfrac{5\sqrt[4]{x}}{4}$

(2 marks)

7 Find a function that differentiates to:

(a) $2x^4$	(b) $-x^{-2}$
$\int 2x^4\,dx$ $= \frac{2x^5}{5} + c$	$\int (-x^{-2})\,dx$ $= \frac{-x^{-3}}{-3} + c$ $= \frac{1}{3x^3} + c.$

(4 marks)

8 If $\frac{dy}{dx} = (x-2)^2$ and the y-intercept is $^-3$, find the equation for y.

$\int (x^2 - 4x + 4)\,dx \Rightarrow \frac{x^3}{3} - \frac{4x^2}{2} + 4x + c$

y int (-3) ∴ $c = -3.$

$\therefore y = \frac{x^3}{3} - 2x^2 + 4x - 3.$

(2 marks)

9 The graph of $y = ab^x$ passes through points (1,1) and (2,5)

a) Find the values of a and b.

$1 = ab$ (1,1)

$5 = ab^2$ (2,5)

$a = 0.2$ $b = 5$

b) Let $b^x = 10^z$

Take the logarithms of both sides (base 10) to find an expression for z in terms of x.

$\log_{10} b^x = \log_{10} 10^z$

$\log_{10} b^x = z$

$z = x \log_{10} b$

(3+2= 5 marks)

10 An estimate for the population of the Earth, P in billions is $P = 4 \times 2^{\frac{(t-1975)}{32}}$ where t is the year.

a) Evaluate P for: (to 3 decimal places where necessary)

i) $t = 1975$

$P = 4 \times 2^0 = 4$ billion.

ii) $t = 2005$

$P = 4 \times 2^{\frac{2005-1975}{32}} = 7.661$ billion

b) In what year will the population of the Earth be twice that in 1997?

$P = 4 \times 2^{\frac{1997-1975}{32}} = 6.4419\ldots$

$2(6.4419\ldots) = 4 \times 2^{\frac{x-1975}{32}}$ solve for x.

$x = 2029$

(1 + 1 + 3 = 5 marks)

11 Evaluate the solution of the equation $x^3 - 7x + 3 = 0$ by carrying out two iterations of Newton's Method, starting at $x_0 = 0$. Give your answer to 4 decimal places.

$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$ $f'(x) = 3x^2 - 7$

$x_1 = 0 - \frac{0^3 - 7(0) + 3}{3(0)^2 - 7}$

$= 0.4286$

$x_2 = 0.4408$.

(4 marks)

12 For the curve with equation $y=\frac{1}{3}x^3+x^2-3x+6$

a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx}=x^2+2x-3.$$

b) What are the co-ordinates of the points when the gradient is zero?. <u>Note: Algebraic working is expected to be shown for this question.</u>

$$0=x^2+2x-3$$
$$0=(x+3)(x-1)$$
$$x=-3 \ \& \ x=1$$
$$(-3,15) \ \& \ (1,\tfrac{13}{3})$$

c) Prove the type (nature) of the turning points:

-4	-3	-2	1	2.
5	0	-3	0	5
/	—	\	—	/

$(-3,15)$ Max

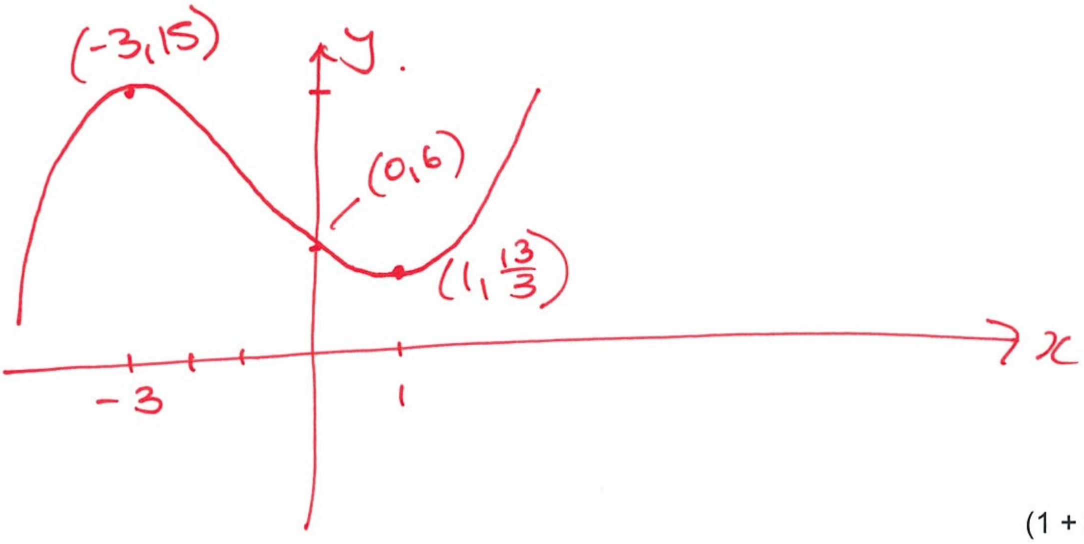
$(1,\frac{13}{3})$ Min.

d) Where does the graph cross the *y* axis?

Y int $(0,6)$

e) Hence sketch the graph of $y=\frac{1}{3}x^3+x^2-3x+6$ (you do NOT need to find the *x* - intercept(s)).

Mark all relevant points with their co-ordinates.



(1 + 3 + 2 + 1 + 3 = 10 marks)

13 The tide level on a pier pylon can be modeled by the equation $H = 4 + 2\sin\frac{\pi}{6}t$ where *H* is the height in metres of the tide from the bottom of the pylon and *t* is time in hours after midnight.

a) What is the period of this tide?

$P = \frac{2\pi}{n} = \frac{2\pi}{\pi/6} = 2\cancel{\pi} \times \frac{6}{\cancel{\pi}} = 12.$

b) What is the maximum and minimum heights of the tide level and when do they occur in the first 24 hours?

Max $2+4 = 6$ metres

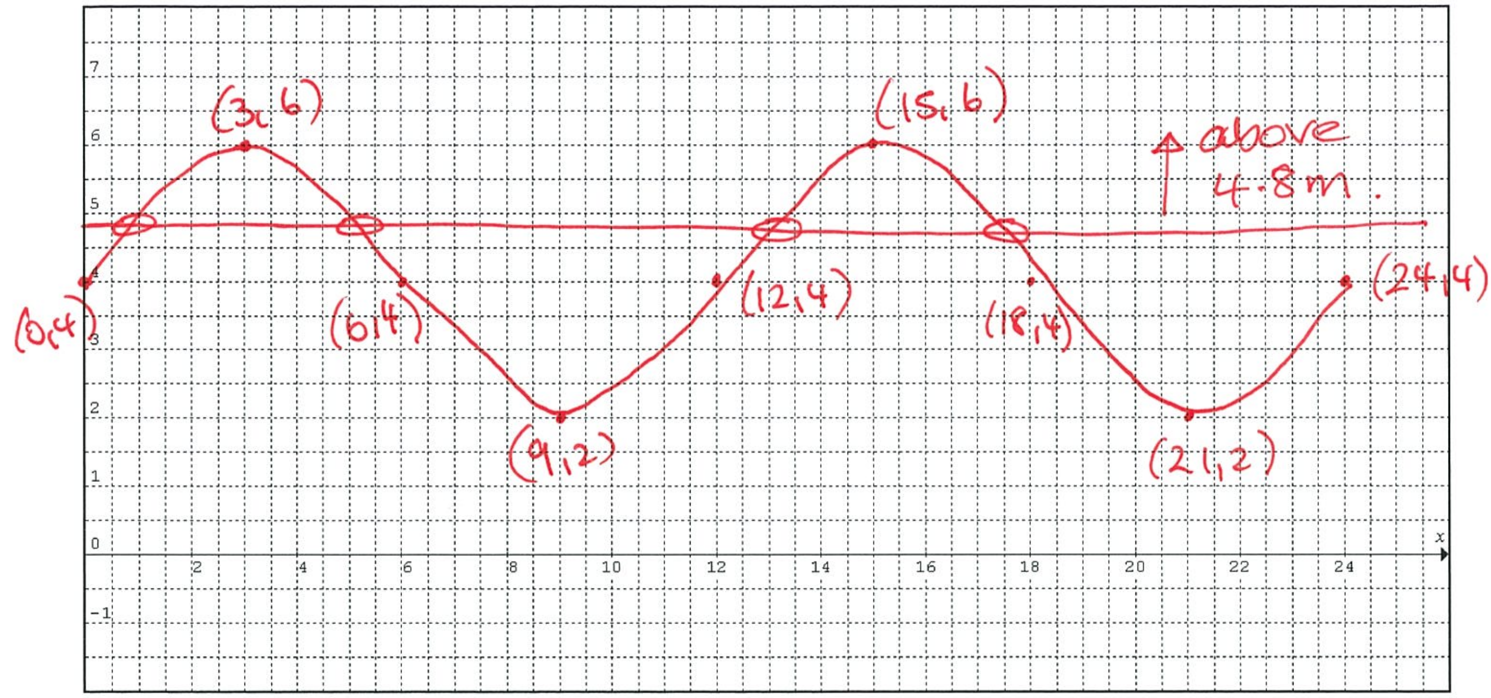
Min. $-2+4 = 2$ metres.

c) What is the water level on the pier at 4 am and at 4 pm? Give your answers as exact and correct to 2 decimal places.

4pm $\Rightarrow$ $t = 16.$

$H = 4 + 2\sin\left(\frac{\pi}{6} \times 16\right) \approx 5{\cdot}73$ metres.

d) Sketch the graph of the tide function showing the first 24 hours.



e) Show on the graph when the tide level is above 4.8m.
At what times in the first 24 hours is the tide level above 4.8 m on the pylon?

$t = 0.7859....$
$t = 5.214......$

between 12:47am & 5:13am.

$t = 12.785....$
$t = 17.214...$

between 12:47pm & 5:13pm.

(1 + 3 + 3 + 3 + 2= 12 marks)

END OF SECTION B

EXAM 2

SECTION A

ANSWER SHEET

Student Name......................................

Teacher (circle one) JOR VNA

Homegroup..

Question	Answer (A – E)	Question	Answer (A – E)
1.	E	9.	D
2.	C	10.	C
3.	A	11.	D
4.	E	12.	A
5.	C	13.	C
6.	D	14.	C
7.	A	15.	E
8.	D	16.	C