PROBABILITY

TECH-FREE TEST 1

Writing time: 30 minutes

Structure of test

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	20

Question 1

The discrete random variable, X, has the probability distribution:

x	-2	-1	0	1	2
$\Pr(X = x)$	$2p$	p^2	$\frac{p^2}{2}$	$\frac{p+2}{3}$	p^2

Find the value of p.

2 marks

Question 2

Marty is attempting a Mathematical Methods topic test on probability. The test consists of 20 multiple choice questions, and each question has 5 options. Unfortunately, Marty has not studied for the test and as such, he must guess every question at random.

a. Find the standard deviation for the number of questions he answers correctly.

2 marks

b. What is the probability that Marty will get 95% or more in this test?

3 marks

Question 3

a. A and B are mutually exclusive events where $\Pr(A) = \Pr(B) = p$. Find $\Pr(A \mid B)$.

2 marks

b. C and D are independent events. Show that $\Pr(C \cap D) = \Pr(C)\Pr(D)$.

2 marks

Question 4

The amount of cookies (in kilograms) that Dan consumes in a day has the probability density function,

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 2 \\ 5k(x-2), & 2 < x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the value of k.

3 marks

b. Evaluate μ, the mean amount of cookie consumption per day.

3 marks

c. Calculate the probability, $\Pr(|X - \mu| < 1)$.

3 marks

END OF TEST

PROBABILITY
TECH-FREE TEST 2

Writing time: 30 minutes

Structure of test

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	20

Background

Mr. Li is an elite sportsman competing in a 7 day tournament hosted by Non-Trivial Sports Inc. He competes in the Darts, Vertical Jump and Triple Jump events on each day of the tournament.

Question 1

Mr. Li would like to drive his car every day to the stadium, but since he shares his car with his brother, he must ride his bike sometimes. If he drives his car on one day, there is a 30% chance he will drive the next day. If he rides his bike on a particular day, he has a 60% chance of riding again the following day. On Day 1 of the tournament, he drives his car.

a. What is the probability he drives his car on the first 3 days?

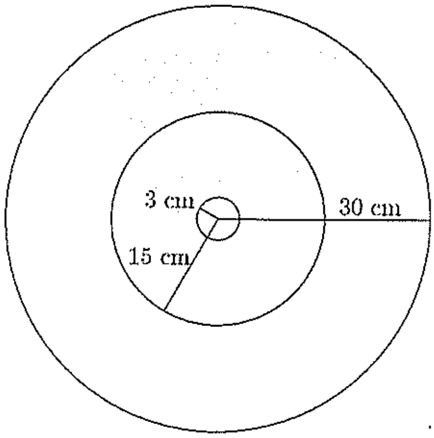
1 mark

b. Calculate the probability that he rides his bike on day 3.

3 marks

Question 2

Mr. Li is not very experienced at darts, so when he throws, each dart is equally likely to land at any point on the board shown below. The dartboard consists of three circular sections with radii of 30 cm, 15 cm, and 3 cm respectively. None of his throws land on the edge of a region.



100 points is awarded for throwing the dart in the centre circle, 25 points for the middle circular section and 10 points for the outer circular section.

a. Define the probability distribution for points scored with one throw.

3 marks

b. What is the probability of Mr. Li scoring more than 50 points with two throws.

3 marks

Question 3

Mr. Li's vertical jump is normally distributed with a mean of 75 cm and a standard deviation of 3.

a. What is the approximate probability that he will jump over 81 cm?

2 marks

b. If he makes one jump per day throughout the tournament, calculate the probability he will jump over 75 cm on 5 days. Give your answer in the form $a(x)^b(1-x)^c$.

3 marks

Question 4

Mr Li is a triple jump specialist and he will win if he jumps over 13 m in both the heat and the final on any given day (i.e. every day there is a heat and final). He has learnt that if he jumps over 78 cm in the vertical jump, he will always jump over 13 m, but has no chance of winning if he does not jump this height. However, he also discovers that when he rides his bike to the stadium, the mean of his vertical jump drops to 74 cm and the standard deviation becomes 2; there is no change when he drives his car.

What is the probability he wins the triple jump event on day 2, correct to 3 decimal places? You may assume that $\Pr(-1 \leq Z \leq 1) = 0.68$, and that $\Pr(-2 \leq Z \leq 2) = 0.95$.

5 marks

END OF TEST

PROBABILITY

TECH-ACTIVE TEST 1

Writing time: 60 minutes

Structure of test

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	11	11	11
2	2	2	29
			40

SECTION 1 - Multiple-choice questions

Question 1

If the following table represents a probability distribution then the possible values of k are

x	-2	-1	0	1	2
$\Pr(X = x)$	$\frac{1}{2}k$	k^2	$2k^2$	$3k^2$	$\frac{1}{2}k$

A. $k = \frac{\sqrt{2}}{4}$

B. $k = -\frac{1}{2}$ and $k = \frac{1}{3}$

C. $k = -\frac{1}{2}$

D. $k = \frac{\sqrt{2}}{2}$

E. $k = \frac{1}{3}$

Question 2

If a fair six sided die is rolled 3 times then the probability of obtaining at least 1 occurrence of a 4 is given by

A. $\frac{3!}{2!1!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 + \frac{3!}{3!0!}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0$

B. $1 - \frac{3!}{0!3!}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3$

C. $\frac{3!}{0!3!}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3$

D. $\frac{3!}{3!0!}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0$

E. $1 - \frac{3!}{0!3!}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3$

Question 3

The probability that Andrew does homework on a day given that he did homework on the previous day is 0.6, while the probability he doesn't do homework on a day given that he didn't do homework on the previous day is 0.2. If he doesn't do homework on a Monday then the probability that he doesn't do homework on a Wednesday is

A. 0.36

B. 0.72

C. 0.68

D. 0.64

E. 0.32

Question 4

A random variable X can be represented by a normal distribution and Z is represented by the standard normal distribution. If the mean and variance of X is 8 and 16 respectively, then the probability that X is greater than 10 is equivalent to

A. $\Pr(Z < -1)$

B. $\Pr(Z > 2)$

C. $\Pr(Z < -\frac{1}{2})$

D. $\Pr(Z > 18)$

E. $\Pr(Z > 8)$

Question 5

If the following table represents a probability distribution, then the median m, and variance σ^2 of the distribution is

x	-2	-1	0	1	2	3
$\Pr(X = x)$	0.05	0.05	0.20	0.20	0.25	0.25

A. $m = 1.0$ and $\sigma^2 = 2.01$

B. $m = 1.5$ and $\sigma^2 = 2.01$

C. $m = 1.5$ and $\sigma^2 = 2.40$

D. $m = 1.3$ and $\sigma^2 = 2.40$

E. $m = 1.3$ and $\sigma^2 = 2.40$

Question 6

A binomial random variable X, has a probability distribution which has an expected value of 3 and a standard deviation of $\frac{3}{2}$. The binomial probability distribution is given by

A. $X \sim \text{Bi}\left(2, \frac{2}{3}\right)$

B. $X \sim \text{Bi}\left(3, \frac{1}{4}\right)$

C. $X \sim \text{Bi}\left(6, \frac{1}{2}\right)$

D. $X \sim \text{Bi}\left(12, \frac{1}{4}\right)$

E. $X \sim \text{Bi}\left(16, \frac{3}{4}\right)$

Question 7

A probability density function is given by the function f with the following rule

$$f(x) = \begin{cases} \frac{1}{x+1} & 0 \le x \le e-1 \\ 0 & x < 0 \text{ or } x > e-1 \end{cases}$$

The expected value and median of f correct to 4 decimal places is

A. $\mathrm{E}(X) = 0.7183$ and $m = 0.6487$

B. $\mathrm{E}(X) = 1.0000$ and $m = 0.4246$

C. $\mathrm{E}(X) = 1.0000$ and $m = 0.6487$

D. $\mathrm{E}(X) = 0.7183$ and $m = -2.6487$

E. $\mathrm{E}(X) = 0.4246$ and $m = 0.5675$

Question 8

Australia and Sri Lanka are playing a cricket match. If Australia won the previous game then the probability of them winning the next game is $\frac{1}{2}$. If Sri Lanka wins the previous game then the probability of them winning the next game is $\frac{1}{3}$. The steady state probability that Australia wins a game is

A. $\frac{1}{6}$

B. $\frac{4}{7}$

C. $\frac{3}{7}$

D. $\frac{5}{6}$

E. $\frac{1}{2}$

Question 9

If the following table represents a probability distribution, then the probability that X is greater than or equal to 2, given that it is equal to or greater than 0 is

x	-2	-1	0	1	2	3	4
$\Pr(X = x)$	0.1	0.1	a	0.2	0.15	0.1	0.05

A. $\frac{4}{5}$

B. $\frac{5}{8}$

C. $\frac{3}{10}$

D. $\frac{1}{5}$

E. $\frac{3}{8}$

Question 10

A probability density function is given by

$$f(x) = \begin{cases} \dfrac{12x\left(x^2 - 13x + 40\right)}{1375} & 0 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

Then the mode of the distribution is

A. $x = 2$

B. $x = 3$

C. $x = 1.9469$

D. $x = 2.2218$

E. $x = 4$

Question 11

A random variable X can be represented by a normal distribution with mean $\mu = 5$ and standard deviation $\sigma = 3$. If $\Pr(X \geq a) = 0.8$, then the value of a correct to 4 decimal places is

A. -2.5746

B. 7.5249

C. 2.4751

D. 3.5423

E. 6.5477

SECTION 2 - Extended response questions

Question 1

A company sells two types of soft drink, orange and lemon flavoured. The amount of liquid, X ml in an orange flavoured drink is normally distributed with a mean of 600 ml and a standard deviation of 3 ml. The amount of liquid, Y ml in the lemon flavoured drink follows the probability density function given below

$$f(y) = \begin{cases} \dfrac{3y^2 - 3600y + 1080064}{2048} & 592 \leq y \leq 608 \\ 0 & \text{elsewhere} \end{cases}$$

a. Find correct to 4 decimal places

i. $\Pr(X \geq 603)$

ii. $\Pr(597 \leq Y \leq 606)$

1 + 2 = 3 marks

b. Find the mean value of liquid in the lemon flavoured drink, that is $\mathrm{E}(Y)$.

2 marks

c. Find the value of a if $\Pr(Y \leq a) = 0.85$, correct to 2 decimal places.

2 marks

d. If 10 bottles of Orange soft drink are picked at random, find the probability that at least 2 of them have less than 595 ml of liquid correct to 4 decimal places.

3 marks

e. Find $\Pr(Y \leq 595 | Y \leq 600)$

3 marks

f. If 20 bottles of Orange soft drink and 10 bottles of Lemon soft drink at put in a box. Find the probability of picking a bottle containing Orange soft drink given that the bottle contains less than 595 ml correct to 4 decimal places.

3 marks

Question 2

Will is the free-kick taker for his soccer team. The probability that Will scores the goal given that he scored on his previous attempt is 0.7, while the probability that he scores given that he did not score on his previous attempt is 0.45.

a. If Will scores on his first attempt, then find the probability that he does not score on his 5^{th} attempt, correct to 4 decimal places.

2 marks

b. If Will does not score on his first attempt, then find the probability that he scores on his next three attempts, correct to 2 decimal places.

1 mark

c. Find the steady state probability that Will scores a goal.

1 mark

d. Will's previous goal was successful. If he has 3 shots at goal, find the expected number of goals he scores (not including the previous attempt), correct to 2 decimal places.

3 marks

Will's friend Michael is the goal keeper for their team. He is successful if he stops goals from being scored against them. The probability that Michael blocks any shot is 0.4.

e. Find the probability that Michael blocks exactly 2 out of 5 shots, given that he blocks at least 1 shot.

3 marks

f. If the probability of blocking at least 2 shots has to be greater than 0.7, find the minimum total number of shots that he should attempt to block.

3 marks

END OF TEST

PROBABILITY

TECH-ACTIVE TEST 2

Writing time: 60 minutes

Structure of test

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	11	11	11
2	3	3	29
			40

SECTION 1 - Multiple-choice questions

Question 1

If the following table represents a probability distribution, then the values of $\mathrm{E}\left(X^2\right)$ and $\mathrm{Var}\,(2X+3)$ are respectively

x	-3	-2	-1	0	1	2	3
$\Pr(X=x)$	0.2	0.1	0.2	0.2	0.05	0.05	0.2

A. -0.25 and 4.39

B. 4.45 and 4.39

C. 4.45 and 17.55

D. -0.25 and 17.55

E. -0.25 and 17.55

Question 2

A continuous random variable X is given by

$$f(x) = \begin{cases} \dfrac{1}{72}\left(x^2 - 6x + 18\right) & 0 \le x \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

Then the value of $\mathrm{Var}\,(X)$ is

A. 3.60

B. 8.60

C. 6.00

D. 3.00

E. 6.60

Question 3

Sam is playing a game in which he can either win or lose. The probability of winning on any game is 0.3. What is the minimum number of games that Sam is required to play if the probability of him winning at least 2 games is greater than 0.90?

A. 12

B. 11

C. 10

D. 9

E. 8

Question 4

If two events A and B are independent and $\Pr(A \cap B') = 3\Pr(A \cap B)$ then

A. $\Pr(A) = \Pr(B)$

B. $\Pr(A) = \frac{1}{2}$

C. $\Pr(A) = \frac{1}{4}$

D. $\Pr(B) = \frac{1}{2}$

E. $\Pr(B) = \frac{1}{4}$

Question 5

The time take for a racing team to do a pit stop, that is to change all 4 wheels on a car is normally distributed with a mean of 4.6 seconds and a standard deviation of 0.6. The middle 50% of pit stops will be completed between

A. 4.40 and 4.80 seconds

B. 4.36 and 4.84 seconds

C. 0.00 and 4.60 seconds

D. 4.20 and 5.00 seconds

E. 4.60 and 9.20 seconds

Question 6

A probability density function is given by the function f, where is given by

$$f(x) = \begin{cases} k^2(\cos(x) + 1) & 0 \leq x \leq \pi \\ 0 & \text{elsewhere} \end{cases}$$

Then the value(s) of k are

A. $-\frac{2}{\sqrt{\pi}}$ and $\frac{2}{\sqrt{\pi}}$

B. $\frac{1}{\sqrt{\pi}}$

C. $-\frac{1}{\sqrt{\pi}}$

D. $-\frac{1}{\sqrt{\pi}}$ and $\frac{1}{\sqrt{\pi}}$

E. $\frac{2}{\sqrt{\pi}}$

Question 7

A discrete random variable X has a probability distribution according to the following table

x	-4	-2	0	2	4
$\Pr(X = x)$	a	0.3	b	0.1	0.1

The mean of the distribution is -1.0. Then the values of a and b are

A. $a = 0.15$ and $b = 0.35$

B. $a = 0.20$ and $b = 0.30$

C. $a = 0.25$ and $b = 0.25$

D. $a = 0.10$ and $b = 0.10$

E. $a = 0.15$ and $b = 0.00$

Question 8

Matt is playing a game of cricket. When the ball comes in his direction, he can either catch or drop the ball. The probability that he catches the ball on any one hit is 0.6. Then the probability that out of 6 balls, he catches at least 3 of them is

A. 0.1792

B. 0.2765

C. 0.4557

D. 0.8208

E. 0.6054

Question 9

The probability that Rahad attends lectures given that he attended the day before is 0.75. While the probability that he attends lectures given that he didn't the previous day is 0.5. If Rahad attends lectures on Monday, then the probability that he attends lectures on Friday is

A. 0.3320

B. 0.6680

C. 0.6670

D. 0.3330

E. 0.6641

Question 10

A continuous random variable X is given by

$$f(x) = \begin{cases} \dfrac{1}{e^2 - 3}\left(e^{2-x} - 1\right) & 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Then the probability that X is greater than 1 correct to 4 decimal places is

A. 0.5000

B. 0.8363

C. 0.9661

D. 0.5485

E. 0.1637

Question 11

A random variable X can be represented by a normal distribution and Z is represented by the standard normal distribution. If the mean and variance of X is 5 and 4 respectively, then the probability that X is greater than 1 is equivalent to

A. $\Pr(Z > 2)$

B. $\Pr(Z > -1)$

C. $\Pr(Z < -2)$

D. $\Pr(Z < 2)$

E. $\Pr(Z > 0)$

SECTION 2 - Extended response questions

Question 1

Dennis takes the train to university every day. On some days he misses the train completely, and the probability that he makes it onto his train each day is p, where $p > 0$. Let the random variable X be the number of times that Dennis makes it onto his train on the way to university.

a. Dennis goes to university 5 days a week, for a five week period. If $\Pr(X > 22) = 4\Pr(X = 25)$ then find the possible value(s) of p.

3 marks

b. **i.** If instead the expected number of days that he makes the train is 23, then show that the value of p is $\frac{23}{25}$ and find the standard deviation of the distribution, correct to 2 decimal places.

ii. Once Dennis does get on a train, the average time (in minutes) taken for the journey to university from the departure of the train that he gets on is normally distributed with a mean of μ and standard deviation of σ. If we represent this time by the random variable Y, then it holds that

$$\Pr(X = 25) = \Pr(Y \leq 34.2)$$
$$\Pr(X \geq 22) = \Pr(Y \geq 34.5)$$

Using the value of p found in question 1 bi, find the values of μ and σ correct to 4 decimal places.

iii. If a train takes less than 34 minutes than it is considered to be on time. Find the probability that the train journey takes less than 30 minutes given that the train runs on time, correct to 4 decimal places.

2 + 4 + 3 = 9 marks

Question 2

During the summer holidays, Jake gets invited to a lot of events. The probability that Jake turns up to an event given that he turned up to the last one is p, while the probability that he turns up to an event given that he didn't turn up to the previous event is $p + \frac{3}{20}$.

a. If the long run probability that Jake does show up to an event is $\frac{7}{23}$, then show that the value of p is $\frac{1}{5}$.

3 marks

b. If Jake does not go to the first event, then find the probability that he goes to the 15^{th} event that he is invited to, correct to 4 decimal places.

3 marks

c. If the probability that Jake doesn't attend to the 4^{th} event is 0.6980 then find whether or not he attended the first event.

2 marks

Question 3

The number of items X, that a customer buys at a convenience store is given by the probability distribution below

x	0	1	2	3	4
$\Pr(X = x)$	0.3	a	0.2	0.05	b

a. If the standard deviation of X is 1.06184 , then find the values of a and b, correct to 2 decimal places.

3 marks

b. The average price of an item if the customer buys 1 or 2 items is \$2.00 while the average price of an item if the customer buys 3 or 4 items is \$1.50. If Y is the cost of item(s) to the customer then draw out the discrete probability distribution for Y. Assume the customer buys at least 1 item.

4 marks

c. Assuming the customer buys an item, find the expected price that the customer pays and the median price m, correct to 2 decimal places.

2 marks

END OF TEST