CONNECT EDUCATION

YOUR GUIDE TO BECOMING A CASIO CLASSPAD GURU.

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About the author.

Yanik Ratilal, Senior Lecturer at Connect Education, is the author of this guide. Yanik achieved a raw study score of 50 in Maths Methods when he was in Year 12. He now studies a Bachelor of Commerce and Commerce Specialist in finance and actuarial studies.

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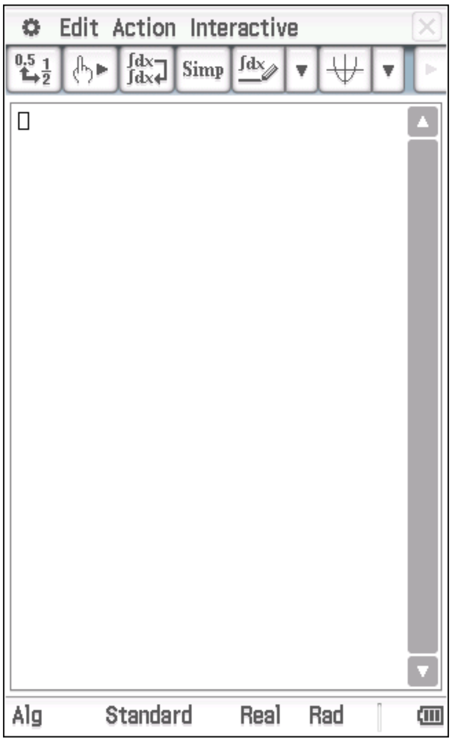
SETTING UP AND NAVIGATING WITHIN THE CALCULATOR

Before we begin exploring the various commands on the calculator, it's important we first ensure your calculator is set up properly and that you know how to navigate around it.

Settings

In the **Main** application, take a look at the bottom bar of your screen.

In this area, you can adjust how your calculator processes equations and how it presents answers. Below are the settings that I recommend.



The following three settings are particularly important, so I've given them a special mention:

- **Calculation mode: Standard**
  The calculator will spit out answers in their exact form, useful for most questions. If you need a decimal, change the mode to Decimal or tap the first button in the top bar of your screen.

- **Number type: Real**
  In general, I recommend to always use Real mode unless you need complex numbers in Specialist Maths. Otherwise, you may get some weird answers.

- **Angle: Radian**
  Use Radian mode, or else your trig functions will look very weird.

Once you've adjusted your settings, your CAS will always remember them every time you come back to the Main application.

Clearing your screen

Whether you're in the Main or Graph & Table applications, you can quickly start from scratch by clearing your screen. First, tap the screen you want to clear, then go to Edit, Clear All.

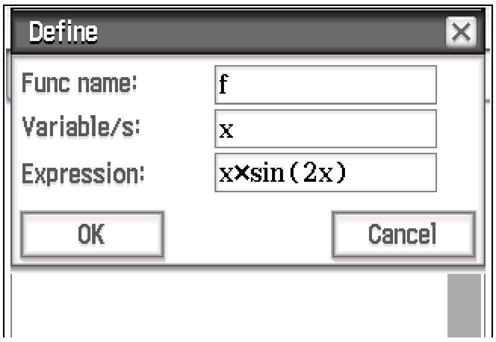
GENERAL CALCULATIONS

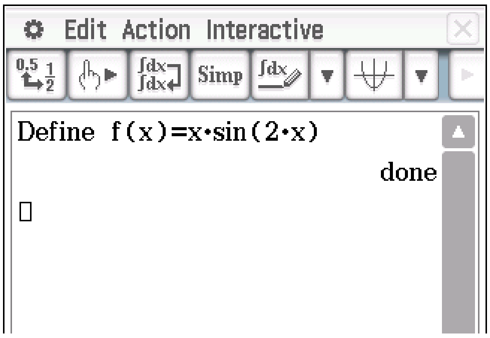
This section goes through a bunch of useful commands on the calculator that are applicable to many types of questions involving different kinds of functions.

Defining functions

Arguably one of the most time-saving commands on the calculator is Define. This lets you 'save' a function so that you don't have to repeatedly type it into the calculator or copy/paste equations and waste time modifying them using the arrows.

To define a function, go to Interactive, Define. In the entry lines, choose a letter that you'd like to represent the function, followed by (x) or (t) or whatever variable your function has. Then, just type in the equation of the function and tap OK.





TIP Your SACs and the exam will most likely have several functions called f because it is the letter conventionally used to denote functions. The problem is that you can't save all these equations to the same letter – each new equation you define as f will replace the previous equation. To get around this problem, you can include a number after the letter. For example, if there is a function f in Question 1, call it $f1$. If there's another in Question 3, call it $f3$. That way, you can keep all equations on the calculator at the same time, allowing you to go back and re-use them if need be.

Recalling defined functions

Forgotten what you had defined a letter as? Go to Settings, Variable Manager, then double tap main. It'll give you a list of functions you've already defined – all you have to do is double click on the letter you're interested in and it'll tell you what equation the letter represents.

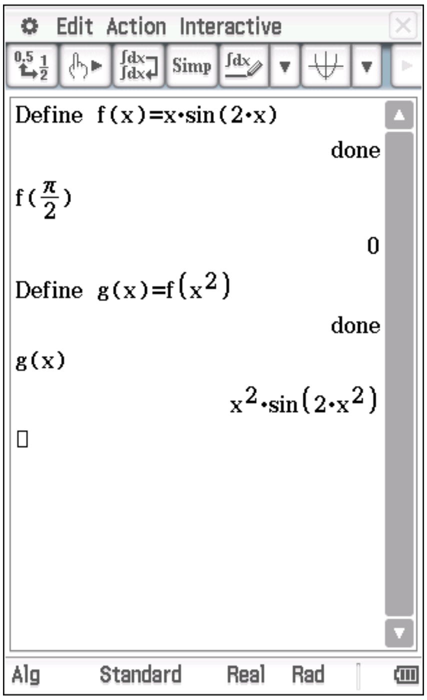
Clearing definitions

If you've finished an exercise or SAC and you don't need the defined functions anymore, you can clear them from your calculator. Go to Settings, Variable Manager, main. From here, select all the functions you want to clear, then go to Edit, Delete.

Substituting values into equations

In Exam 2, there's no point wasting time trying to sub numbers into equations by hand and running the risk of getting your arithmetic incorrect. Instead, there are two ways you can efficiently sub values into equations on the calculator.

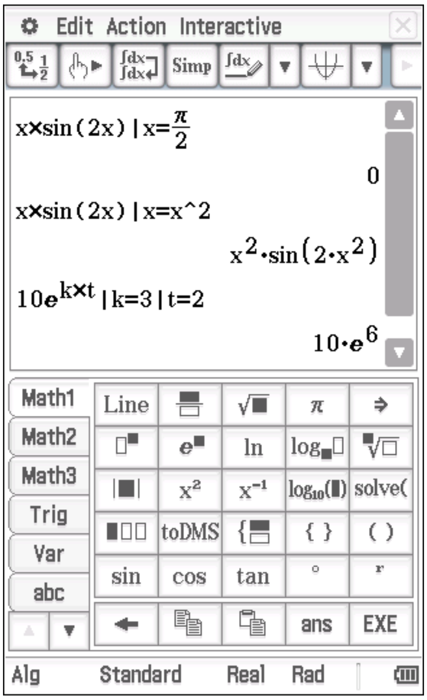
Subbing values into defined functions



The beauty about defining a function is that you can just use function notation to sub values into it. That is, instead of writing x inside the brackets, replace it with the value you'd like to sub in, as seen in the example on the right.

This can also be applied to composite functions – rather than replacing the x with a number, you can replace it with a whole equation (such as x^2), as seen in the example on the right.

Subbing values into equations 'on the fly'



If you're dealing with a question where you won't need to re-use the same equation, then defining the equation isn't all that necessary. Instead, you can sub values in by typing out the equation, then using the 'given' symbol — this looks like a vertical bar. You can access this symbol through your keyboard by going to Math3. After this 'given' symbol, type $x =$ followed by the value you'd like to sub in. If you need to sub multiple values in one line, use the 'given' symbol for each value you need to sub in.

When it comes to finding the equation of a composite function, you can still use $x =$ followed by the new function, even if it's in terms of x. Although this doesn't make sense if we "sub $x = x^2$" on paper, the CAS accepts this method.

Algebraic commands

These commands are useful when it comes to finding solutions or manipulating equations.

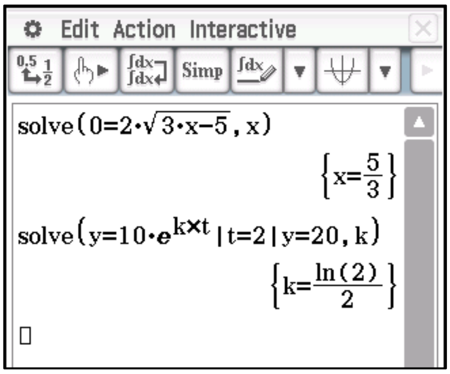
Solving equations

In the Main application, type out the equation you'd like to solve. Highlight the entire equation with your stylus, or instead tap Edit, Select All, then go to Interactive, Advanced, solve. In the box that appears you should see that the entry for Equation is already filled with your highlighted equation. If you're using a different variable to x, be sure to change it in the Variable entry. It's important that you highlight the entire equation as you may get an error if you don't. Once that's done, tap OK.

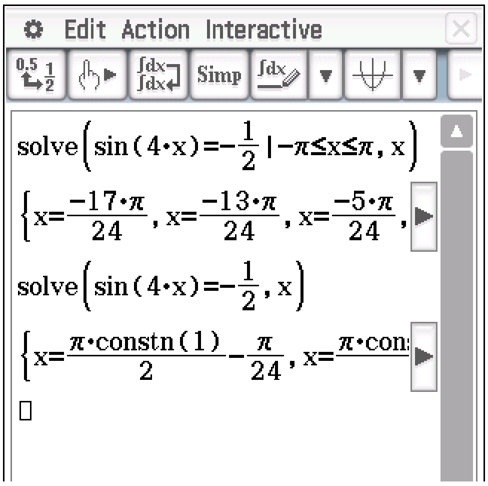
NOTE If the equation has letters representing constants (e.g. the k in $y = 10e^{kt}$) then you must include a multiplication sign between the letter and the variable or else the calculator will treat them as one variable.

If you need to **sub a coordinate** into an equation and then **solve** for a constant, you can do this cool thing where you:

- Type in the full equation.
- Insert a 'given' sign in Math3 of the keyboard.
- Then type what the x and y values equal (in this case, t and y).



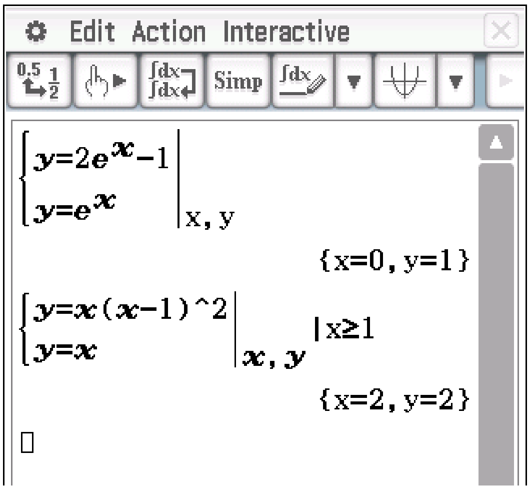
NOTE The 'given' signs must be inside the 'solve( )' command.



The 'given' symbol can also be used to restrict the domain of an equation – this is especially useful for finding specific solutions to trigonometric equations. In the example on the left, we restricted the domain to $[-\pi, \pi]$, which means the calculator only gives us solutions within this interval. You can insert the 'less than' or 'bigger than' signs through Math3 as well!

If you don't restrict the domain for a trigonometric equation, the calculator will give you a general solution. You'll see a 'constn' in the answer followed by a number (in this case 'constn(1)') – the number will increase the more you solve trigonometric equations, but in all cases it just represents an integer. Remember, for general solutions to trigonometric equations, we always say $n \in \mathbb{Z}$, where $\mathbb{Z}$ represents integers.

Solving simultaneous equations



To solve two equations simultaneously, we first have to go to **Math1** in the keyboard and select the icon. Type in your two equations in the first two lines next to the big bracket. At the bottom of the vertical line, type in each of the variables you want to solve for, each separated by a comma.

If there is a domain restriction in the question, or you need to sub something in, you can still do it by using the 'given' symbol. Make sure that the 'given' symbol isn't on the same line as the variables you want to solve for – make sure that it comes after that so it's above the variables, just like in the example above.

If you have more equations to solve simultaneously, tap the icon twice, three times or four times (or more) and the CAS will give more lines in the bracket to use.

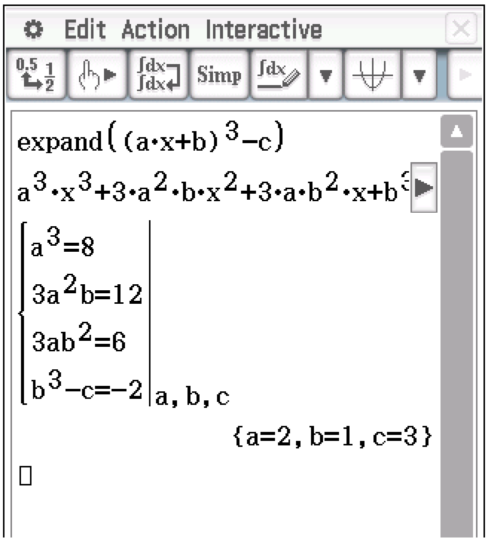
Expanding

As the name suggests, the Expand command allows you to expand a factorised function. This is handy in many situations – we'll discuss two of these through examples below. In general, type out the expression you would like to expand, then highlight the entire thing using **Edit**, **Select All**. Then, go to **Interactive**, **Transformation**, **expand**, and the CAS will spit out the expanded form.

Equating coefficients

Consider the function, $f(x) = 8x^3 + 12x^2 + 6x - 2$. If $f(x)$ can be expressed in the form of $f(x) = (ax + b)^3 - c$, find the values of $a, b,$ and c where $a, b,$ and c are real, positive constants.

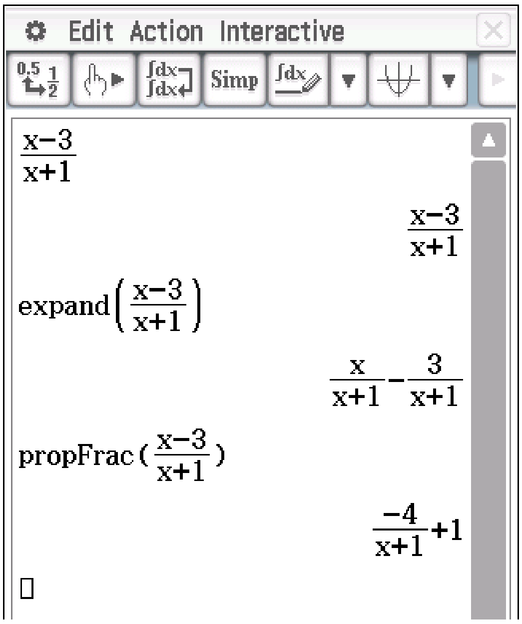
To find the value of these letters, we just have to expand the equation, then make the coefficients equal to the corresponding coefficients in the other equation.



Simplifying fractions

Consider the function $g(x) = \frac{x-3}{x+1}$. Express $g(x)$ in the form $g(x) = a - \frac{b}{x+1}$ and, hence, state the equation(s) of any asymptote(s).

When you get funny looking equations like the one above, where there's an expression containing x on the top and bottom of the fraction, what you want to do is divide the numerator by the denominator to simplify it. The problem is that when you just enter $x - 3$ divided by $x + 1$ on the calculator, it doesn't actually do the division… Even worse, the ordinary Expand command won't cut it. Instead, we should use a special expanding command just for fractions. Type in the fraction, highlight it all then go to **Interactive**, **Transformation**, **Fraction**, **propFrac**. As you can see above, $g(x) = 1 - \frac{4}{x+1}$, which means the asymptotes are $y = 1$ and $x = -1$.



Factorising

The Factor command is not used all that much, but it does come in handy when you need to express an equation in a different form to meet what the question requires.

For example, the following question was asked in the **2002 VCAA paper**:

The equation of the normal to the curve with equation $y = x\sin(x)$ at the point on the curve with x coordinate π, is:

A. $y = -(x - \pi)\pi$

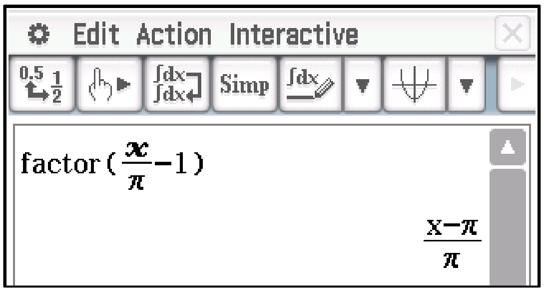
B. $y = (x - \pi)\pi$

C. $y = -\frac{1}{\pi}(x - \pi)$

D. $y = \frac{1}{\pi}(x - \pi)$

E. $y = \frac{-1}{x\sin(x)}$

The problem here is that when you obtain the equation of the normal using the calculator (this is explained in the Differentiation section), the answer you get is $y = \frac{x}{\pi} - 1$, which isn't in the multiple choice options. When this happens, try the Factor command – in this case, we get option D.



SKETCHING GRAPHS

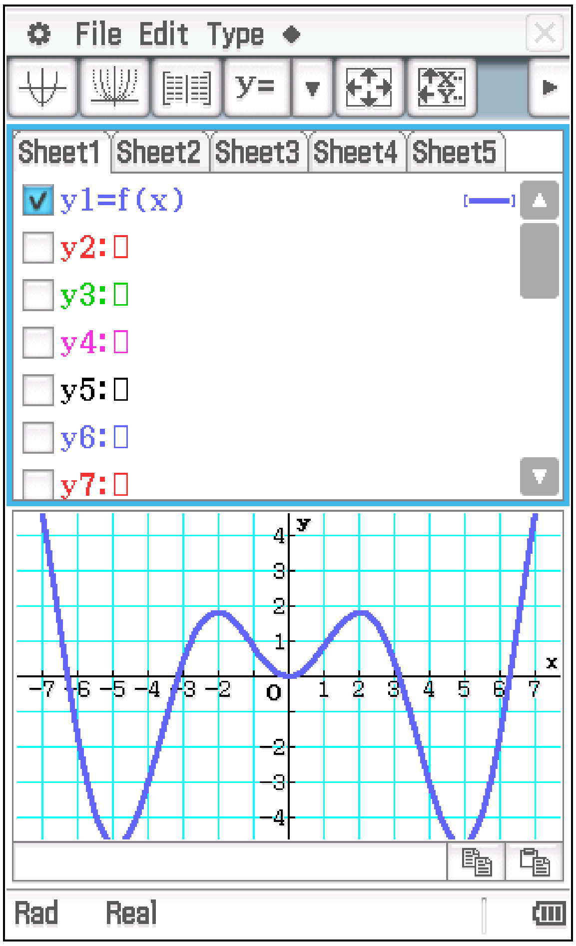
Visualisation is key to understanding what's going on in a question and for checking your answers, so being able to efficiently sketch a graph is very powerful.

Entering equations

When you enter the Graph & Table application, your screen will split into two windows: the top is where you enter your equations, and the second is where graph is sketched. Note that we always have to use x as the variable in the equation.

To tell the CAS what equation to sketch, tap the check box next to your equation and press the  sketch icon in the top left of the screen.

Keep in mind that you can also enter defined equations here. Even if you initially defined an equation in terms of t or some other variable, you can still graph it by writing x now inside the bracket, as you can see in the example above.



Adding another equation

Simply type your equation into another empty line, tick the check box and tap the sketch icon in the top left.

Modifying equations

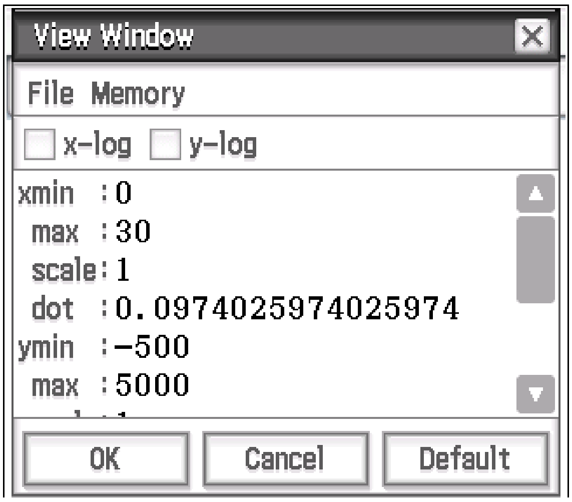
After you've entered an equation, if you need to change it you can do so by going back to the equation entry window. Tapping the sketch icon again will graph the new equation.

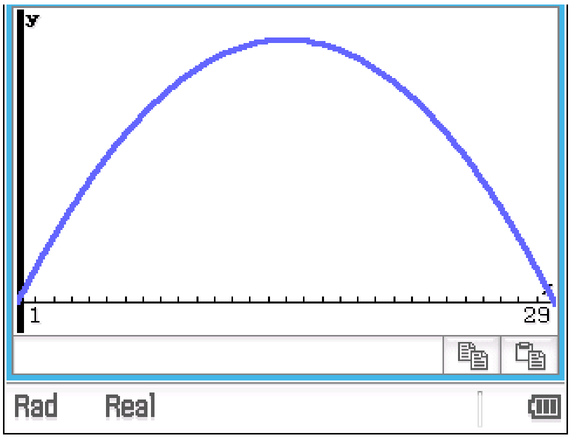
Window settings

Often, when you enter an equation, you can't 'see' it properly because the window zoom isn't suitable. You can fix this by:

- Tapping the graph window, then going to **Zoom**, **Zoom In**/**Zoom out**. This just zooms in/out from the centre of the graph. You can drag the graph window to move the graph and looks around.

- Tapping the graph window and selecting the  icon in the top bar of the screen. Alternatively, you can get to the same window by tapping the graph window, then going to **Settings**, **View Window**. This is the method I prefer because it gives you more control over exactly what you see. For example, if there's a domain restriction, you can specify that here.





In the example above, since the equation is $y = -20x(x - 30)$, we know that the x-intercepts are going to be 0 and 30, so we can set the xmin to 0 and xmax to 30. With the ymin, I've set it to -500 so I can still see the x-axis. With the ymax, you sometimes have to play around until you find a number that gives you a good view of the graph. In this case, 5000 did the trick.

NOTE If you can't see the scale of any axis properly, you can adjust the 'scale' option in either the x or the y-axes. Adjust them until it makes them visible.

Resetting the window's zoom

To get the original zoom back or to set the zoom back to default, go to **Zoom**, **Initialize**.

Analysing graphs

While you can find most features on graphs using the Main application, it's sometimes helpful to use the Graph & Table application instead as it's easier to visualise what's going on.

Maximum/minimum of graphs

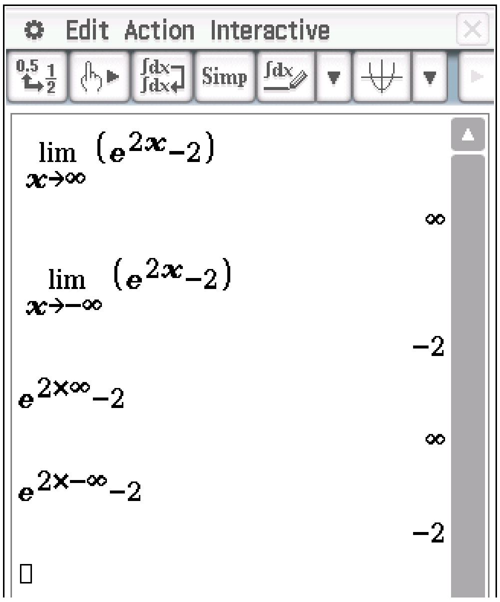
These can be found by first tapping the graph window to select it, then going to **Analysis**, **G-Solve**, **Min**/**Max**. If you have multiple graphs sketched, then the CAS will prompt you to select which graph you want to use. Keep pressing the up or down arrow keys until your desired graph is flashing, then hit **EXE**. If there are many local maxima or minima, you can use the arrows keys until you find your required stationary point.

Intersections of two graphs

This feature can be found by going to **Analysis**, **G-Solve**, **Intersection**. You can use the arrow keys to scroll through the different intersections if there are more than one.

Finding asymptotes

In simple situations, you can look at the Graph screen and 'see' what the equations of the asymptotes are. However, when you're dealing with more complicated equations or trigonometric functions, it's difficult to read the scale on the x- and y-axes correctly. This is where the Main application can help.



Horizontal asymptote

The horizontal asymptote represents a y-value that cannot exist. The graph will *approach* this y-value, but never reach it. Again, we can exploit this fact to find the equation(s) of the horizontal asymptote(s).

In your keyboard, go to Math2 and select the icon, and input  $x \to \infty$ and $x \to -\infty$, or you can directly sub $\pm\infty$ into your equation. In the example, you can see that the horizontal asymptote of the exponential graph is $y = -2$.

NOTE Often the horizontal and vertical asymptotes can be quickly found just by looking at the equation of the function. That's why I recommend trying this first, and using the above calculator commands if you get stuck or if you have time to go back and check your answers.

Graphing composite functions

SACs and the exams commonly ask for the domains and ranges of composite functions. Visualising the graphs is the best way to avoid silly errors.

Let's explore this through an example:

Let $g\colon [0,\ \infty) \to R,\ g(x) = \sqrt{x+4}$

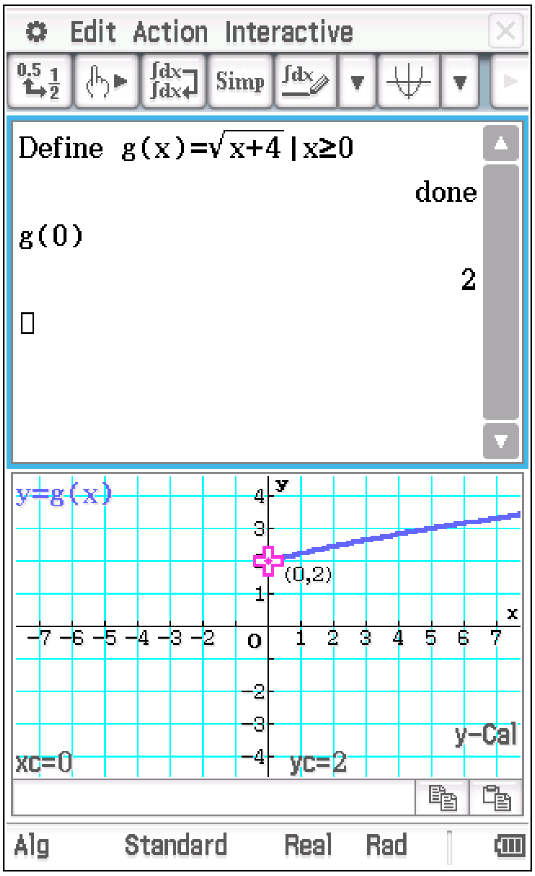
a) State the range of g.

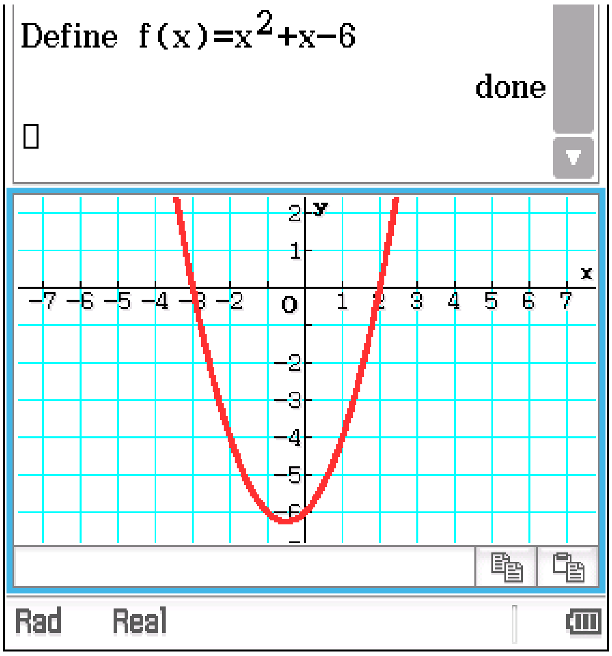
b) Let $f\colon (-\infty,\ p] \to R,\ f(x) = x^2 + x\ - 6$, where $p < 0$.

i. Find the largest possible value of p such that $g \circ f$ exists.
ii. State the domain of $g \circ f$.
iii. State the range of $g \circ f$.

Firstly, since we have an equation we'll be using repeatedly, we should define it straight away and include the domain restriction.

When finding the range of g, students often only think about the shape of a root graph, but forget to consider the domain restriction. That's where graphing the function really helps! (Alternatively, you can sub in $x = 0$ to see at what y-value the root graph starts.) Hence, $\boldsymbol{ran\ g = [2, \infty)}$.





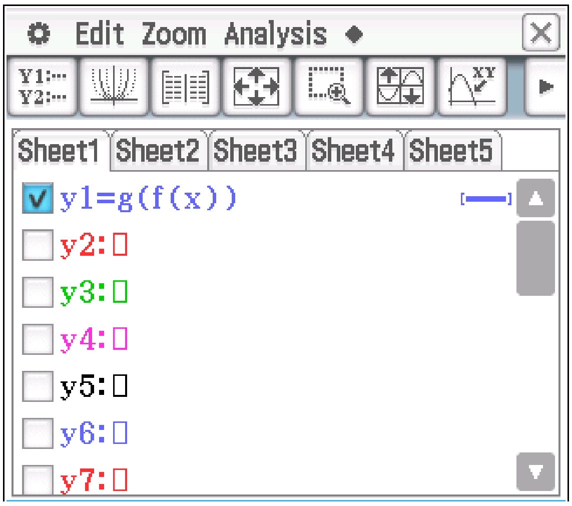
In Part (b), we have a parabola with a domain that starts from the left and stops at an x-value called p.

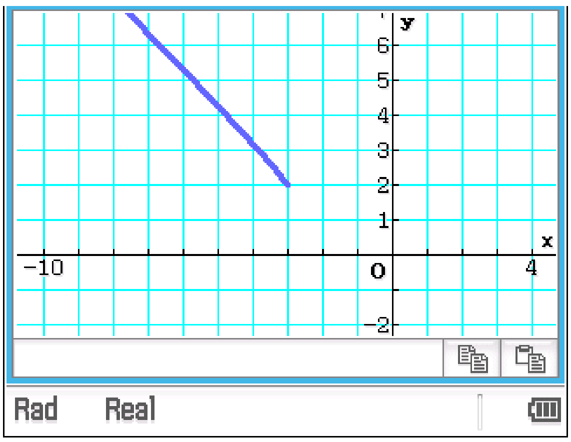
For $g(f(x))$ to exist, recall that the range of the 'inside' function (in this case, $f(x)$) has to fit inside the domain of the 'outside' function (which is $g(x)$). Since the domain of $g(x)$ is $[0, \infty)$, this means we need to restrict the domain of f so that it's range is $[0, \infty)$. Again, visualisation is key here!

As you can see from the graph above, the range of f will be $[0, \infty)$ if the domain is either $x \in (-\infty, -3]$ or $x \in [2, \infty)$. In this question, since the domain of f has already been defined as $(-\infty, p]$, this means $\boldsymbol{p}$ **is -3**.

To answer Part (b)ii, you just recall the rule that the domain of a composite function is the same as the domain of the 'inside' function. So, $dom\ g \circ f = (-\infty, -3]$.

Part (b)iii is a little trickier though because the range depends on the equation of the composite function, as well as the domain restriction. The easiest way to answer it is… you guessed it, visualisation! Define $f(x)$ (including its domain restriction) in the Main application, then open the Graph & Table application and enter $g(f(x))$. From the scale on the axes, you can see that the range of the composite function begins at 2 and continues upwards (if you're unsure, you can always trace the graph using **Analysis**, **Trace** to locate the endpoint of the graph. Hence, $ran\ g \circ f = [2, \infty)$.



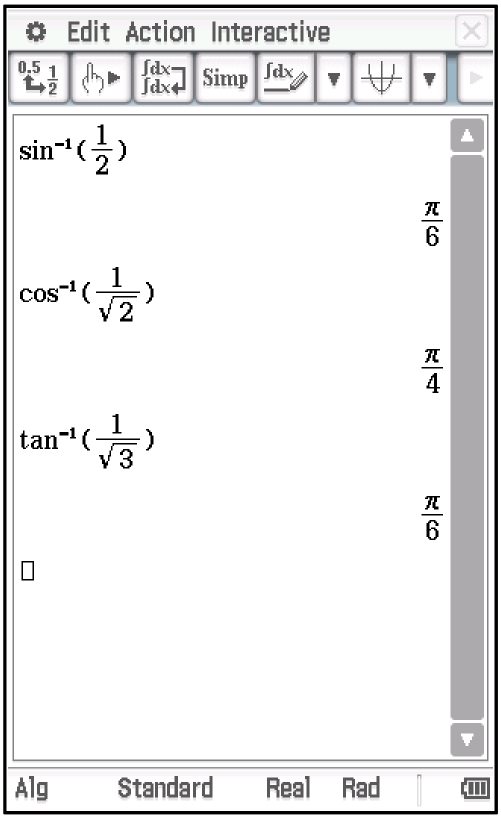


TRIGONOMETRY

Past VCAA questions have often involved trigonometric functions and triangles, so it's a good idea to be able to confidently calculate various angles on the calculator.

Calculating angles

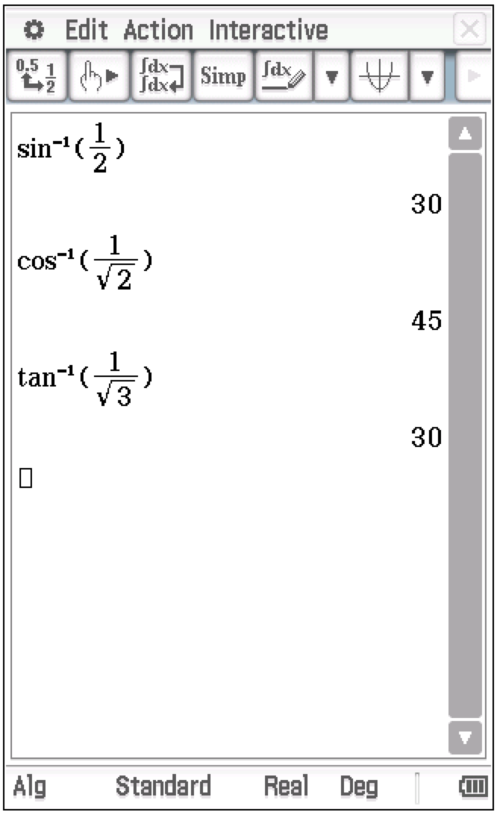
When you're trying to find the value of one angle in a triangle, the easiest way to do so is with the inverse sin, cos, and tan functions. The 'inverse' here means that we can use these functions to calculate an angle. (Normal sin, cos, and tan functions instead use angles to calculate the ratio of the opposite/hypotenuse or adjacent/hypotenuse or opposite/adjacent.) You can access the inverse functions in the **Trig** section of your keyboard and selecting the appropriate trigonometric function that has a -1 in it. Some examples using exact values are shown above, but below we will consider a more difficult question from VCAA.



Radians to degrees conversion

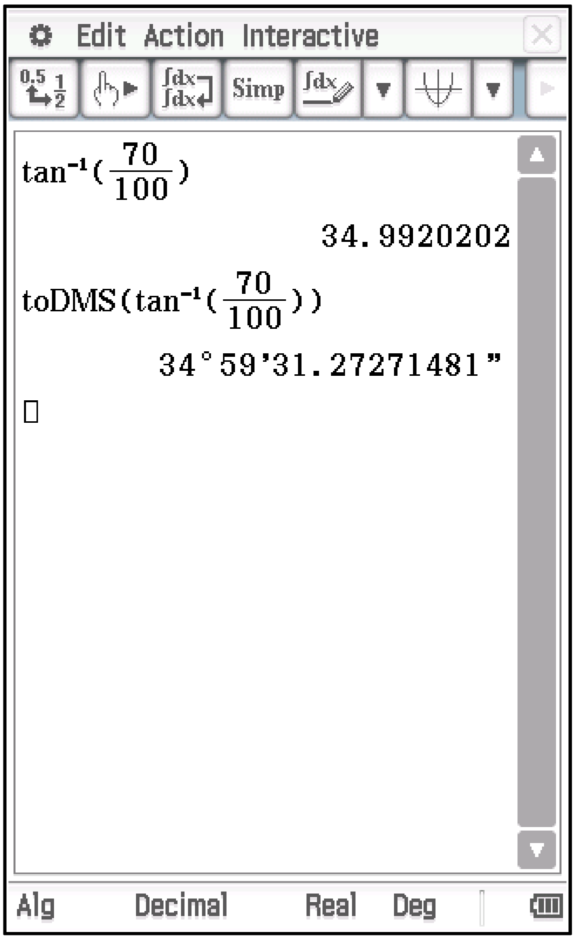
In the Settings section above, we said you should set your calculator to radians. This still holds here, but what if the question asks for the answer in degrees?

All you have to do is change the angle mode from Rad (radians) to **Deg** (degrees in the bottom settings bar of your screen.



To convert your angles to decimals, simply type out the calculation or angle you want to convert, then tap the decimal conversion key in the top bar of your screen. Alternatively, you can change your output mode from Standard to **Decimal** in the bottom settings bar of your screen. Using this, all answers given will be in decimals automatically. If you do this, don't forget to turn it back to Standard once you're done!

You can easily get angles in degrees, minutes and seconds by typing out your calculation or angle, highlighting it all then going to **Interactive**, **Transformation**, **DMS**, **toDMS**.

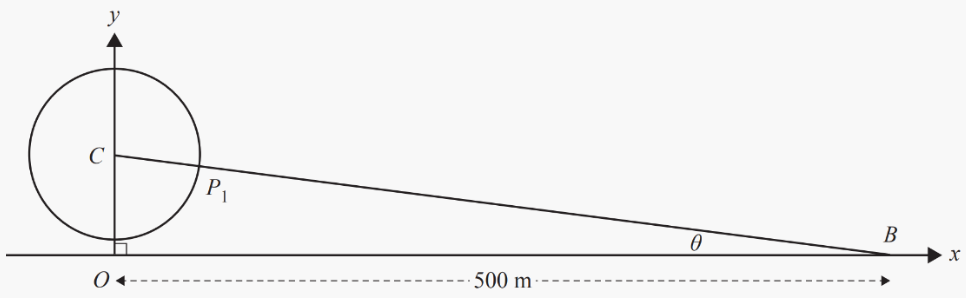


Let's apply all of the above to the following **2017 VCAA** question. (Note: I've picked out only the parts of this question that are relevant.)

Sammy enters a capsule on the Ferris wheel from a platform above the ground. The Ferris wheel is rotating anticlockwise. The capsule is attached to the Ferris wheel at a point P. The height of P above the ground, h metres, is modelled by $h(t) = 65 - 55\cos\left(\frac{\pi t}{15}\right)$, where t is the time in minutes after Sammy enters the capsule. Sammy exits the capsule after one complete rotation of the Ferris wheel.

b) For how much time is Sammy in the capsule?

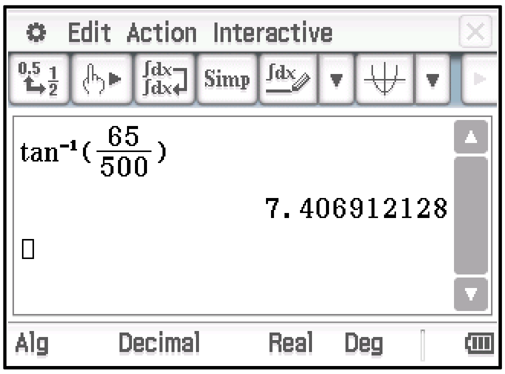
As the Ferris wheel rotates, a stationary boat at B, on a nearby river, first becomes visible at point P_1. B is 500 m horizontally from the vertical axis through the centre C of the Ferris wheel.



d) Find θ in degrees, correct to two decimal places.

For Part (b), students often get confused due to the wording of the question. It's basically asking how long it takes for Sammy to go around the Ferris wheel once. Since we're given a trigonometric equation, $h(t)$, which tells us the height of the capsule over *time*, we can use this equation to determine how long it takes for the capsule to go around and get back to where it started. In other words, we're after the period of the function. $Time\ (period) = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{15}} = 30\ minutes$

In Part (d), we're given a pretty confusing diagram and asked to find the value of θ. What you should take note of is that the θ is part of a triangle. Whenever you're asked to find the value of an angle in a triangle, you should always consider whether you can use the inverse sin, cos, or tan functions. Here, we know the adjacent side is 500 m, and we can figure out the opposite side – it's just the height of the centre of the Ferris wheel, 65 m, which we can get from $h(t)$. So, the angle in radians is given by $\tan^{-1}\left(\frac{65}{500}\right)$. If we change our mode to Decimal and Degree straight away, we'll get our answer, 7.41° (two decimal places), straight away.

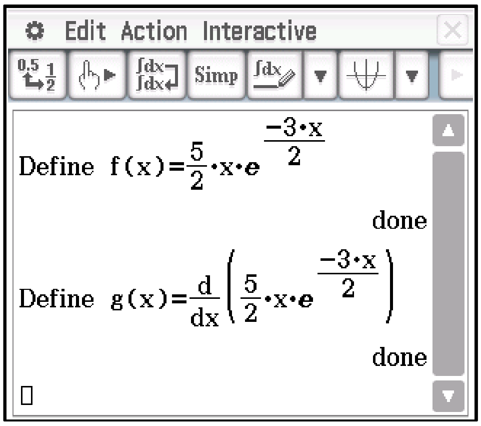


DIFFERENTIATION

Questions requiring differentiation appear multiple times in the calculator exam, so by mastering the following tips and commands, you can buy yourself a lot of precious time in the exam.

Defining derivative equations

Hopefully by now, having read through the previous pages, you can see how useful it is to define functions – the derivative function is no different.



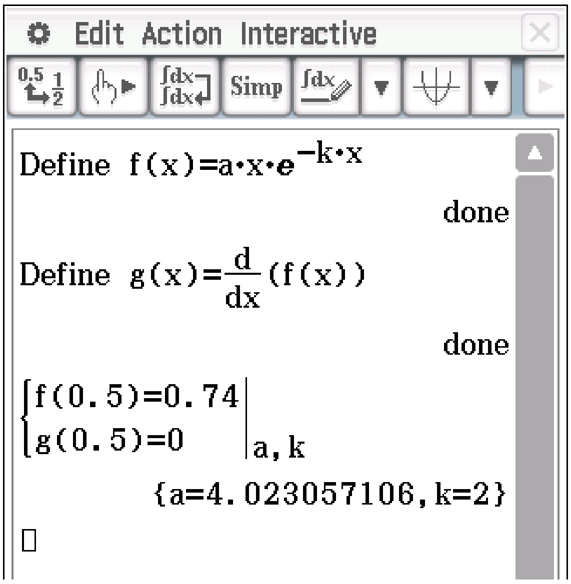
For example, in the screen on the right, we defined $g(x)$ as being the derivate of $f(x)$. We can do this by using the $\frac{d}{d\square}\square$ differentiation icon found in your keyboard under **Math2**, then defining the expression by highlighting it all then going to **Interactive**, **Define**.

Example (adapted from VCAA 2014, Exam 2)

The concentration, c, of medicine in a person's blood (in milligrams per litre) at time t hours after administration is given by $c(t) = Ate^{-kt}, t \geq 0$, where A and $k \in \mathbb{R}^+$.

If the maximum concentration of medicine in the blood was 0.74 mg/L at 0.5 hours after administration, find the value of A and k, correct to the nearest integer.

Firstly, as always, define the equation. It's easier to stick with f as the function's name and x as the variable than using c and t. Also be sure to use the letters a and k from the **Var** section of your keyboard, not the abc section. Now, whenever you have two unknowns in a question (A and k), you should straight away think about setting up two simultaneous equations.



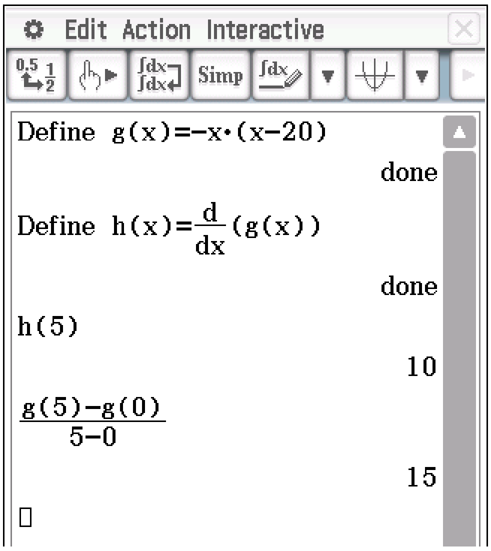
We're told that (0.5, 0.74) is a coordinate on the graph, so that gives us one equation. To get the other equation, we can use the fact that this point is a maximum. This means the derivative equals zero when t is 0.5. Therefore, we can define the derivative equation, and using the point (0.5, 0) to get our other equation. Finally, we just solve them simultaneously!

Instantaneous vs average rate

This trips students up every year. The **instantaneous rate** is the rate at *one point* on a curve, so it's found by subbing the x-value into the **derivative**. The **average** rate of change, on the other hand, is just the **rise over run** between *two points* on a curve.

For example, say we were looking at a curve with the equation $g(x) = -x(x-20)$. If we wanted to know the **instantaneous rate** at $x = 5$, we would just sub 5 into the derivative of g, which we've defined as being $h(x)$.

In contrast, if we wanted to find the **average** rate of change of g over the interval $x \in [0, 5]$, then we need to calculate rise over run instead, which is given by the formula $\frac{y_2 - y_1}{x_2 - x_1}$. Since we've defined g, we can easily get y_2 and y_1 by using $g(5)$ and $g(0)$.



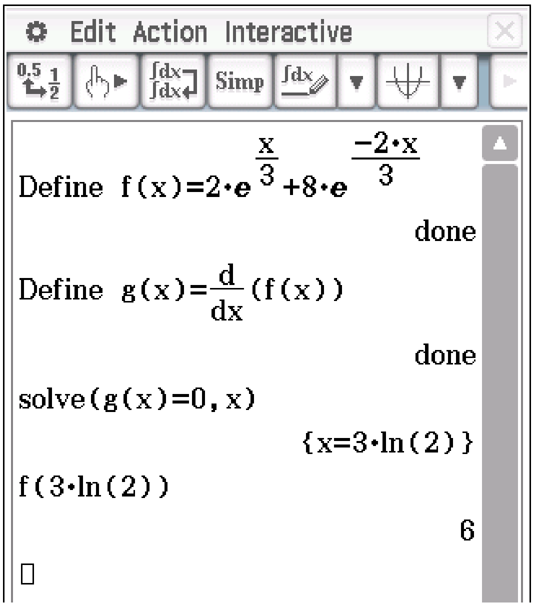
Maxima and minima

Finding the x-value of where the maximum or minimum of a graph occurs can be done two ways, depending on what the question's after.

Making the derivative equation equal zero

This is the traditional way of finding a graph's local maximum or minimum turning point, or stationary point of inflection. Since the gradient of the graph is zero at these points, we just make the derivative equation equal zero and solve for x.

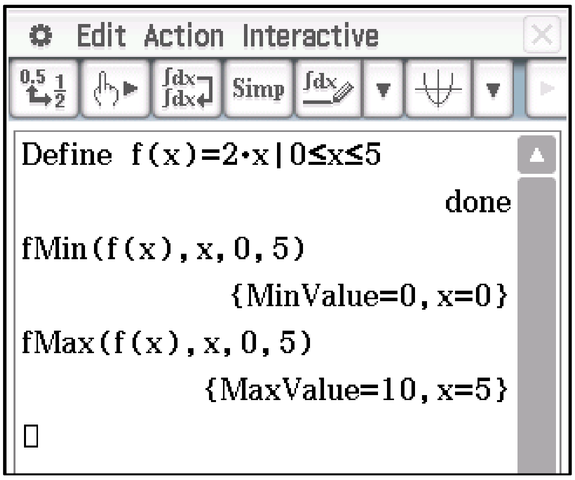
NOTE When you solve $\frac{d}{dx} = 0$, you get the x-value of the turning point or stationary point. To find the y-value, you need to sub the x-value back into the **original** equation – <u>not</u> the derivative! Students often make this mistake, so pay close attention to what equations you're using on your calculator.



Using fMin and fMax

An alternative to the above method is to use the in-built command for finding a function's minimum and maximum. Note that this is the absolute minimum or maximum, so the lowest or highest point on the entire graph. All you do is type out your equation, highlight it all then go to **Interactive**, **Calculation**, **fMin/fMax**, then choose either **fMin** or **fMax**. You'll have to enter a domain in the entry lines 'Start' and 'End'. This is the domain that the CAS will look for the lowest or highest point.

Try it on the equation we defined as $s1(t)$ *above!*



However, you need to practice some caution with this command. You see, it finds the minimum or maximum of the function in general, but not necessarily the minimum and maximum *turning point*. For example, if we consider the equation $f: [0,5] \to R, f(x) = 2x$, we know that because it's a straight line, it doesn't have a maximum or minimum turning point. However, when we use the fMin or fMax command on it, it does give us x-values because it's just telling us where the lowest and highest points on the graph occur.

… So when are fMin and fMax useful?

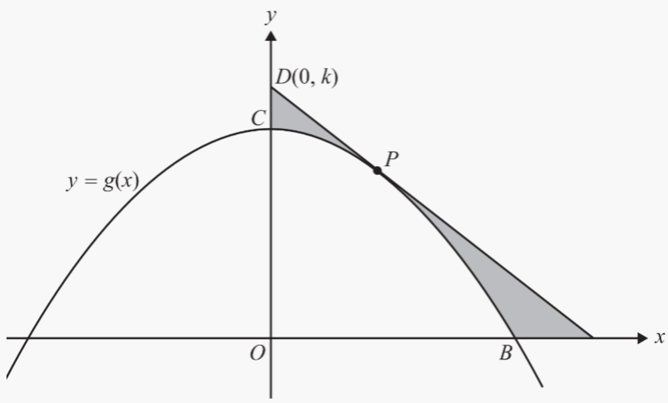
VCAA will sometimes ask questions where they get you to find the maximum and minimum of an equation, but the equation only has one turning point. This often stumps students because they forget that 'maximum' and 'minimum' don't necessarily have to be turning points, but could just be the endpoints. Let's look at an example:

Example (adapted from VCAA 2013, Exam 2)

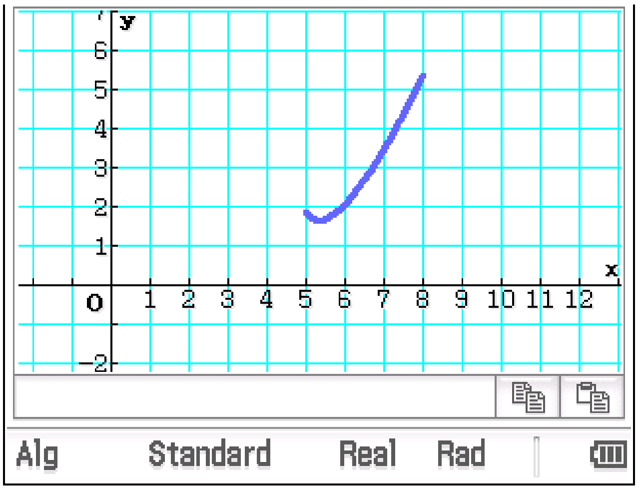
The tangent to the graph of $g(x) = \frac{16-x^2}{4}$ at a point P intersects the y-axis at the point $D(0,k)$, where $5 \le k \le 8$. The area of the shaded region, in terms of k, is given by $A(k) = \frac{k^2}{2\sqrt{k-4}} - \frac{32}{3}$.

a) Find the **maximum** area of the shaded region and the value of k for which this occurs.

b) Find the **minimum** area of the shaded region and the value of k for which this occurs.



Since we're working with the area equation, let's define it on our calculator first. Take note of the domain restriction on k!



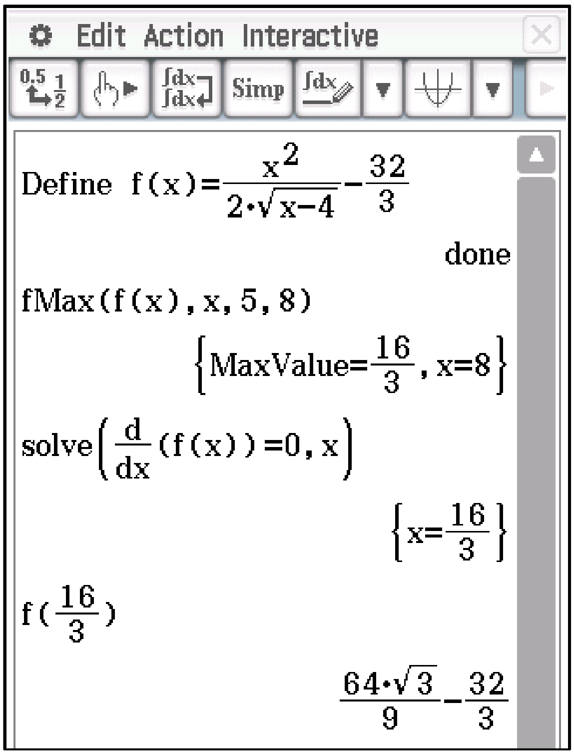
To understand what's going on, it's a good idea to first graph $A(k)$ to see whether the max/min is an endpoint or turning point. As you can see from the graph on the right, the maximum is an endpoint, while the minimum is a turning point – this will influence what method we use to calculate their coordinates:

- For the maximum (endpoint), use fMax because it will tell you the maximum regardless of whether it's a legitimate turning point or not.
- For the minimum (turning point), make the derivative of $A(k)$ equal zero.

In each case, sub the k value you get back into the original equation, $A(k)$, to get the y-coordinate of the point (which represents the area).

$\therefore Max\colon \left(8, \frac{16}{3}\right)$

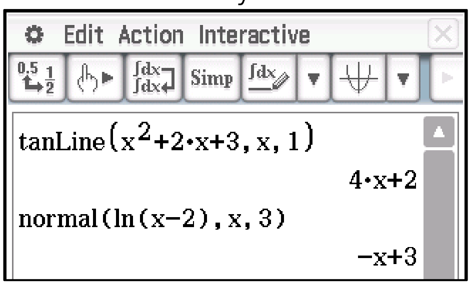
$\therefore Min\colon \left(\frac{16}{3}, \frac{64\sqrt{3}}{9} - \frac{32}{3}\right)$



Tangent and normal equations

As you're probably aware, finding the equation of a tangent or normal can be quite a long process. Luckily, the calculator can do it for us in a flash!

Type out your expression, highlight it all and go to **Interactive**, **Calculation**, **line**, **tanLine** or **normal**. All you have to do is select the x value at which you'd like to find the tangent or normal and you're good to go. In the example below, we've found the equation of the tangent to the curve $y = x^2 + 2x + 3$ at $x = 1$, and the equation of the normal to the curve $y = \log_e(x - 2)$ at $x = 3$.



ANTIDIFFERENTIATION

In this section, we'll go through how to find the antiderivative equation, as well as the difference between setting up an integral for signed area and absolute area.

Finding the equation of the antiderivative

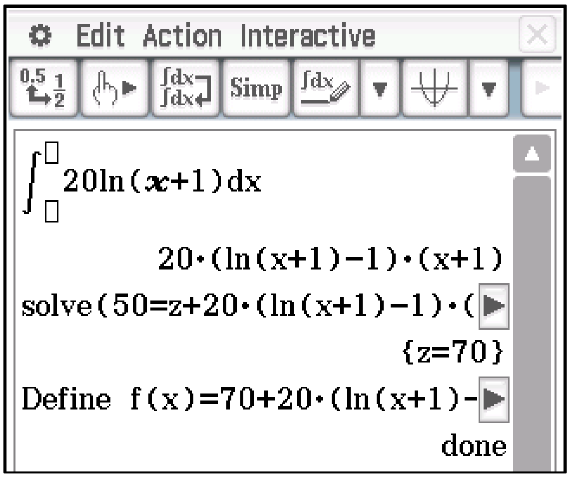
A typical exam question relating to antidifferentiation is when they give you the equation of a derivative, then ask you to find the equation of the antiderivative, given it goes through some point. You can tackle this two ways, which we'll look at through this example:

> To save the quokkas from extinction, fifty of them are introduced to a remote island off the shore of WA at the start of 2000. The population increases steadily with a rate given by the function $R: [0, \infty) \to R,\ R(t) = 20\log_e(t+1)$, where R is the number of quokkas per t years after 1 Jan 2000.
>
> Find an equation for quokka population, P, at time t years after 1 Jan 2000.

When approaching application questions, first always think carefully about the information you're provided and how it relates together. Here, we're given the **rate** of population growth of quokkas – that means $R(t)$ is a **derivative** function. Since we're trying to find the equation for the quokka population, we'll need to antidifferentiate $R(t)$.

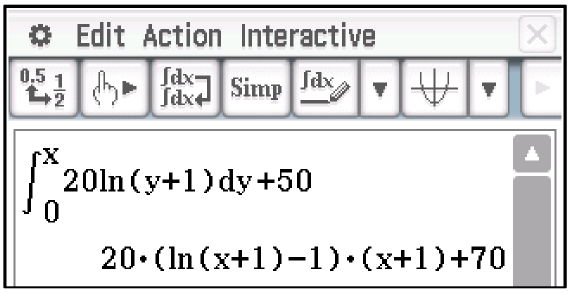
Method 1

We can antidifferentiate $R(t)$ by going into Math2 in our keyboard and selecting the integral icon. Since the calculator doesn't show it, remember that you need to include a $+c$ at the end of the antiderivative. Now we can sub in a point to figure out what this $+c$ is. We know that initially there are fifty quokkas, so our point is (0, 50). As you can see from the screen-shot below, $c = 70$, so $P(t) = 20(t+1)(\log_e(t+1) - 1) + 70$. Note that I've used z instead of c as we can access this quicker.



Method 2

By setting up the integral a bit differently, you can actually get the value of c in your answer straightaway. The format of the integral is as follows: $\int_{x_1}^{x} f(x)\,dx + y_1$. This tells the calculator to antidifferentiate $f(x)$ in terms of x as per usual, but then it subs in x and the point (x_1, y_1). The answer you'll get is an equation in terms of x, with the c value already figured out at the end.



If you're using this method, it's super important to put things in the right places. The x_1 is always the bottom terminal, while y_1 goes outside the integral. Keep in mind that the CAS doesn't like when the same variable is in the integral and terminals. Hence, we have to place in another temporary variable inside the integral for this.

For those who are curious to find out the mechanics behind this method, I've outlined where it comes from below:

Imagine we have a derivative represented by $f(x)$. The antiderivative, $F(x)$, would be given by:

$$F(x) = \int f(x)\,dx + c$$

To find the value of c, we would sub in a point (x_1, y_1) and solve for c.

$$\therefore F(x_1) = \int f(x_1)\,dx + c$$

$$y_1 = \int f(x_1)\,dx + c, \qquad as\ F(x_1) = y_1$$

$$c = -\int f(x_1)\,dx + y_1$$

We can then sub this c value back into $F(x)$ and modify the integrals:

$$F(x) = \int f(x)\,dx - \int f(x_1)\,dx + y_1$$

$$F(x) = [F(x)]_{x_1}^{x} + y_1$$

$$F(x) = \int_{x_1}^{x} f(x)\,dx + y_1$$

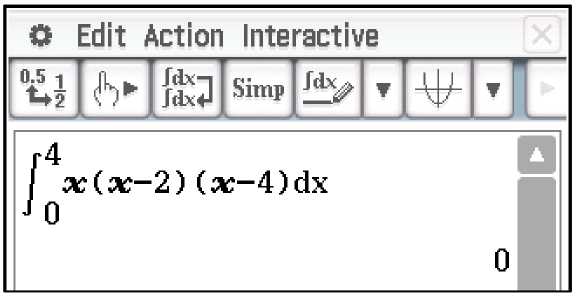
Using integrals to calculate area

There are two types of areas you can be asked to calculate – signed and absolute – and the method for doing each is different.

The **signed area** involves taking the area bounded by a graph and the x-axis, *regardless of whether that area is negative or positive*. On the other hand, when finding the **absolute area**, you do have to be aware of any 'negative' area and make it positive.

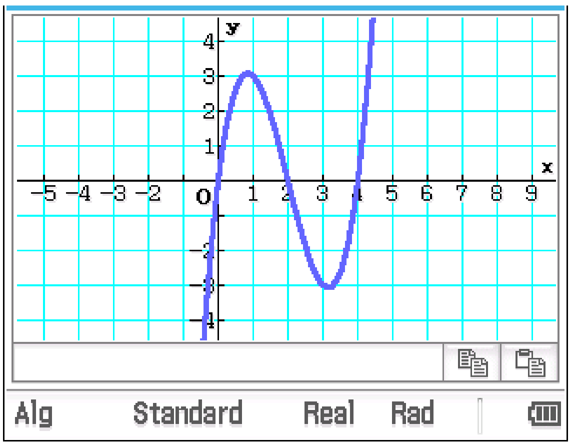
For example, let's say we had the graph of $y = x(x-2)(x-4)$ and we were interested in the area bounded by the graph between $x = 0$ and $x = 4$.

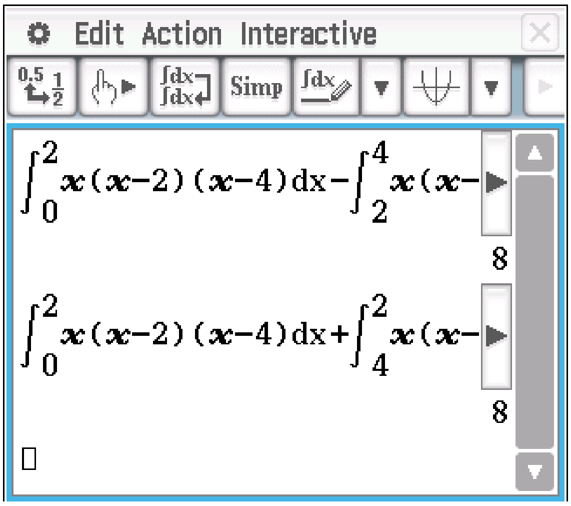
If we were calculating the **signed area**, we wouldn't bother checking whether the area was above or below the x-axis – we would just integrate between $x = 0$ and $x = 4$.



However, if we were after the **absolute area**, we'd need to graph the equation first and calculate area above the x-axis and area below the x-axis separately. As you can see from the graph below, the area between $x \in [2,4]$ is negative. We can make it positive by either:

- Multiplying the integral by -1 (as multiplying two negatives creates a positive)
- Swapping the terminals





The moral to take away from the above is that the signed area does not represent the true area bounded by the graph and the x-axis – what happens is that the negative area actually subtracts from the positive area, resulting in a lower value than expected. In contrast, the absolute area gives you the actual area (as it involves making any negative area positive), so it is the one that crops up in the exam more often.

PROBABILITY

It's one of students' most despised/feared topics. That's why by making it your strength, you can lap up more marks in the exam that other students would avoid or struggle with.

Binomial distributions

When you have a situation where there are only two outcomes (typically either success or fail), but the attempts can be repeated any number of times without changing the probability of those outcomes, then the situation can be described by a **binomial distribution**. Below, we'll look at some common questions that get asked in relation to binomial distributions and how to calculate them.

BinomialPdf vs binomialCdf

When you head to **Interactive**, **Distributions/Inv. Dist**, **Discrete**, you'll notice that there are two types of binomial commands – **Pdf** and **Cdf**. These abbreviations stand for **Probability** Distribution Function and **Cumulative** Distribution Function, respectively. From the names, we can logically infer that the Pdf command gives us the probability of obtaining *one* particular outcome (e.g. 4 successes out of 6 attempts), whereas the Cdf command will *accumulate* or *add* probabilities together (e.g. 4 *or more* successes out of 6 attempts). It's crucial you pick the right one.

Example (VCAA 2008, Exam 2) [GA exam]

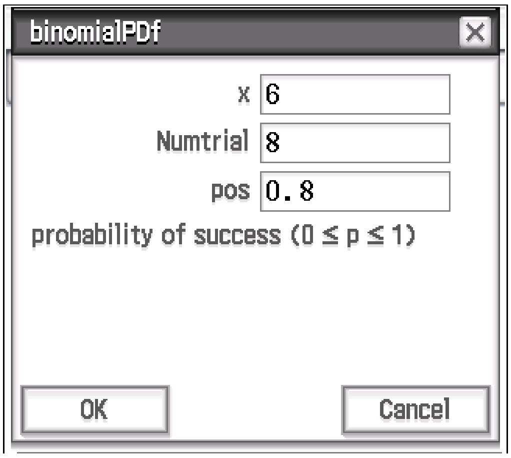
Sharelle is a goal shooter for her netball team. During her matches, she has many attempts at scoring a goal. Assume that each attempt at scoring a goal is independent of any other attempt. In the long term, her scoring rate has been shown to be 80% (that is, 8 out 10 attempts to score a goal are successful).

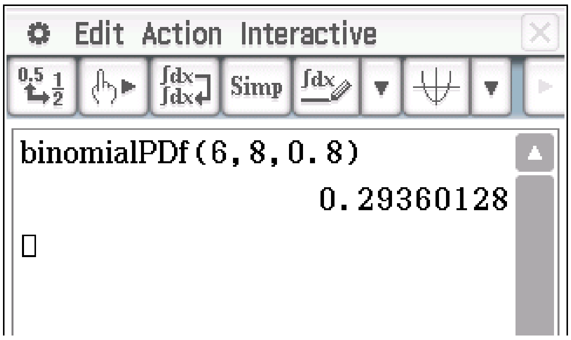
ii. What is the probability, correct to four decimal places, that exactly 6 of her first 8 attempts at scoring a goal in a match are successful?

iii. What is the probability, correct to three decimal places, that her first 4 attempts at scoring a goal are successful, given that exactly 6 of her first 8 attempts at scoring a goal in a match are successful?

First, in probability questions you must identify the type of distribution. Here there are many repeated attempts, and the outcomes are only success/fail, so this in a binomial distribution.

For Part (ii), we're asked for exactly 6 of 8 goals to be successful – the word 'exactly' here is a big hint that we need to use binomialPdf. When filling out the pop-up box for Binomial Pdf, make sure you input the probability as a decimal. The 'X Value' refers to the number of successes you want.

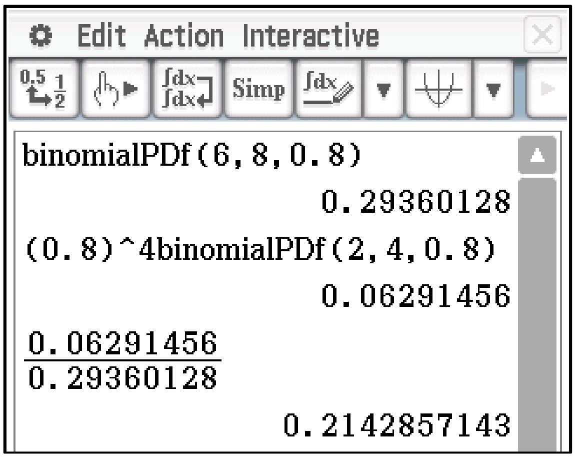




Part (iii) is challenging. The word 'given' is a dead giveaway that this question involves conditional probability. When you're dealing with conditional probability, always identify the outcome you *already know has happened*, and the outcome *you're finding the probability of*. In this case, *we know* that Sharelle has scored 6/8 goals, so that's what's given (it goes on the bottom of our fraction). What we want to know is the probability that the first four in a row are successful. Bear in mind that we still need Sharelle to score 6 goals overall, so we need the first four in a row to be successful, and then any 2 of the next 4 to be successful. The way we'd write this down on our exam paper is as follows:

$$\Pr(first\ 4, then\ 2\ out\ of\ 4) = (0.8)^4 + \binom{4}{2}(0.8)^2(0.2)^2 = 0.062915$$

Then we construct a fraction to find the conditional probability: $\frac{0.62915}{0.2936} = 0.214$



Finding sample size

Sometimes, we know there's a certain probability of an outcome, but we don't know how many attempts we should have to make sure we actually observe the outcome. This comes up in research a lot. For example, you might know from health records that 5% of people develop a certain disease in their lifetime; if you want to observe a group of people over their lifetime to see what factors could be contributing to developing the disease, you want to make sure you pick a sample size large enough such that the chance of at least one person developing the disease is greater than 95%. Otherwise, if your sample size is too small and no one develops the disease, then all your research has been for naught.

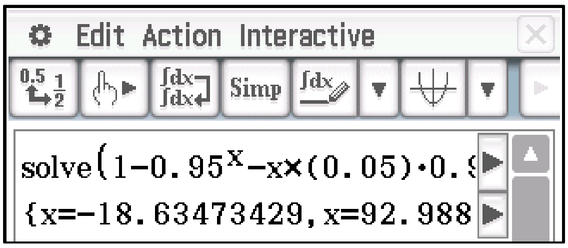
Imagine we have the above situation (where 5% of people will develop a particular disease), and we want to find a sample size such that the probability of at least 3 people getting the disease is more than 95%.

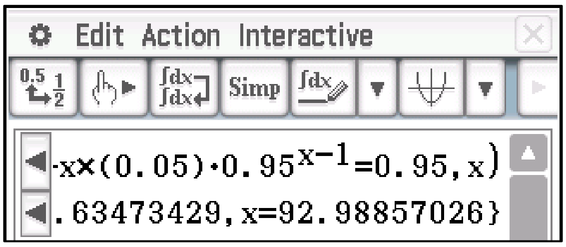
That means we want $\Pr(X \geq 2) > 0.95$, where X is the number of people with the disease. Solving this is very hard because we wouldn't know when to stop adding our probabilities together! Using the complement law, we can write this equation another way, which is easier to solve: $1 - \Pr(X = 0) - \Pr(X = 1) > 0.95$.

Now we just enter the equation on our calculator, using the format $\Pr(X = x) = \binom{n}{x}(p)^x(1-p)^{n-x}$. To figure out what the value of $\binom{n}{x}$ is, you can go to **Catalog** in your keyboard, scroll to the letter **n** and double tap **nCr(**. Enter the sample size (n), followed by a comma and the number of successes ($0, 1,$ and 2 in this case). If the CAS doesn't give nice answers, go back to Pascal's triangle and work out the pattern. In our case, we use $\binom{n}{0} = 1$ and $\binom{n}{1} = n$.

NOTE The calculator doesn't solve equations well when there's an inequality. Instead, use an equals sign on the calculator, but write an inequality sign on your exam paper.

The calculator gives us $n = 92.99$, meaning we need at least 93 people.





Continuous distributions

Data that follows a continuous distribution can be easily identified because they're always presented as a hybrid function.

Setting up integrals

To find the probability of an outcome for a continuous variable, all you have to do is set up a definite integral – just as you would for finding the area under a graph. Something to be weary of is if the continuous distribution is defined by multiple equations – make sure you're using the right equation over the appropriate domain!

Example

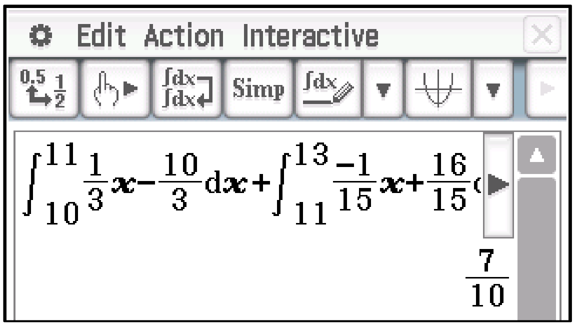
The continuous random variable, T, the time taken for a driver to get to work, has a pdf with the rule:

$$T(x) = \begin{cases} \frac{1}{3}t - \frac{10}{3} & if\ 10 \le t \le 11 \\ \frac{-1}{15}t + \frac{16}{5} & if\ 11 \le t \le 16 \\ 0 & otherwise \end{cases}$$

Find the probability that the time taken to get to work is less than 13 minutes.

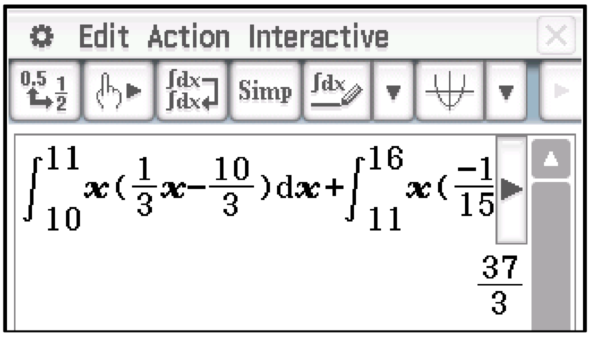
Since the pdf begins at $t = 10$, we'll need to integrate over the interval [10, 13). However, the first equation is only defined over [10, 11], so we'll need to use the second equation for the interval (11, 13). Hence, we need two integrals:

Note that for continuous variables, it doesn't matter whether or not the terminal values are inclusive or exclusive – that is, we'd construct the same integrals regardless of whether the question wanted the probability for 'less than 13' or 'less than or equal to 13'.



Mean

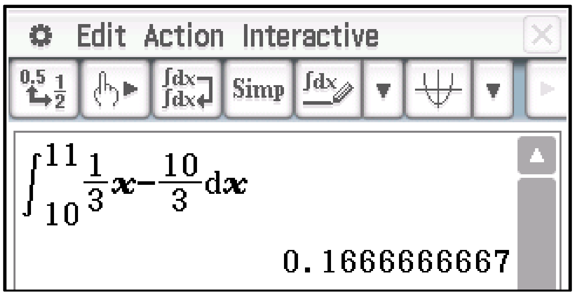
As defined on the formula sheet, the mean for a continuous distribution is found by integrating x times the function over the whole domain. For example, for the above distribution, the mean would be calculated as shown.



Median

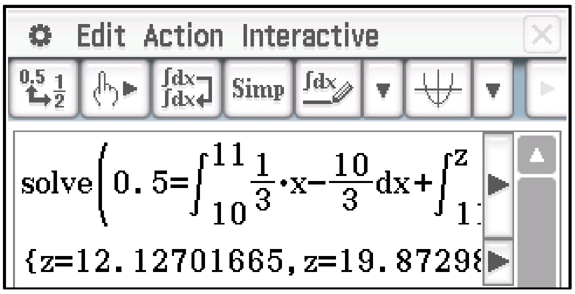
The median is the ‘middle’ value of the data, so it’s essentially the data value that gives you a probability of 50% (0.5). Therefore, to find its value you have to integrate under the graph from the start of the domain to m (the median), and make this integral equal 0.5 to solve for m.

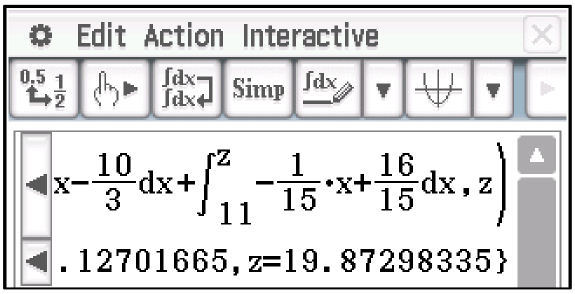
When the continuous random variable is defined by multiple equations, you have to integrate under the first equation first to see whether it’s below or above 0.5. If it’s above 0.5, you can just replace the top terminal with m and solve. If it’s below 0.5, this means the median is in the next equation, so you would put the m in the next integral, as shown below.



Here, when we integrate under the first equation, the area is less than 0.5. This means the median is in the next equation. Therefore, we now integrate underneath the first equation and up to m in the second equation, and make the sum equal 0.5 so we can find m.

Since $m \in [10,16]$, the median is 12.13 minutes (two decimal places). Note that I’ve used z here as this is an easier letter to reach on our physical keyboard than m.

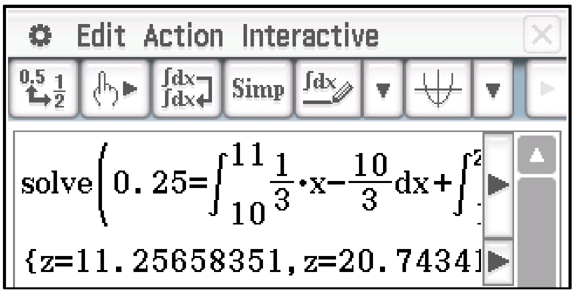


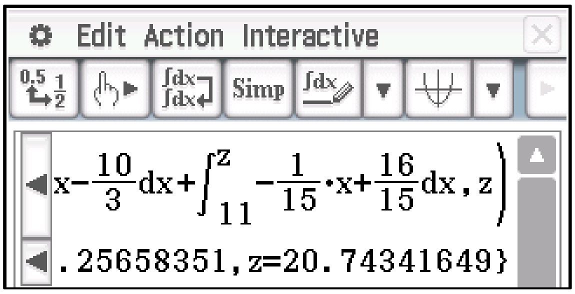


Percentiles

Finding percentiles works in a very similar way to finding the median. The difference is that this time the area isn't 0.5. Rather, if you're trying to find the 10th percentile, you'd make the area equal 0.1. Or if you're trying to find the 80th percentile, you'd make the area 0.8.

Just as before, we integrate our equation(s) from the lower boundary up to some value p which represents the data value that gives you the area you're after. For example, the calculation you'd use to find the 25th percentile (also called the 'lower quartile') is given below.



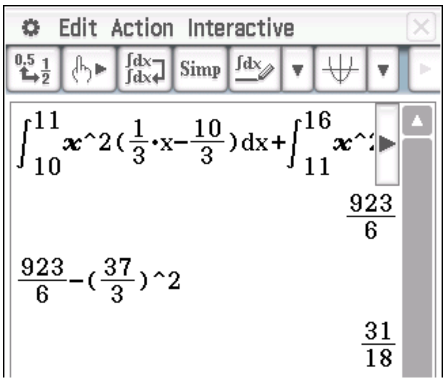


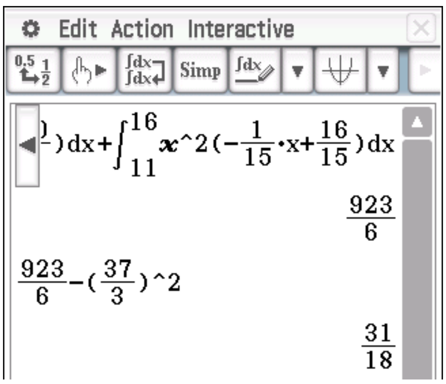
Variance

The formula for variance is a tad confusing, so it's crucial you pay close attention to your formula sheet and enter it on the calculator correctly. Instead of entering it all in one go, it can help to evaluate each part of the variance separately. That is, since the variance formula is $Var(X) = E(X^2) - [E(X)]^2$, it may be easier to first find $E(X^2)$, and then subtract $[E(X)]^2$ from it.

TIP The $[E(X)]^2$ term is just the mean squared. Questions will often get you to find the mean in an earlier part before finding the variance, so you can just use your answer from before (just like we're doing here).

The standard deviation is just the square root of the variance, so if asked to find it you would just evaluate $\sqrt{Var(X)}$. In this case, $SD = \sqrt{\frac{31}{18}} = \frac{\sqrt{62}}{6}$.





Normal distribution

The normal distribution is a special type of continuous distribution. When data is normally distributed, this means it's perfectly symmetrical, with the mean, median, and mode all being the same value. Examples of this include the distribution of heights and ATAR scores – in each case, there is a central value that represents the most common height/ATAR, and as you go further away from the centre (either left or right), the probability of observing a very low or very large height/ATAR gets smaller.

NormPdf vs normCdf

When you go to **Interactive**, **Distribution/Inv. Dist**, **Continuous**, you'll notice that there's both **normPdf** and **normCdf**. The difference is subtle, but crucial.

- **NormPdf** lets you enter one x-value (one data value). The probability that the calculator spits out is all the area to the left of the x-value you enter (i.e. $\Pr(X < x)$). This can be annoying to remember, and can be costly if the question actually wants you to find the area to the right of the x-value (i.e. $\Pr(X > x)$).

- **NormCdf**, on the other hand, gets you to enter a lower and upper bound. If you need to find area to the left of a point, go from $-\infty$ up to the point, or if you need the area to the right of a point, go from the point up to ∞. Therefore, it does the same job as Normal Pdf, but gives you more control! That's why I recommend you **always use Normal Cdf**, and never Pdf.

Example (VCAA 2009, Exam 2)

The Bouncy Ball Company (BBC) makes tennis balls whose diameters are normally distributed with mean 67 mm and standard deviation 1 mm. The tennis balls are packed and sold in cylindrical tins that each hold four balls. A tennis ball fits into such a tin if the diameter of the ball is less than 68.5 mm.

a) What is the probability, correct to four decimal places, that a randomly selected tennis ball produced by BBC fits into a tin?

BBC management would like each ball produced to have a diameter between 65.6 and 68.4 mm.

b) What is the probability, correct to four decimal places, that the diameter of a randomly selected tennis ball made by BBC is in this range?

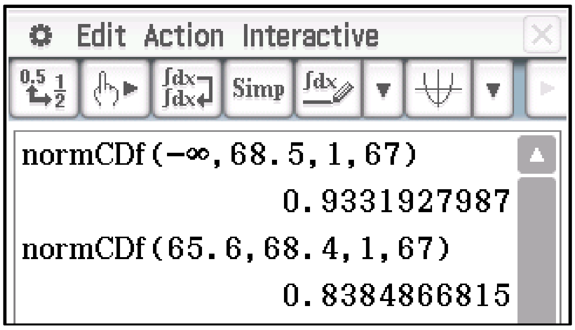
c) What is the probability, correct to four decimal places, that the diameter of a tennis ball which fits into a tin is between 65.6 and 68.4 mm?

For Part (a), a tennis ball fits in a tin if it's less than 68.5 mm. In mathematical terms, we're finding $\Pr(X < 68.5)$. Therefore, our interval is $(-\infty, 68.5)$.

Using normCdf, our 'Lower' entry would be $-\infty$ and our 'Upper' entry would be 68.5. Then we just need to fill in the standard deviation then mean.

For Part (b), our interval is $(65.6, 68.4)$.

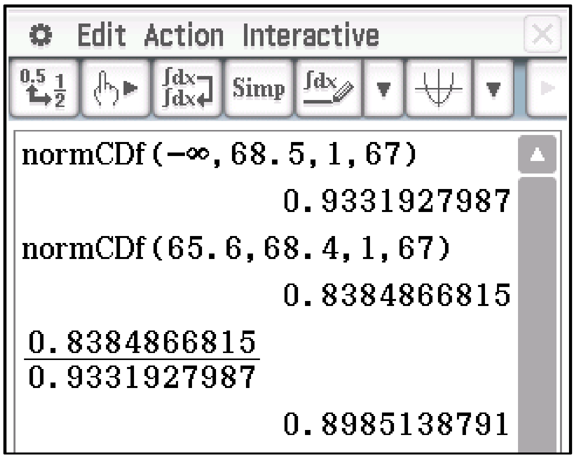
Both Part (a) and (b) can be done using normCdf.



NOTE In the pop-up box that appears, note that standard deviation comes first then the mean. Even though on paper we write the mean first, be careful to input the values in the correct order. Also keep in mind that we need to input the standard deviation, not the variance. So, if you're given the variance, be sure to square root it before putting it into the calculator.

Part (c) is a tricky conditional probability question… a lot of students don't realise it's conditional probability because it's not stated explicitly. However, the fact that the question says that the ball fits in the cylinder, and *then* asks us to find the probability that its diameter is between 65.6 and 68.4 mm makes it conditional. All we do is divide $\Pr(65.6 < X < 68.4)$ by $\Pr(X < 68.5)$.

$\therefore \Pr(65.6 < X < 68.4 | X < 68.5)$ is 0.8985 (four decimal places).



Inverse normal

Found under **Interactive**, **Distribution/Inv. Dist**, **Inverse**, **invNormCdf**, this command lets you enter an area/probability and tells you what data value produces it.

What you have to be cautious of is that the 'Tail settting' drop-down menu affects how the calculator works out the answer.

- If the tail is set to 'Left', then you'll be calculating a value of x such as $\Pr(X < x) = 0.7$. This means that you're finding the data value such that the probability all the way from the left side of the bell curve gives 0.2, for example.

- If the tail is set to 'Right', then you'll be calculating a value of x such as $\Pr(X > x) = 0.3$. This means that you're finding the data value such that the area under the bell curve from the right side is 0.3, for instance.

- If the tail is set to 'Center', then you'll be calculating a value of x such as $\Pr(-x < X < x) = 0.4$. This means that, starting from the middle of the bell curve, the area stretching out in both directions is 0.4, for example.

This Inverse Normal command is particularly useful in questions involving the **standardisation formula**. Before we jump into that, it's important you remember that the **Standard Normal curve** is often represented by Z and is defined as having a mean of 0 and a standard deviation of 1. We can take any other Normal curve and *standardise* it to the Standard Normal curve by using this transformation:

$$z = \frac{x - \mu}{\sigma}$$

What this means is that you can convert an x-data value on a non-standard curve into a z-data value on the Standard curve by subtracting the mean from the x-value and dividing by the standard deviation.

TIP This is a good formula to have in your Summary book as it's not on the Formula Sheet!

So, why is this equation useful? Well, when the mean or standard deviation are unknown, we can rearrange the formula and solve for them. Let's look at this through the following example, which continues from the question above.

Example continued (VCAA 2009, Exam 2)

BBC management wants engineers to change the manufacturing process so that 99% of all balls produced have a diameter between 65.6 and 68.4 mm. The mean is to stay at 67 mm but the standard deviation is to be changed.

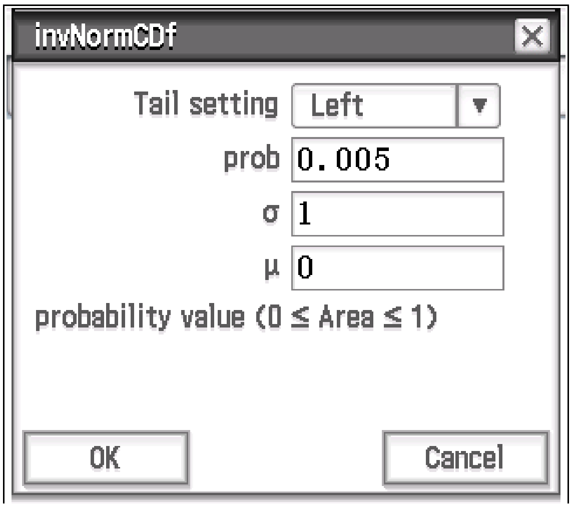
d) What should the new standard deviation be (correct to two decimal places)?

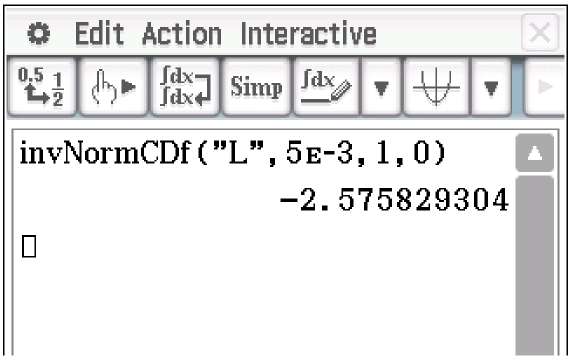
Translating this to mathematical notation, what this question is saying is that $\Pr(65.5 < X < 68.4) = 0.99$. What we've written down is too tricky because there are two x-values involved. To simplify it, we can just consider the area to the left of 65.5, which is equal to 0.005. (I got this by doing (1-0.99) divided by 2.) Therefore, $\Pr(X < 65.5) = 0.005$.

In order to use the standardisation formula to solve for σ, we need to find the value of z. Remember, z just represents the value on the Standard Normal curve that gives you the equivalent area that an x-value gives you on another curve. In this case, we need to find the z-value that gives us an area of 0.005 from the left side of the bell curve. We can use the Inverse Normal command for this!

The calculator tells us that $z = -2.5783$, so we can sub this into the standardisation formula and solve for σ.

Therefore, $\sigma = 0.58$.





STATISTICS

A lot of the commands used in Statistics are the same as those described above in Probability. In Statistics, the major difference is that we're dealing with **samples** that only serve to *represent* a population. That means the sample proportions and sample means we calculate are only **estimates** of what the true population proportions/means are. A classic example is the proportion of males and females in a sample. According to biology, theoretically the population proportion of males and females should each be 0.5. However, when you randomly select people from the population, you might not necessarily exactly have a 50:50 split of females to males.

Calculating confidence intervals

Because sample proportions/means are only estimates, we use confidence intervals to get a range of values that is likely to contain the *true* population proportion/mean.

The equation on the formula sheet for this looks quite scary…

$$Approximate\ confidence\ interval = \left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

However, all this is really saying is that the interval is given by your sample proportion or mean, plus and minus the degree of confidence (z), multiplied by the sample standard deviation. The reason why the p values have **hats** is to represent that these have been calculated from a **sample**, and are therefore only estimates.

When you're using the equation by hand, it's important to note that the z value changes depending on the degree of confidence. For 95% confidence, the z value is 1.96, which you can get by using invNormCdf of 0.95 with Center tail setting. For 98% confidence, z is 2.326 (found by doing Inverse Normal of 0.98 with Center tail setting).

If you're asked to find a confidence interval in a calculator-active exam, we'll have to type out the formula every time. In saying that, there is a neat trick we can use that gives us both the left and right points of the confidence interval in one go.

In your keyboard, head to Math2 and tap on the [■□] icon. This will bring up a 1x2 matrix, though don't worry, this will have nothing to do with matrices. Using x for $\hat{p}$ and y for n (you could use z, too, since these are all accessible on the physical keyboard), we can type in the negative formula in the left square and the positive formula into the right square. Then, use the 'given' symbol to sub in our value of $\hat{p}$ and n. We'll see this in the example below.

Example

A chocolate factory needs to estimate the proportion of their chocolate bars that are underweight so that appropriate remedies can be made. In a random sample of 100 bars, 9 of them are underweight.

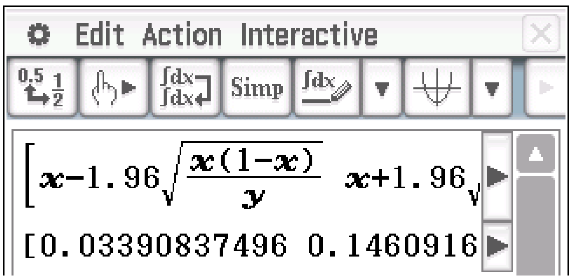
a) Find a point estimate for p, the proportion of bars that are underweight, correct to 2 decimal places.

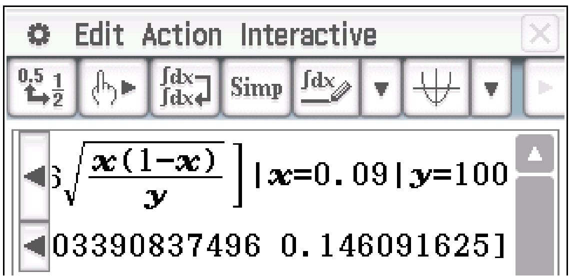
b) Calculate a 95% confidence interval for p (correct to 4 decimal places) and interpret it.

For Part (a), we're asked for an **estimate** of the proportion. Therefore, all we do is divide the number of underweight bars by the total number of bars in the sample, and ensure we write $\hat{p}$ because it is just an estimate of the true proportion.

$$\hat{p} = \frac{9}{100} = 0.09$$

For Part (b), we use the matrix shortcut from before. Type out the negative formula into the first square matrix, then highlight and drag it to the second square – you just have to change the negative in the second square to a positive. Then, using the 'given' symbols, we obtain two numbers: the first being the first part of confidence interval and the second being the other part, just like a real confidence interval! Using this, we get a confidence interval of $(0.0339, 0.1461)$ to four decimal places. To make our lives that much easier, once we've typed out the negative formula, you can highlight and drag it into the second square, then just change the negative to a positive.





That's a wrap!

What a blast we've had with our Casio CAS. Knowing it inside out is the key to saving time and effort in calculator-allowed SACs and exam. Don't be afraid to experiment with it – with this giving you a good foundation, you might even find new commands and shortcuts that will make your Methods journey a breeze. Make the effort to do questions with both CAS and without CAS, even if it's supposed to be done without, and you'll learn for yourself when it's best to use the tech or just do it by hand. Keep at it!

Thank you for reading!

– Yanik Ratilal

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