SET 1 EXAM 2

Reading time: 15 minutes

Writing time: 120 minutes

Structure of examination

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			80

Note: Formula Sheet is NOT supplied. You will need to use your own!

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, **one** bound reference, **one** approved cas calculator and one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape

Materials supplied

- Question and answer book

Instructions

- Complete all multiple-choice questions by circling your choice on the book.
- Complete all extended-response questions in the spaces provided.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1 - Multiple-choice questions

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question

Question 1

The value(s) of m for which the pair of equations $(m+2)\,x+3y=(m+3)$ and $4x+(2m-1)\,y=5$ have infinite solutions are

A. $2, -\frac{7}{2}$

B. $-\frac{7}{2}$

C. 2

D. $\mathbb{R}\setminus\{2\}$

E. $\mathbb{R}\setminus\left\{-\frac{7}{2}, 2\right\}$

Question 2

If $2x+c$ is a factor of $2x^3+7x^2+cx$ and $c \in \mathbb{R}\setminus\{0\}$, then the value of c is

A. -1

B. 0

C. 1

D. 3

E. 5

Question 3

The line that is normal to $f(x)=x^3+5x^2+2x-8$ at $x=-1$ is

A. $y=\frac{1}{5}(x-29)$

B. $y=\frac{1}{5}(x+1)$

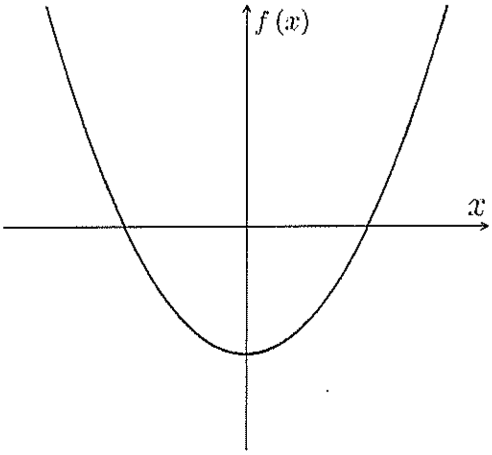
C. $y=-5x-11$

D. $y=-5x+29$

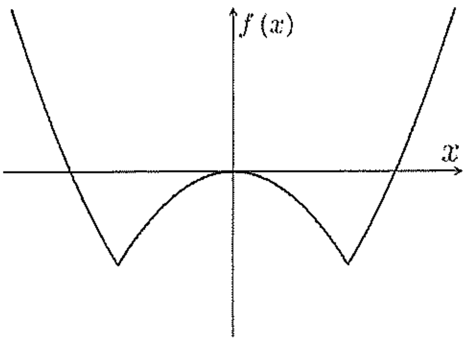
E. $x=-1$

Question 4

If a function, $f(x)$ is represented by the following graph, and $g(x) = |x| - 4$, then a possible graph of $g(f(x-2))$ could be



A.



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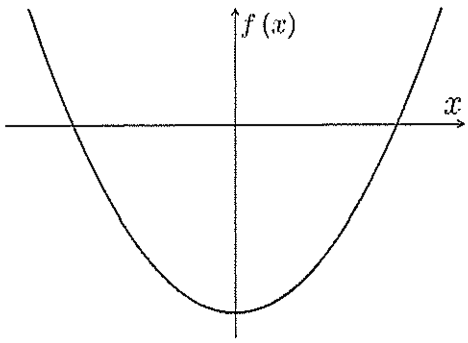
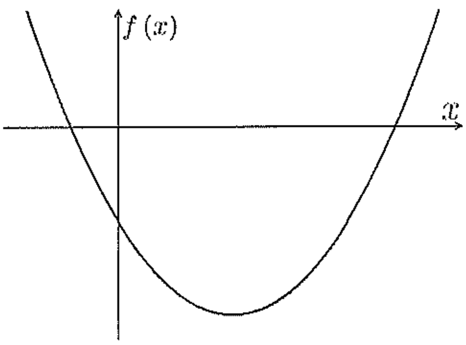
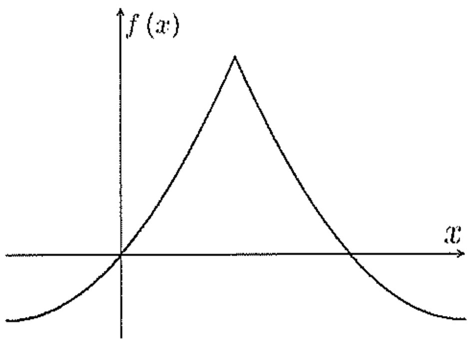


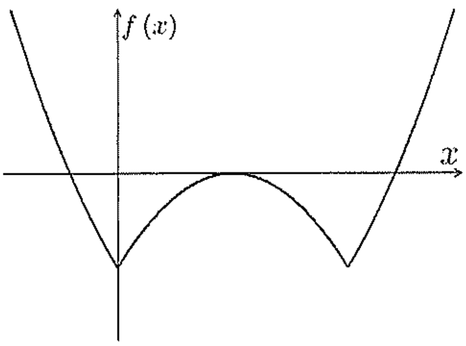
C.



D.



E.



Question 5

If $g(x) = \dfrac{1}{\sqrt{1-(f(x))^2}}$, then $g'(x)$ is equal to

A. $\dfrac{f(x)f'(x)}{\left(1-(f(x))^2\right)^{\frac{1}{2}}}$

B. $\dfrac{f(x)}{\left(1-(f(x))^2\right)^{\frac{3}{2}}}$

C. $\dfrac{f'(x)}{\left(1-(f(x))^2\right)^{\frac{3}{2}}}$

D. $\dfrac{f(x)f'(x)}{\left(1-(f(x))^2\right)^{\frac{3}{2}}}$

E. $\dfrac{f'(x)}{\left(1-(f(x))^2\right)^{\frac{3}{2}}}$

Question 6

The system of linear equations

$$\begin{aligned} x + 3y - 2z &= 1 \\ 4z - x &= 2 \\ x + 8y &= 4 \end{aligned}$$

can be represented in matrix form by

A. $\begin{bmatrix} 1 & 8 & 0 \\ 1 & 3 & -2 \\ -1 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

B. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 3 & -2 \\ 4 & -1 & 0 \\ 1 & 8 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 3 & -2 \\ 4 & -1 & 0 \\ 1 & 8 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

D. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 8 & 0 \\ 1 & 3 & -2 \\ -1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 8 & 0 \\ 1 & 3 & -2 \\ -1 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

Question 7

If a continuous random variable X is normally distributed with a mean of 3 and standard deviation of 4, then the probability that X is less than or equal to 2 is closest to

A. 0.4013

B. 0.4751

C. 0.2525

D. 0.0002

E. 0.3086

Question 8

If the function $f : [a, \infty) \to \mathbb{R}, f(x) = 2x^3 - 3x^2 - 12x + 5$ has an inverse function, then the minimum value of a is

A. -2

B. -1

C. 0

D. 1

E. 2

Question 9

If $f(x) = \frac{1}{2}\log_e(2x^2 + 1)$ and $f(4x - 1) = \log_e(y)$, then y is equal to

A. $\sqrt{32x^2 + 1}$

B. $\sqrt{32x^2 - 16x + 3}$

C. $32x^2 + 1$

D. $\log_e(32x^2 - 16x + 3)$

E. $32x^2 - 16x + 3$

Question 10

The period, amplitude and range of $f(x) = 4\cos\left(\frac{\pi}{2}(x + 2)\right) + 1$ respectively are

A. 4, 4, $[-3, 5]$

B. $\frac{\pi}{2}$, 4, $[-3, 5]$

C. 4, 2, $[-4, 4]$

D. 2, 2, $[-5, 3]$

E. 2, 2, $[-4, 4]$

Question 11

The exact value for the area bound by the curve $f(x) = \log_e(x + 1) + 1$, the y-axis and the line $y = 2\log_e(2) + 1$ is

A. $3 - 2\log_e(2)$

B. $2\log_e(2) + 1$

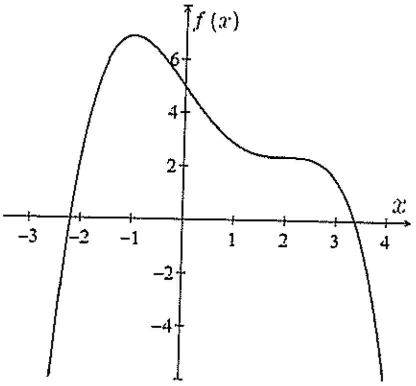
C. $-2\log_e(2) + 3 - \frac{1}{e}$

D. $e^2 - 3 - \frac{1}{e}$

E. $8\log_e(2)$

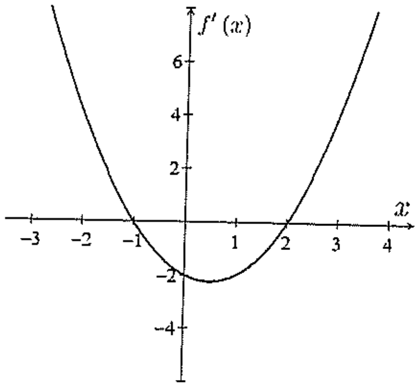
Question 12

The graph of a function $f(x)$ is drawn below.

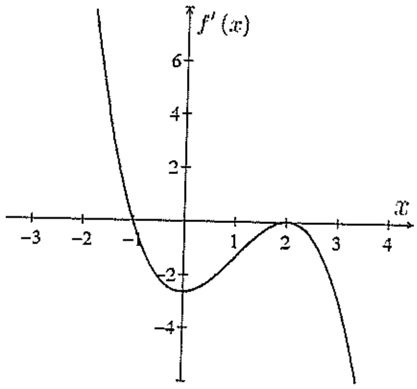


The graph of the derivative function, $f'(x)$, could be

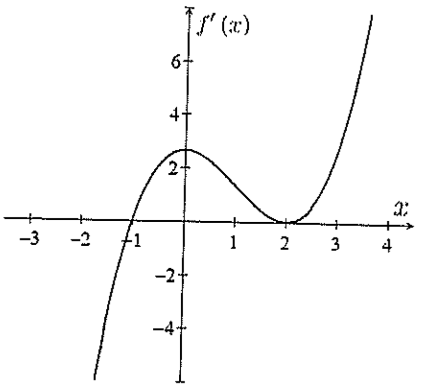
A.



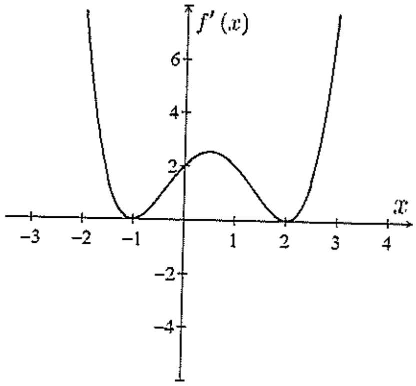
B.



C.



D.



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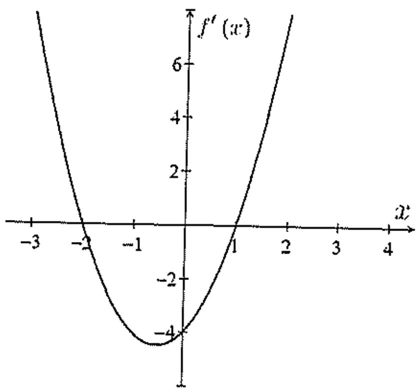


Question 13

A continuous random variable X has a probability distribution given by the following function

$$f(x) = \begin{cases} ke^{-\frac{1}{4}(x-5)} - 1 & \text{for } \{x : f(x) \geq 0\} \cap \{x : x \geq 0\} \\ 0 & \text{elsewhere} \end{cases}$$

If $k > 0$, then the possible value(s) of k, correct to four decimal places, are

A. 0.0260 only

B. 0.1286 and 0.5394

C. 0.6023 only

D. 0.3581 only

E. 0.0260 and 1.0986

Question 14

The general solution to $2\cos\left(2x - \frac{\pi}{2}\right) - \sqrt{3} = 0$ is

A. $x = \frac{(3n+1)\pi}{3}, n \in \mathbb{Z}$

B. $x = \frac{(12n \pm 1)\pi}{6}, n \in \mathbb{Z}$

C. $x = \frac{(3n \pm 1)\pi}{3}, n \in \mathbb{Z}$

D. $x = \frac{\pi(3n + (-1)^n)}{3}, n \in \mathbb{Z}$

E. $x = \frac{(12n \pm 1)\pi}{6}, n \in \mathbb{R}$

Question 15

The average value of the function $f(x) = \frac{1}{2}x(2x-3) + 1$ over the interval $[0, 2]$ is

A. $\frac{5}{3}$

B. $-\frac{1}{3}$

C. $-\frac{1}{6}$

D. $\frac{5}{6}$

E. 0

Question 16

If $f(x) = 2\log_e(2x)$ then which of the following statements is **false**?

A. $f(x) + f(y) = f(2xy)$

B. $f(x) - f(y) = f\left(\frac{x}{2y}\right)$

C. $f(x-y) = f(x+y)$

D. $f(x^y) = 2\log_e(2) + 2yf\left(\frac{x}{2}\right)$

E. $e^{f(x)} = 4x^2$

Question 17

As guests enter their city library, the security guard may ask to check whether their bag is small enough (by their specifications) to enter to the library. The probability that the security guard will ask the guest to check their bag is 0.8. It is known that 50 guests entered the library in the last hour. Given that the security guard checked the bags of at least 35 guests, the probability that the security guard checked at least 40 of them is closest to

A. 0.5255

B. 0.5559

C. 0.9692

D. 0.5836

E. 0.6021

Question 18

Brenden and Thushan are racing each other. If Brenden beat Thushan in the previous race, then the probability that that Brenden wins again is x. If Thushan beat Brenden in the previous race, then the probability that Thushan wins again is y. It is known that $x + y = 0.8$, and in the long-run the probability of Brenden and Thushan winning is equal. If the event that Brenden wins is taken as a success (that is, Brenden occupies the top of the initial state column matrix), then a transition matrix for the situation is

A. $T = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$

B. $T = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

C. $T = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}$

D. $T = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}$

E. $T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Question 19

One white ball, two red balls and three black ball are placed in a bag. If two balls are drawn without replacement, then the probability that no white balls are drawn is

A. $\frac{3}{9}$

B. $\frac{4}{9}$

C. $\frac{5}{9}$

D. $\frac{1}{3}$

E. $\frac{2}{3}$

Question 20

The maximal domain of the function $f(x) = \dfrac{1}{\sqrt{9-x^2}} + \log_e(x+1)$ is

A. $\mathbb{R} \backslash \{-3, 3\}$

B. $(-1, 3)$

C. $[-3, 3]$

D. $x > -1$

E. $[-1, 3]$

Question 21

$f(x)$ is differentiable for all x, and the following is known about $f(x)$:

- $f'(x) > 0$ for $x < -2$
- $f'(x) = 0$ for $x = -2$
- $f'(x) > 0$ for $-2 < x < 3$
- $f'(x) = 3$ for $x = 0$
- $f'(x) < 0$ for $x > 3$

Which of the following statements is true?

A. There is a local maximum at $x = -2$

B. There is a stationary point of inflection at $x = 3$

C. There is a local minimum at $x = 3$

D. There is a stationary point of inflection at $x = -2$

E. There is a local maximum at $x = 0$

Question 22

If a function is represented by $f(x) = |\log_e(x)|$ then which of the following is true?

A. $f(x)$ is increasing for $x \in \mathbb{R}^+$

B. $f(x)$ is continuous for $x \in \mathbb{R}$

C. $f'(x)$ is not continuous for $x \in \mathbb{R}^+$

D. $f(x) > 0$ for $x \in \mathbb{R}^+$

E. $f(x)$ has one stationary point

SECTION 2 - Extended response questions

Instructions for Section 2

Answer **all** questions in the spaces provided
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider the function $f(x) = \frac{1}{16}(3x-a)^2(bx-2)^2 - 4$.

a. Find the stationary points of f in terms of a and b.

4 marks

Let n be the number of stationary points that f has.

b. **i.** If $b = 2$, find the possible values of n.

ii. Find the value(s) of a if f is to have 3 stationary points.

iii. If $a = 9$, find the three stationary points of f, and state the nature of each.

2 + 1 + 3 = 6 marks

c. i. The domain of f is restricted to $[c, \infty)$ so that its inverse is defined. If $a = 3$ and $b = 2$, find the smallest value of c.

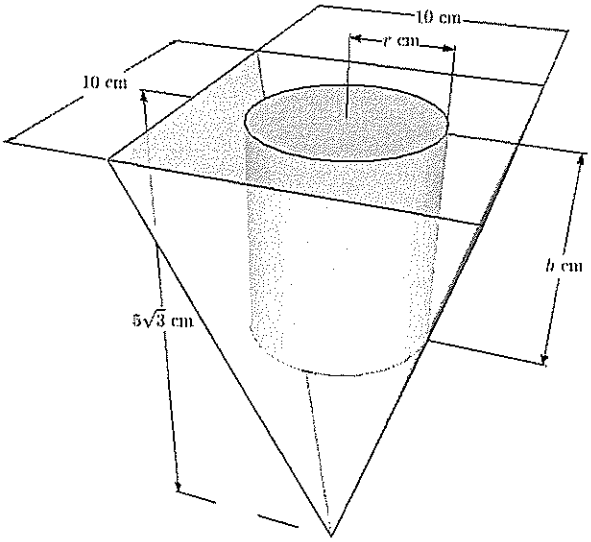
ii. Find the inverse function of f, f^{-1}, and state its domain using the value of c found in **part c.i.**

iii. Find the point(s) of intersection of f and f^{-1}, correct to 2 decimal places.

$1 + 2 + 2 = 5$ marks

Question 2

A cylinder is inscribed in an inverted square-based pyramid as in the diagram below.



a. Find an expression for h in terms of r.

2 marks

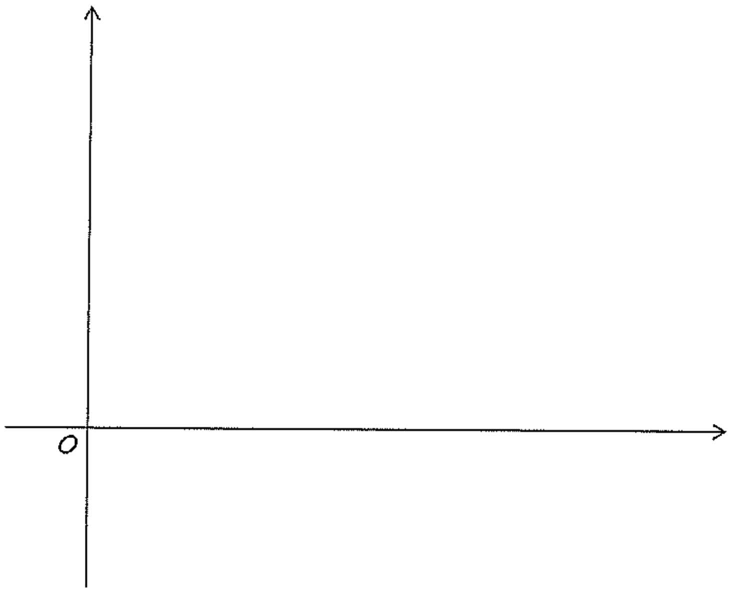
b. Find an expression for the volume, V cm^3, of the cylinder in terms of r only, and state a suitable domain for V.

2 marks

c. Find the maximum volume of the cylinder and the value of r for which this occurs.

3 marks

d. Sketch the graph of V cm^3 vs r cm, showing all intercepts and the point of maximum volume.



3 marks

The cylinder is now removed from inverted pyramid.

e. Water is now poured into the inverted pyramid at a rate of $10\,\text{cm}^3/\text{s}$. If h_w is the height of the water in the pyramid and x is the base width of the surface of the water, find the rate that the height of the water in the pyramid is increasing when $h = 5$ cm.

4 marks

Question 3

The probability that Rohit plays an addictive computer game given that he played the game on the previous day is 0.75. On the other hand, the probability that he does not play on a particular day given that they didn't play on the previous day is 0.4.

a. i. If the probability that Rohit doesn't play the game on the first day is 0.6, then find the probability that Rohit does play the game on the 5^{th} day correct to 4 decimal places.

ii. Find the long-run probability that Rohit will play the game.

3 + 1 = 4 marks

Rohit's friend, Steph, also plays the same game. The probability that she plays on any particular day is independent of any other day.

b. i. If the probability that she plays on a particular day is 0.75, then find the probability that Steph plays the game on 4 out of 5 days given that she plays for at least 3 days, correct to 4 decimal places.

ii. Find the minimum number of days that she must play so that the probability that she plays for at least 2 days is greater than 0.75.

$3 + 2 = 5$ marks

The combined number of hours that Rohit and Steph play the game in a day is given by the continuous random variable Y, which is represented by the function below.

$$f(y) = \begin{cases} \dfrac{1}{50}\log_e(y) & 1 \le y \le a \\ 0 & \text{elsewhere} \end{cases}$$

c. i. Show that the value of a correct to 4 decimal places is 22.9629.

ii. Find the mean and standard deviation of the distribution correct to 2 decimal places.

2 + 3 = 5 marks

Question 4

Two countries are fighting a battle. The gradient of the battle line is given by

$$f'(x) = \frac{1}{30}x\left(4x^2 - 3(c+10)x + 20c\right) \text{ for } 0 < x < 10$$

The area of land that the first country occupies is the area between the curve and the x-axis from $x = 0$ to the first positive x intercept. The area of land that the second country occupies is the region between the curve and the x-axis, from the first positive x-intercept to the second positive x-intercept. The curve of f is the boundary between the occupied and unoccupied territories. The rest of the territory is unoccupied or in dispute. All distances are in kilometres.

a. If $f(x)$ passes through $(0,0)$, find $f(x)$.

2 marks

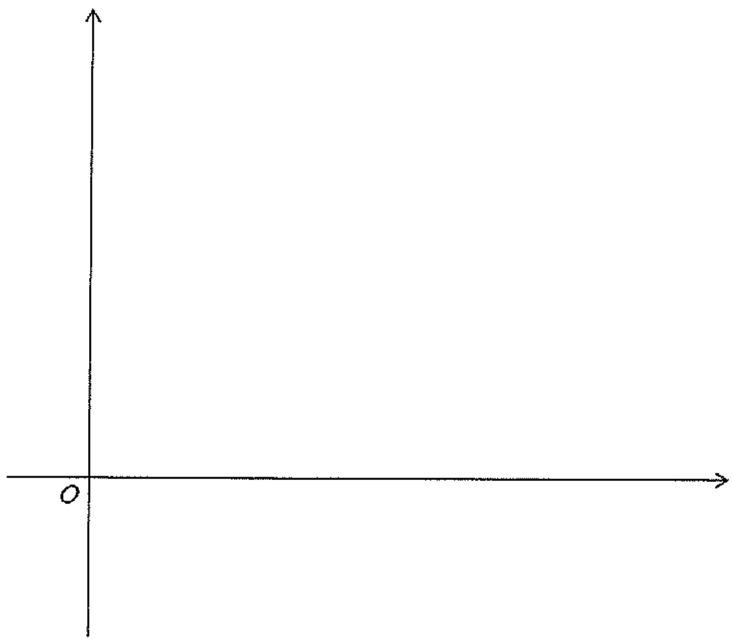
b. Both countries occupy territories of equal areas. Show that the value of c is 6.

3 marks

c. Due to weather conditions, a battle station at the origin can fire anywhere between $(0, 0)$ and the parabola with equation $y = \frac{1}{10}x^2 - 8$. The enemy's territory, which overlaps this parabola, is being fired at. Find the percentage of the enemy's territory that they can fire into, correct to 2 decimal places.

3 marks

d. Sketch the functions $f(x)$ and $g(x)$ for $0 \leq x \leq 10$, showing axis intercepts, endpoints, and stationary points, giving coordinates to two decimal places where appropriate.
Shade the region found in **part c.**



3 marks

e. Since their homes are being disturbed, a group of peasants have to run from the origin to another part of their territory. The route they travel takes them out of their territory before re-entering at a point on the curve. The linear path that they run along is tangential to the curve at the point where it meets the curve. Find the point at which they proceed back into their territory, and the distance they they run to reach this point, correct to 2 decimal places.

3 marks

END OF EXAMINATION