SET 3 EXAM 1

Reading time: 15 minutes

Writing time: 60 minutes

Structure of examination

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Note: Formula Sheet is NOT supplied. You will need to use your own!
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book.
- Working space is provided throughout the book.

Instructions
- Complete all responses in the spaces provided.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Differentiate $\frac{1}{\sqrt{1-x^2}}$ with respect to x.

2 marks

b. Let $f(x) = \sin(2x)\cos(2x)$. Find $f'\left(\frac{\pi}{8}\right)$.

2 marks

Question 2

a. Find an antiderivative of $\sin(4-2x)$ with respect to x.

1 mark

b. Evaluate $\int_0^a \frac{1}{x+a}\,dx$ for $a > 0$.

2 marks

Question 3

Consider the following functions

$$f : D \to \mathbb{R},\ f(x) = \log_e(\log_e(x)) \quad \text{and}$$
$$g : (b, \infty) \to \mathbb{R},\ g(x) = \frac{1}{4}x^2$$

a. Find D given that it is the maximal domain of $f(x)$.

2 marks

b. If $f(g(x))$ is defined over the domain of g, find the smallest possible value of b.

2 marks

Question 4

The probability density function for a continuous random variable X is given by

$$p(x) = \begin{cases} 1 - \cos\left(\dfrac{2\pi x}{k}\right) & \text{if } 0 \leq x \leq k \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k.

3 marks

Question 5

Solve the equation $\sin^2(x) = \cos^2(x)$ for $x \in [0, 2\pi]$.

2 marks

Question 6

James plays a game of darts in which he must score as many consecutive bull's-eyes as possible. If he hits a bull's-eye, he is allowed another throw, and if he misses, the game ends. He wins $10 for each bull's-eye he hits.

If the probability of James hitting a bull's-eye is 0.1, find the probability that James will win more than $20 if he plays one game.

3 marks

Question 7

A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$

The transformation T maps the points $(1, 1)$ and $(2, 2)$ to the points $(-2, 2)$ and $(3, 1)$, respectively.

a. Find the values of a, b, c and d.

3 marks

b. Hence, or otherwise, find the equation of the image of the line $y = x$ under this transformation.

1 mark

Question 8

Two independent events A and B are such that $\Pr(A \cap B) = \frac{1}{4}$ and $\Pr(A \cap B') = \frac{5}{8}$.

a. Find $\Pr(A)$ and $\Pr(B)$.

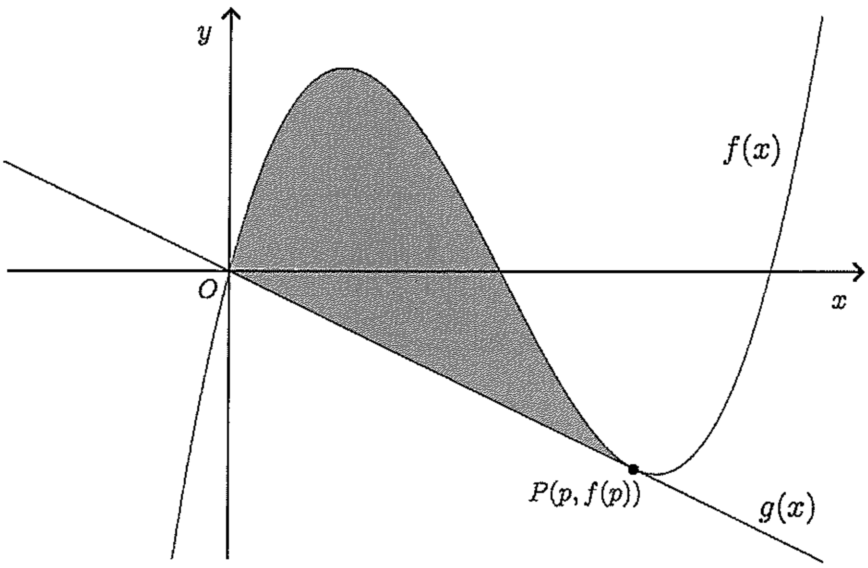
3 marks

b. Show that A and B' are independent events.

2 marks

Question 9

Consider the following diagram.



The rules for the two functions are

$$f(x) = x(x-2)(x-4) = x^3 - 6x^2 + 8x \quad \text{and}$$
$$g(x) = -ax$$

The graph of g is tangent to the graph of f at the point P as shown. Both a and p are positive real numbers.

a. Show that $a = 1$ and $p = 3$.

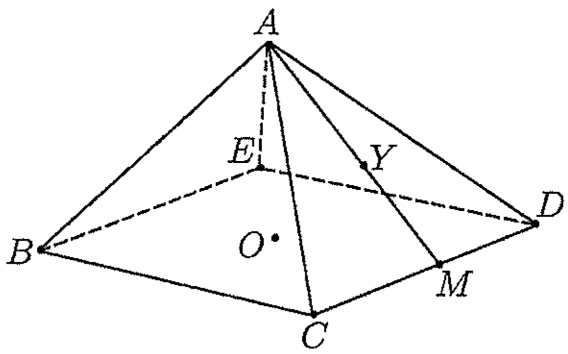
3 marks

b. Find the area of the shaded region.

3 marks

Question 10

ABCDE is a square-based pyramid whose edges are all equal to x metres in length. O is the centre of the base of the pyramid, and M is the midpoint of CD.



a. Find the lengths of AM and AO in terms of x.

2 marks

b. Find the volume, V m^3, of the pyramid, in terms of x.

1 mark

c. Y is the point on the line joining A and M that is closest to O. Find the length of OY if $x = \frac{\sqrt{30}}{2}$ m.

3 marks

END OF EXAMINATION