ALGEBRA
TECH-FREE TEST 1

Writing time: 30 minutes

Structure of test

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	20

Question 1

For the simultaneous linear equations

$$\begin{aligned} 2mx + 3y &= 8 \\ x + my &= m \end{aligned}$$

a. Find the value(s) of m for which these two lines have the same gradient.

2 marks

b. Hence, determine the value(s) of m for which the equations have;

i. no solutions.

ii. a unique solution.

$2 + 2 = 4$ marks

Question 2

Solve the following equation for $x \in [\pi, 2\pi]$.

$$4\sin^2(x) + (2 - 2\sqrt{3})\sin(x) = \sqrt{3}$$

4 marks

Question 3

Consider the functions $f(x) = \tan^2(x)$ and $g(x) = \tan(x)$.

Show that $f(x)g(x) = \dfrac{1}{g(x)} - g(x)$. State the values of x for which this is true.

4 marks

Question 4

Solve the following equation for $x \in \mathbb{R}$.

$$|\log_e(2x+1)| = 9$$

2 marks

Question 5

Let $f(x) = e^{x^2}$ and $g(x) = 3x^2 + 1$.

a. Write down the rule for $g(f(x))$.

1 marks

b. Find the rule for the inverse relation of the result found in **part a**, $g(f(x))^{-1}$.

2 marks

c. State the domain of the inverse function found in **part b.**

1 mark

END OF TEST

ALGEBRA

TECH-FREE TEST 2

Writing time: 30 minutes

Structure of test

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	20

Question 1

Consider the function $f(x) = \dfrac{x-3}{x+1}$.

a. Find the rule for the inverse of this function, f^{-1}.

3 marks

b. State the domain of this inverse function.

1 marks

Question 2

Solve the equation $\log_3(x) = 4\log_x(3)$ for x. Comment on the validity of all solution(s) found.

4 marks

Question 3

Consider the functions $f(x) = \dfrac{1}{x} - 3$ and $g(x) = -ax$.

Find the value(s) of a for which $f(x)$ and $g(x)$ have no intersection.

4 marks

Question 4

Show that $(a+3b)^2 - 12ab \geq 0$ for all $a, b \in \mathbb{R}$.

3 marks

Question 5

Find the values of x that satisfy $|x-1|^2 - 2|x-3| \geq 3$.

5 marks

END OF TEST

ALGEBRA

TECH-ACTIVE TEST 1

Writing time: 60 minutes

Structure of test

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	11	11	11
2	3	3	29
			40

SECTION 1 - Multiple-Choice Questions

Question 1
The number of solutions to the equation $f : \mathbb{N} \to \mathbb{R}, f(x) = 4x^5 - 6x^4 + 3x^3 - 2x + 6 = 0$ is

A. 5

B. 4

C. 3

D. 1

E. 0

Question 2
Which of the following is a solution to the following functional equation for all $x, y \in \mathbb{R}$:

$$\frac{f(x) + f(y)}{f(x) - f(y)} = 1$$

A. $f(x) = -2x + 1$

B. $f(x) = 0$

C. $f(x) = x^2$

D. $f(x) = e^{2x}$

E. none of these

Question 3
The solution(s) to the equation $\log_e(x+5) + \log_e(x^2) = -4\log_e(x)$, correct to two decimal places, is/are

A. nonexistent

B. $x = -5.00,\ -0.78,\ 0.74$

C. $x = -5.00,\ 0.00$

D. $x = 0.74$ only

E. $x = -0.78,\ 0.74$

Question 4
Consider the following system of simultaneous linear equations:

$$\begin{aligned} 4x - my &= 6 \\ nx + 3y &= 7 \end{aligned}$$

For what value of m does there exist an n such that these simultaneous equations have an infinite number of solutions?

A. -3

B. 3

C. -12

D. $-\frac{7}{6}$

E. $-\frac{18}{7}$

Question 5
A possible domain of the function $f(x) = \log_e\left(\sqrt{3} + 2\sin(3x-2)\right)$ is

A. $\left[\frac{2}{3}, \frac{\pi+2}{3}\right)$

B. $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

C. $\left(0, \frac{2\pi}{3}\right)$

D. $(2, \pi+3)$

E. $\left[0, \frac{2\pi}{3}\right]$

Question 6
The function $f : \mathbb{R} \to \mathbb{R}, f(x) = 2x^3 + 18x^2 + ax + 27$ will have an inverse exactly when

A. $a < 54$

B. $a \in \mathbb{R}$

C. $a > 324$

D. $a \geq 54$

E. $0 < a < 54$

Question 7
Correct to two decimal places, the solutions to the equation $2\sin(3x-2) - x = x^2 - 3x - 3$ are

A. nonexistent

B. $x = -1.25, 3.42$

C. $x = 1.25, 3.42$

D. $x = -1.12, 3.80$

E. $x = -1.24, 3.42$

Question 8
Consider the following equation:

$$2x = e^y + e^{-y}$$

Which of the following equations is equivalent to the above?

A. $x = 2(e^y + e^{-y})$

B. $\log_e(2x) = 0$

C. $y = \log_e\left(x \pm \sqrt{x^2-1}\right)$

D. $y = \log_e\left(x^2 + x - 1\right)$

E. none of the above

Question 9
The general solution to the equation $2\tan(2x) = 1$ is

A. $x = \frac{1}{2}n\pi + \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right),\ n \in \mathbb{N}$

B. $\frac{1}{2}n\pi + \frac{\pi}{6},\ n \in \mathbb{Z}$

C. $x = \frac{1}{2}n\pi + \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right),\ n \in \mathbb{Z}$

D. $\frac{1}{2}n\pi \pm \frac{\pi}{6},\ n \in \mathbb{N}$

E. none of these

Question 10
Which of the following functional equations is it possible for $f(x) = 2x^n$ to satisfy for at least one positive value of n?

A. $f(f(x)) = x$

B. $f(xy) = f(x) + f(y)$

C. $f(x+y) = f(x) + 2f(y)$

D. $f(f(f(f(x)))) = 2^{40}x^{81}$

E. none of these

Question 11

The number of real solutions to the equation $\sin^3(x) - 3\sin^2(x) + 3\sin(x) = 1$ is

A. 0

B. 1

C. 2

D. 3

E. none of these

SECTION 2 - Extended-Response Questions

Question 1

Consider the function $f : \mathbb{R} \to \mathbb{R}, f(x) = x^4 - 3x^3 - 6x^2 + px + q$.

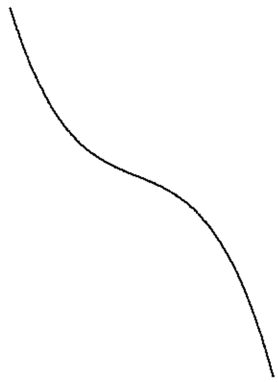
a. Determine the minimum number of stationary points in this function and explain your answer.

2 marks

b. Determine all pairs (p, q) such that $f(x)$ has exactly one stationary point.

4 marks

Now, let us consider the function $g : \mathbb{R} \to \mathbb{R}$, $g(x) = ax^3 + 6x^2 - cx + d$. One possible shape of the graph is below:



c. Given that the shape of $y = g(x)$ is as shown above, determine all values that a, c and d can take. Also determine a relationship between a and c, with c as the subject.

4 marks

Question 2

Consider the following functions:

$$f : \mathbb{R} \to \mathbb{R}, f(x) = 2^{x+4}$$

$$g : \mathbb{R} \to \mathbb{R}, \ g(x) = 2e^{x+3} - 5$$

a. If $p(x) = 5^x$ and $q(x) = 6^x$, show that $p(x) = q\left((\log_6(5))x\right)$.

2 marks

b. Determine a series of transformations to transform $y = f(x)$ to $y = g(x)$.

4 marks

Let us now consider the following functions:

$$r : \mathbb{R} \to \mathbb{R}, r(x) = e^{x+4}$$

$$s : \mathbb{R} \to \mathbb{R}, s(x) = 2e^{x-3}$$

c. Deduce a translation to transform $y = r(x)$ to $y = s(x)$.

2 marks

Consider the following functions:

$$a : (0, \infty) \to \mathbb{R}, a(x) = \log_e(x)$$

$$b : S \to \mathbb{R}, b(x) = \log_e\left(\frac{x^6}{3}\right)$$

d. Deduce a possible transformation to transform $y = a(x)$ to $y = b(x)$, involving at least **one** translation.

2 marks

e. Determine the domain, S, of b, given your answer to **part d.** is valid.

1 mark

Question 3

a. Using algebra, find the general solution to the equation $|\sin(\log_e(2x+5))| = \frac{\sqrt{3}}{2}$ for $x \in \mathbb{R}$.

4 marks

b. Determine the domain of the function $f(x) = |\sin(\log_e(2x+5))|$.

2 marks

c. Consider two adjacent x-intercepts of the graph of $f(x) = |\sin(\log_e(2x+5))|$, $(x_1, 0)$ and $(x_2, 0)$. Explain, in words, whether the stationary point between those two x-intercepts is to the left, to the right, or the same as the point $\left(\frac{x_1+x_2}{2}, f\left(\frac{x_1+x_2}{2}\right)\right)$.

2 marks

ALGEBRA

TECH-ACTIVE TEST 2

Writing time: 60 minutes

Structure of test

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	11	11	11
2	2	2	29
			40

SECTION 1 - Multiple-Choice Questions

Question 1
Which of the following is an exact solution to the equation $5 \times 2^{3x} = 1$?

A. $x = \frac{1}{5}\log_2\left(\frac{1}{3}\right)$

B. $x = 3\log_2\left(\frac{1}{5}\right)$

C. $x = -\frac{1}{3}$

D. $x = \frac{1}{3} \times 2^{\frac{1}{5}}$

E. $x = \frac{1}{3}\log_2\left(\frac{1}{5}\right)$

Question 2
The sum of the solutions to the equation $2\cos(2x) = 1$ for $x \in [0, 2\pi]$ is

A. $\frac{2\pi}{3}$

B. π

C. 2π

D. 4π

E. $\frac{\pi}{3}$

Question 3
The polynomial $P(x) = x^3 + ax^2 + bx + 6$ has a factor $(x + 1)$ and a remainder of 4 when divided by $(x - 1)$. The values of a and b, respectively, are

A. -1 and 1

B. 4 and 1

C. -2 and -1

D. 3 and -1

E. -4 and 1

Question 4
The solution to the inequality $(0.6)^{-x} \leq 3$ is approximately

A. $x \geq 0$
B. $x \geq 2.15$
C. $x \leq 2.15$
D. $x \leq -0.86$
E. $x \leq -2.15$

Question 5
The number of real solutions to the equation $(x^2 - 3x - 4)(x^2 - 6x + 9) = 0$ is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 6
The solution(s) to the equation $\log_3(x - 2) - \log_3(x) = 1$ are

A. $x = -3$, $x = 1$
B. $x = -1$
C. $x = 3$
D. $x = -1$, $x = 3$
E. There are no real solutions for x

Question 7
The coefficient of x^6 in the expansion of $(a + x^3)^5$ is

A. $-a^3$
B. $10a^3$
C. $-10a^3$
D. a^3
E. $10a^2$

Question 8

If $\cos(x) = 0.7$, the value of $\sin\left(\frac{\pi}{2} - x\right) + \cos(x + \pi)$ is

A. -1

B. 0.7

C. $\frac{\pi}{2}$

D. 1.4

E. 0

Question 9

The solution to the equation $2\log_e(x) - \log_e(2x) = y$, in terms of y, is

A. $x = 2e^y$

B. $x = y$

C. $x = \log_e(2y)$

D. $x = 2$

E. $x = \frac{1}{2}$

Question 10

If $\sin(2x) + \cos(2x) = 0$ for $0 \leq x \leq 2\pi$, the difference between the largest and smallest solution for x is

A. π

B. 2π

C. $\frac{3\pi}{2}$

D. $\frac{7\pi}{4}$

E. 4π

Question 11

For which of the following conditions does the equation $2^{2x} + 2^x + b = 0$ have exactly one real solution?

A. $b < 0$

B. $b > 0$

C. $b \leq 0$

D. $b = \frac{1}{4}$

E. $b = 4$

SECTION 2 - Extended-Response Questions

Question 1

Consider the following function, f, where c is a real constant:

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + c$$

a. Assuming that $c = 0$, show that

i. the function $f(x)$ has three x-intercepts.

ii. the x-coordinates of the stationary points are $x = -1$ and $x = 3$.

$2 + 3 = 5$ marks

b. Find the maximal range of values for c such that the equation $f(x) = 0$ has

i. **one** real solution.

ii. **three** distinct real solutions.

iii. **two positive** distinct real solutions.

$2 + 2 + 2 = 6$ marks

c. Consider the function $g(x) = \frac{1}{3} \times 2^{3x} - 2^{2x} - 3 \times 2^{x} + d$.

i. Find all real value(s) of x which satisfy the equation $g(x) = d$.

ii. Find the value(s) of d such that the equation $g(x) = 0$ has two real solutions.

$3 + 2 = 5$ marks

Question 2

Consider the following system of simultaneous equations, with variables x, y and z:

$$\begin{aligned} x - y - 2z &= -3 \\ tx + y - z &= 3t \\ x + 3y + tz &= 13 \end{aligned}$$

a. Using algebra, show that $x = \dfrac{3t^2 - 21t + 30}{t^2 - 5t + 6}$.

3 marks

b. Find the two values of t such that the given system of simultaneous equations will have no unique solutions. (Hint: You need only analyse a 2-variable set of simultaneous equations. Consider the values of t for which x cannot be defined, and analyse your working in part **a.** to see what happens to your working for those values of t.)

3 marks

c. For which value of t does the system have no real solutions?

3 marks

d. If $t = 0$, solve the system of simultaneous equations for x, y and z.

4 marks

END OF TEST