SECTION A – MULTIPLE CHOICE QUESTIONS [18 marks]

1. Which one of the following statements is **False** for the graph with equation:
$5x-2y=4$?

A It has a y-intercept at the point (0, –2)

B Its gradient is $\frac{5}{2}$

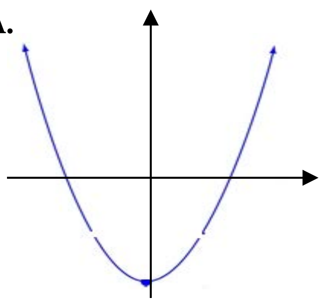
C It would be parallel to the line with equation $3x-2y=0$

D It is not a horizontal line

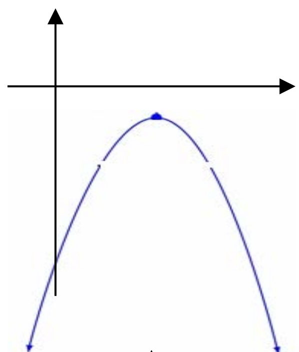
E The angle it makes with the positive x direction is obtuse.

2. Which one of these graphs would have a quadratic rule for which the discriminant is greater than zero?

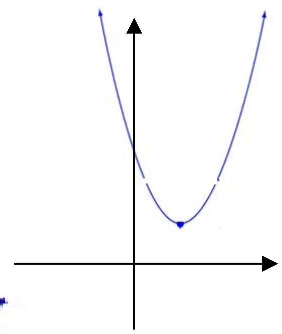
A.



B.



C.



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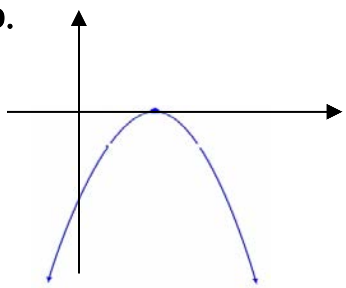
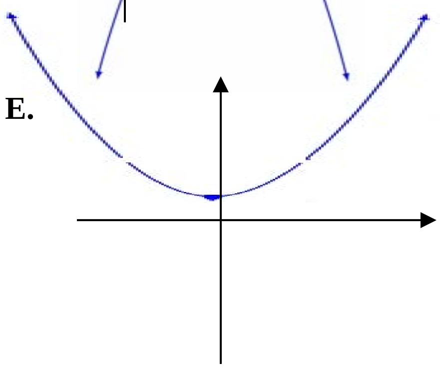


E.



3. A possible value of k if the equation $x^2+kx-k+8=0$ has one real solution is:

A 2

B 8

C -2

D -4

E 4

4. When $4x^3 + kx^2 - 10x - 4$ is divided by $x - 1$ the remainder is 15. The value of k is:

A 25

B $-\frac{1}{4}$

C -4

D 1

E 5

5. The maximal domain of the graph with equation $y = \dfrac{-2}{\sqrt{3x-2}}$ is equal to

A $\{x : x < \dfrac{2}{3}\}$

B $\{x : x > \dfrac{2}{3}\}$

C $\{x : x \geq \dfrac{3}{2}\}$

D $R \setminus \{-\frac{2}{3}\}$

E $R \setminus \{\frac{2}{3}\}$

6. The range of the graph of the relation $x^2 + (y-2)^2 - 4 = 0$ is

A $[-4, 4]$

B $[-2, 2]$

C $[0, -4]$

D $[0, 4]$

E $(-4, 4)$

7. Which one of the following functions does not have an inverse function?

A $y = 3x^2 - 3x, x > 0$

B $y = 5 - x^2, 0 \leq x \leq \sqrt{5}$

C $y = -2x^2, x > 0$

D $y = \frac{4}{x}$

E $y = 3x$

8. The asymptotes of the equation $y+1=\dfrac{2}{x+2}$ is/are ?

A $y=-1$ *and* $x=-1$
B $y=-1$ *and* $x=2$
C $y=-2$ *and* $x=2$
D $y=-1$ *and* $x=-2$
E $y=-2$ *and* $x=-1$

9. The range of the graph of the function $f: [0, 5) \to R, f(x) = (x-2)^4 + 2$ is

A [2, 83)
B [2, 83]
C [18, 83]
D (18, 83)
E (2, 18)

10. The graph of $y=\sqrt{x}$ is reflected in the *y* axis then translated 3 units in the negative *x* direction. The resulting graph has a range of:

A $(-\infty, -3]$
B $[-3, \infty)$
C $[0, \infty)$
D R
E R $\cup$ {0}

11. The function f has rule $f(x)=(x-2)^2+1$. Which one of the following sets is a possible domain for f if the inverse function f^{-1} exists?

A $[1, \infty)$
B $(-\infty, 0]$
C $[0, 5]$
D $(-2, \infty)$
E $[0, \infty)$

12. If $p(x)=x^2-4$ then $p(t-2)$ is equal to:

A t^2-4t-8
B $t(t-4)$
C $t(t-2)$
D t^2
E t^2-8

13. The gradient and angle of inclination to the positive direction of the x-axis of the straight line that passes through the points (1, 6) and (–2, 9) are:

A –1 and 45°
B –1 and 135°
C 1 and 45°
D 1 and 135°
E –1 and 225°

14. The weight of a rabbit in its first 8 weeks is described by the function $W(t)=t^3-3t^2+200$ where W is the weight of the rabbit in grams and t is the time measured in weeks. The average rate of change of weight of the rabbit between weeks 2 and 4 is:

A. 10 grams per week
B. 20 grams per week
C. 196 grams per week
D. 216 grams per week
E. 272 grams per week

15. If $y = 3x^2 - \frac{4}{x} + 6$, the rate of change of *x* at *x* = *a* is:

A. $6x - 4x^{-2}$
B. $6x + 4x^{-2}$
C. $6a - 4a^{-1}$
D. $6a + 4a^{-1}$
E. $6a + 4a^{-2}$

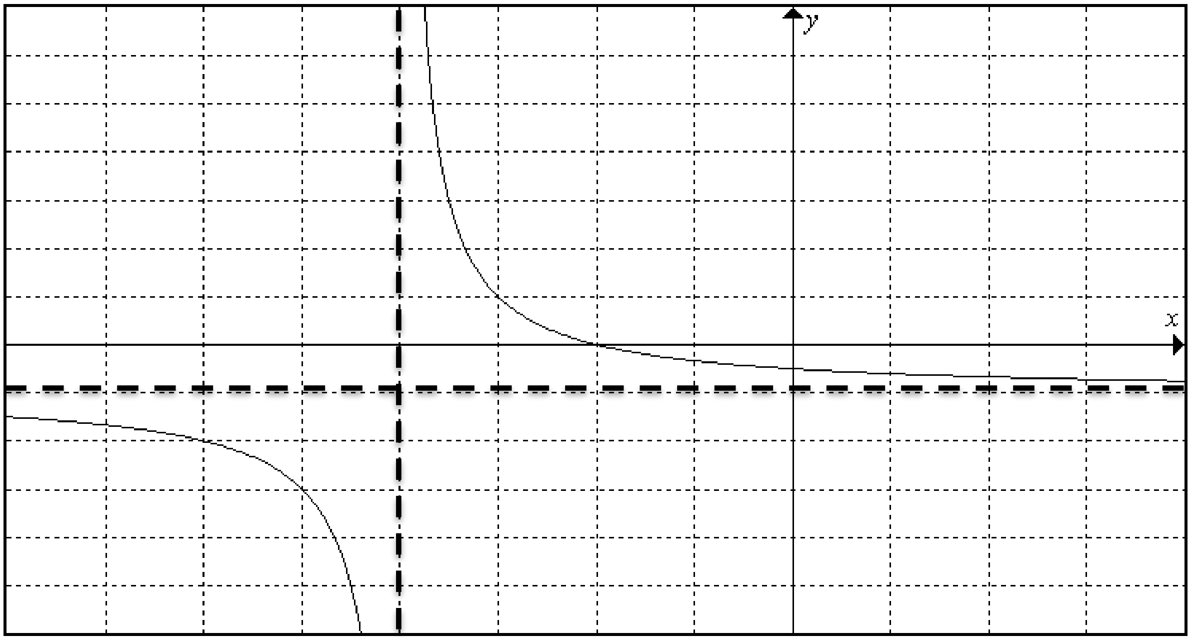
16. If $y = \frac{x^3}{3} - 4x^2 + 7$, then the values of *x* for which $\frac{dy}{dx} > 0$ are:

A. $\{x : 0 < x < 8\}$
B. $\{x : x > 8\}$
C. $\{x : x \le 0\} \cup \{x : x \ge 8\}$
D. $\{x : x < 0\} \cup \{x : x > 8\}$
E. $\{x : x \in \mathbb{R}\}$

17. For the function $f(x) = 2x^2 + 5x - 3$, the gradient of the tangent to the curve at the larger *x* intercept is:

A. 7
B. -7
C. 5
D. $-\frac{5}{4}$
E. -3

18. The given graph below is of a hyperbola. It is best described by which equation? Asymptotes have equations $x = -a, y = -b$ where $a \text{ and } b \in R^+$.



A. $y = \frac{1}{x-a} - b$

B. $y = \frac{1}{x-a} + b$

C. $y = \frac{1}{x+a} - b$

D. $y = \frac{1}{x-b} - a$

E. $y = \frac{1}{x+b} - a$

SECTION C: Extended Response Questions [47 marks]

Question 1 [8 marks]

Consider the triangle with vertices A (1, 2), B (–2,–1) and C (0,–3).

a) Find the distance of the line segment joining A and B.

b) Find the gradient of the line joining A and B.

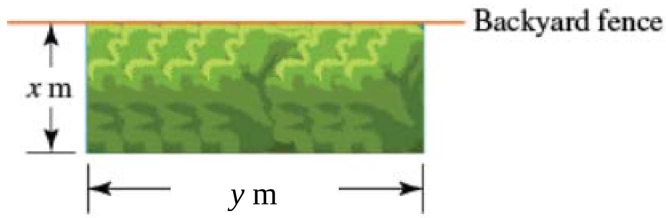
c) Using mathematical calculations show that the lines $\overline{AB}$ and $\overline{BC}$ are perpendicular.

d) Find the equation of the line that is parallel to the line joining the two points A and B and that passes through the midpoint of $\overline{BC}$.

[2 +1 + 2 + 3 = 8 marks]

Question 2 [12 marks]

A gardener has 16 metres of fencing to place around three sides of a rectangular garden bed, the fourth side is bound by the backyard fence. The width of the garden bed is x metres and the length is y metres.



a) Show that an expression for y in terms of x is $y = 16 - 2x$

b) Hence, show that the area, A, in m^2, inside the fence is given by $A = 16x - 2x^2$

c) State the domain of the function found in part **b**).

d) On the set of axes below, sketch the graph of A against x. Indicate clearly on the graph the co-ordinates of any turning point and endpoints over a suitable domain.

e) State the maximum area the garden can have and the dimensions that will deliver this area.

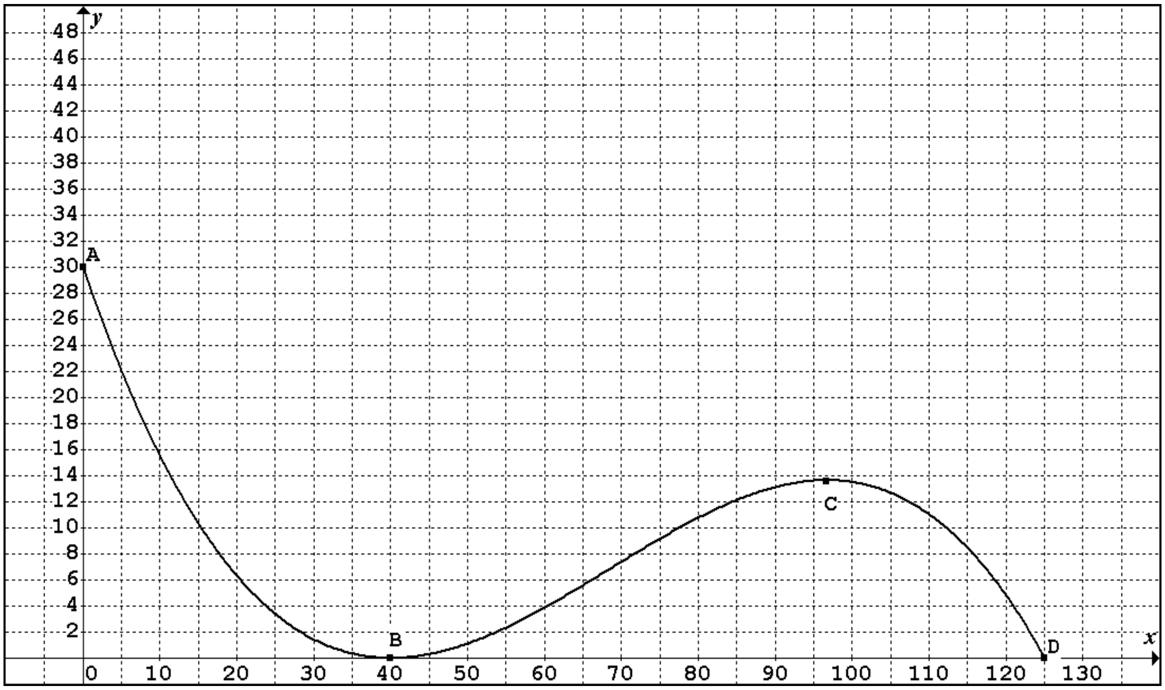
Due to the expense of the garden mix that is to be used, the area inside the fence must be **less than** 30 m^2.

f) Find the possible values of x, which will fulfil this requirement.

[2 + 2 + 1 + 3 + 2 +2 = 12 marks]

Question 3 [11 marks]

The ride below is a roller coaster whose path can be modelled by a cubic equation $y = p(x-q)^2(x-r)$, where y is the vertical height in metres and x is the horizontal distance of the ride from the starting point in metres.



a) State the value of q and r.

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b) What is the starting height of the ride?

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c) Show that the value of $p = -0.00015$ or $\frac{-3}{20000}$.

d) The roller coaster heads down- hill initially to point B, before ascending to a maximum turning point *C*. State the coordinates of *C* correct to 1 decimal place.

e) State the horizontal length of the ride.

f) The ride has two vertical supports, the first is at $x = 0$ and the other at point *C*. What is the total length of steel required for the two supports correct to 1 decimal place?

g) What is the height of the ride at a horizontal distance of 20 m from the starting point?

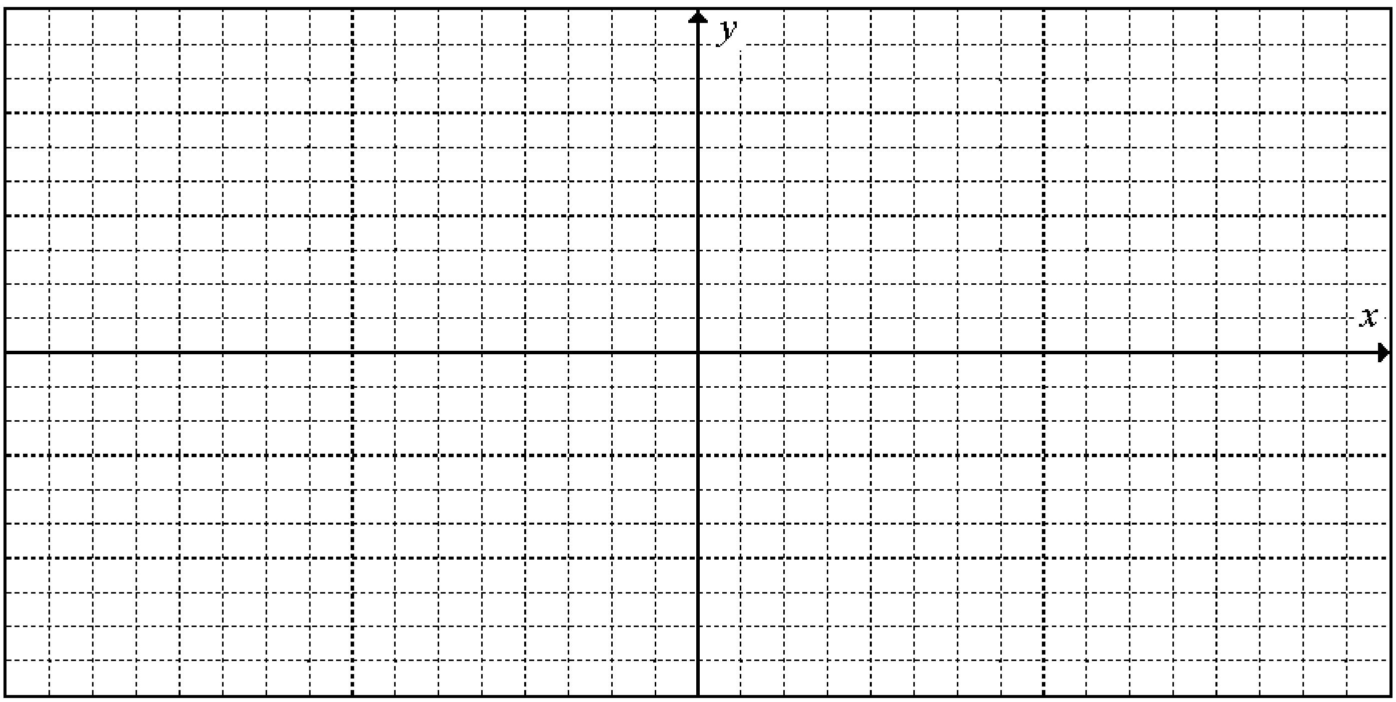
[2+1+3+1+1+2+1=11 marks]

Question 4 [8 marks]

A function, f, is defined as $f: (\frac{-1}{2}, k] \rightarrow R \text{ where } f(x) = x(x-3)^2$

a) i) State the **largest value of *k*** from the left that makes function *f* a one-to-one function.

ii) On the axes below, sketch the one-to-one function *f* using the value of *k* calculated, showing the coordinates of key points.



iii) On the axes above, sketch the graph of the inverse of the one to one function *f*, labelling it clearly and showing coordinates of key features.

b) Find the value(s) of x where $f(x)$ intersects with $(x-3)^2 + y^2 = 9$ within the domain $\left(\frac{-1}{2}, \infty\right)$ correct to 2 decimal places.

[1+3+2+2 = 8 marks]

Question 5 [8 marks]

A rectangular box, made of thin sheet metal and **without a lid**, is of length $2x$ cm, width x cm and height h cm.

(a) Write down, in terms of x and h, the area of sheet metal required to make the box.

2 marks

(b) Given that the area of sheet metal is 600 cm^2, show that $h = \dfrac{600 - 2x^2}{6x}$

2 marks

(c) Hence show that the volume, V cm^3, of the box is given by $V = 200x - \dfrac{2x^3}{3}$

2 marks

(d) Use CAS to find the value of x for which V is a maximum and state the maximum volume correct to two decimal places.

2 marks

END OF EXAM