THE HEFFERNAN GROUP

P.O. Box 1180
Surrey Hills North VIC 3127
ABN 20 607 374 020
Phone 9836 5021
Fax 9836 5025

Student Name.....

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 2

2000

Reading Time: 15 minutes
Writing Time: 90 minutes

Instructions to Students

This exam consists of 5 questions and all of these 5 questions should be answered.
There is a total of **60** marks available.
Students may bring up to two A4 pages of pre-written notes into the exam.

This paper has been prepared independently of the Board of Studies to provide additional exam preparation for students. Although references have been reproduced with permission of the Board of Studies, the publication is in no way connected with or endorsed by the Board of Studies.

Question 1

Let $z_1 = 3 - 4i$ and $z_2 = 2 - i$

a. **i.** Express $\frac{1}{z_1}$ in Cartesian form.

_____

_____

_____

1 mark

ii. Find the value of $\text{Arg}\, z_1$ to the nearest minute.

_____

_____

_____

1 mark

iii. Verify that $\text{Im}\, z_1 + \bar{z}_2 + z_1 z_2 = -10i$

_____

_____

_____

1 mark

iv. Find algebraically the square roots of z_1 and express them in Cartesian form.

_____

_____

_____

_____

_____

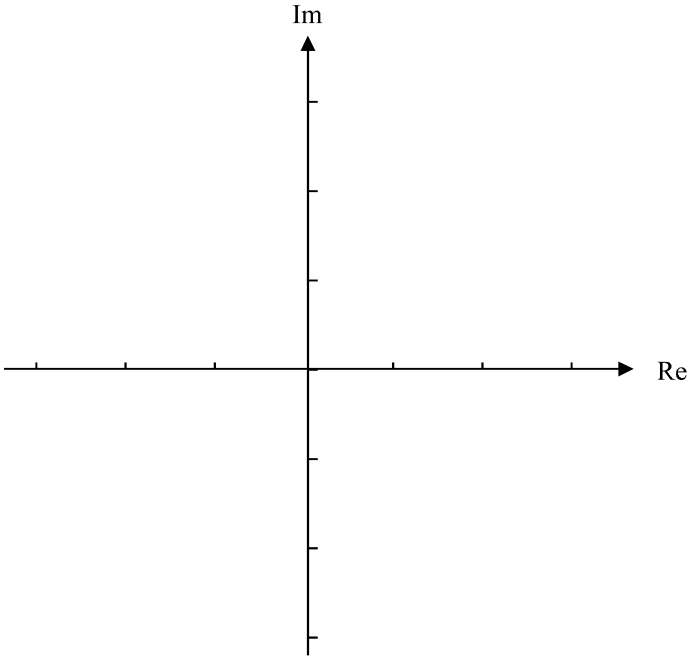
_____

_____

_____

3 marks

b. **i.** On the complex plane below sketch S where $S = \{z : |z - 2 + i| = |z + 2 - i|\}$



1 mark

ii. Find algebraically the Cartesian equation of S.

1 mark

iii. Given that $z_4 = 3cis\frac{\pi}{3}$, state with reasons whether or not $z_4 \in S$.

1 mark

iv. Write down that subset of the complex plane for which the points are equidistant from the points z_4 and z_1

2 marks

Total 11 marks

Question 2

Consider the function $f : [1, 4] \to R$ where $f(x) = \frac{16}{x^2} - 1$

a. Find the coordinates of the x intercept of the graph of f.

1 mark

b. The point (m, n) lies on f and the gradient of f at this point is -1. Find the values of m and n correct to 1 decimal place.

2 marks

A sketch of a vase is made. In a cross sectional view, the sides of the vase have the shape of the graph of *f(x)* and of its reflection in the *y* axis. The cross sectional area of the vase is shaded in Figure 1 below.

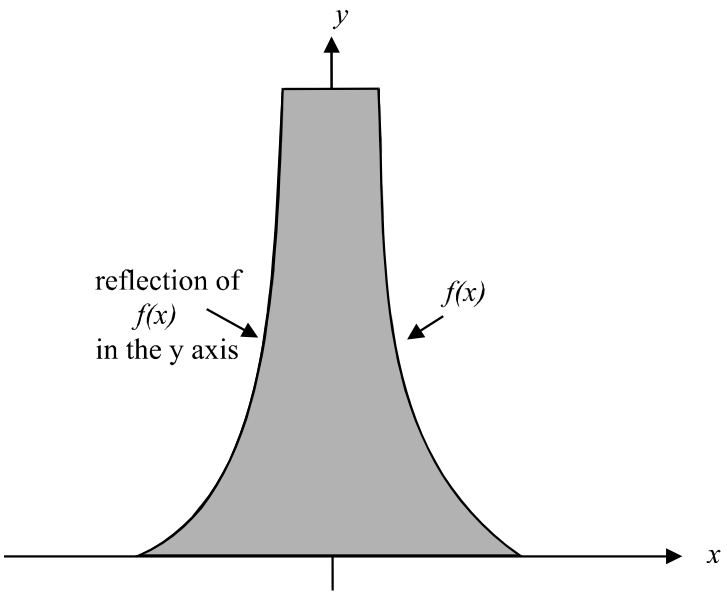


Figure 1

c. Find this shaded area.

3 marks

d. That part of the vase which holds the water is formed by rotating the function

$$g[0, 1] \rightarrow R \text{ where } g(x) = 14x^2 + 1$$

about the *y* axis.
The cross sectional area of the clay vase is shaded and is shown in Figure 2 below.
All measurements are in centimeters.

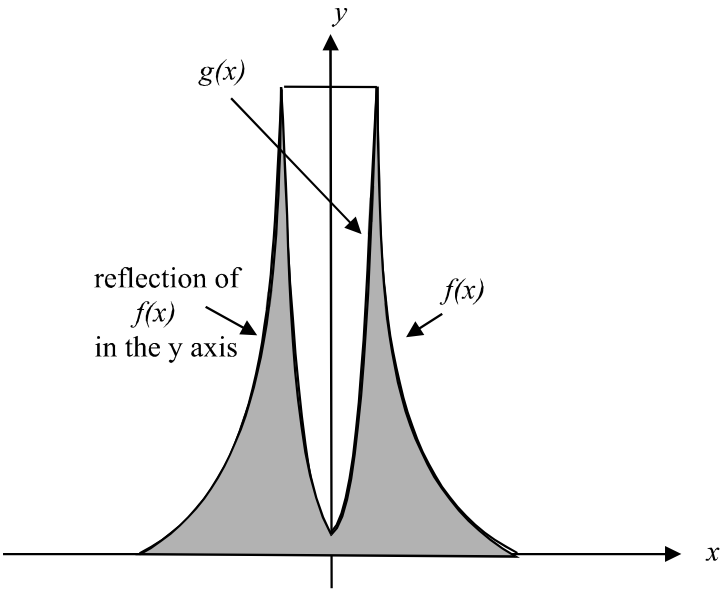


Figure 2

Find the volume of clay used to make the vase, expressing your answer as an exact value.

4 marks
Total 10 marks

Question 3

Particle A moves so that its position $\underset{\sim}{r}$ (metres) from the origin at time t (secs) is given by

$$\underset{\sim A}{r} = (\sqrt{2}\sin(2t)+1)\underset{\sim}{i} + \sqrt{2}\cos(2t)\underset{\sim}{j} \qquad t \geq 0$$

a. **i.** Find the distance of particle A from the origin at time t.

1 mark

ii. Find the furtherest distance that particle A can be from the origin and state the value(s) of t when this occurs.

2 marks

b. Show that speed of particle A is constant.

2 marks

A second particle B moves so that its position $\underset{\sim}{r}$ (metres) from the origin at time t (secs) is given by

$$\underset{\sim B}{r} = \left(\frac{4}{\sqrt{3}}\cos(2t)\right)\underset{\sim}{i} + (4\sin t\cos t)\,\underset{\sim}{j} \qquad t \geq 0$$

c. **i.** Find the Cartesian equation of the path of particle B.

_____

_____

_____

_____

_____

_____

_____

2 marks

ii. Write down the domain and range of the Cartesian relation found in part **i.**

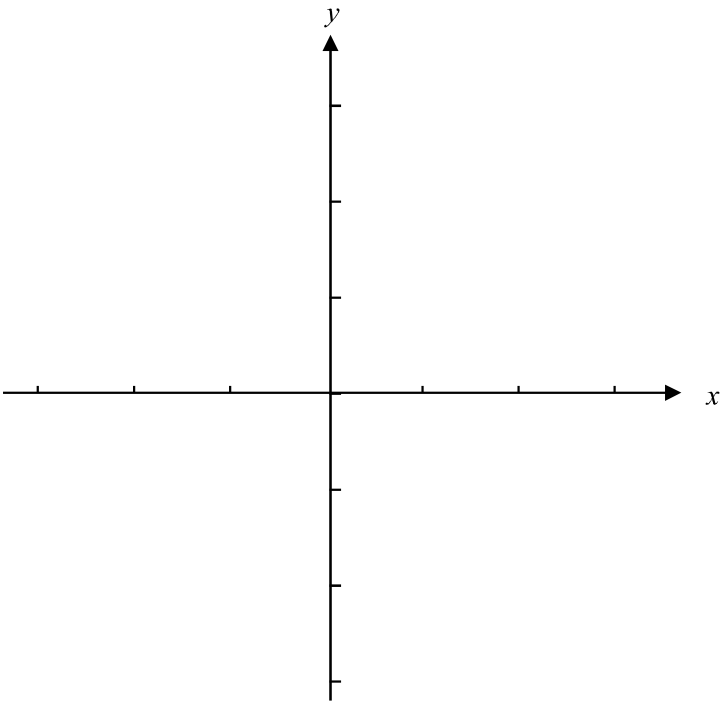
_____

_____

_____

2 marks

iii. Sketch the path of particle B on the set of axes below.



1 mark

d. Show that the acceleration and the position of particle B are always parallel.

2 marks

e. Show that the particle A and particle B do not collide in the interval $t \in [0,\ 0.1]$

2 marks

Total 14 marks

Question 4

A suitcase on wheels has a mass of 20 kg and rests on the carpeted floor of an airport. The coefficient of friction between the suitcase and the carpet is 0.5

The suitcase has a handle which extends from one of its corners and locks into position at an angle of θ with the horizontal as indicated in Figure 1 below.

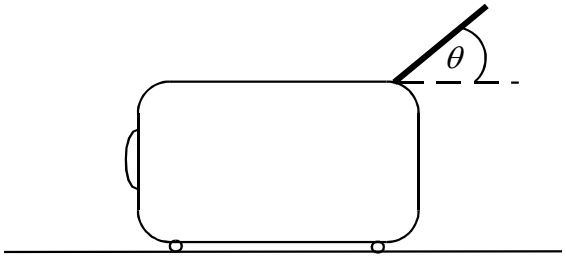
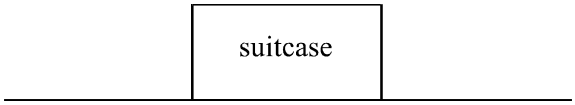


Figure 1

A man exerts a pulling force of P newton on the handle.

a. Show on the diagram below the 4 forces, including the normal force N, acting on the suitcase.



1 mark

b. When the pulling force P is equal to $\frac{20\sqrt{2}g}{3}$ newton, the suitcase is on the point of moving.

Show that θ equals 45°.

_____

_____

_____

_____

_____

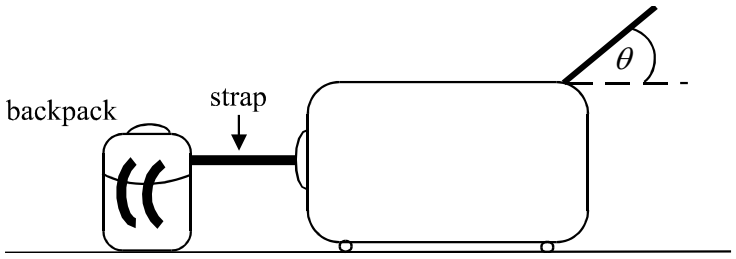
_____

_____

_____

2 marks

The man decides to take off his backpack which has a mass of 8 kg and is uncomfortable and attach it to the suitcase using a strap. The coefficient of friction between the backpack and the carpet is 0.6



c. Suggest why the coefficient of friction between the backpack and the carpet is greater than the coefficient of friction between the suitcase and the carpet.

_____

_____

_____

1 mark

d. **i.** The man exerts a pulling force on the handle of the suitcase of P newton.
Show the forces acting now on the suitcase and the backpack on Figure 2 below.

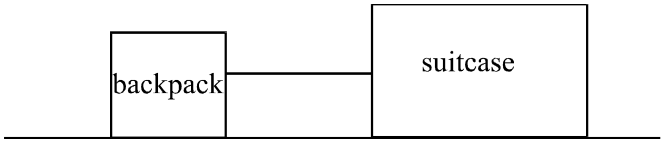


Figure 2

2 marks

ii. Show that if the pulling force P, is 90 newton and the tension in the strap connecting the suitcase and the backpack is 20 newton, that the suitcase and the backpack are not about to move.

_____

_____

_____

_____

_____

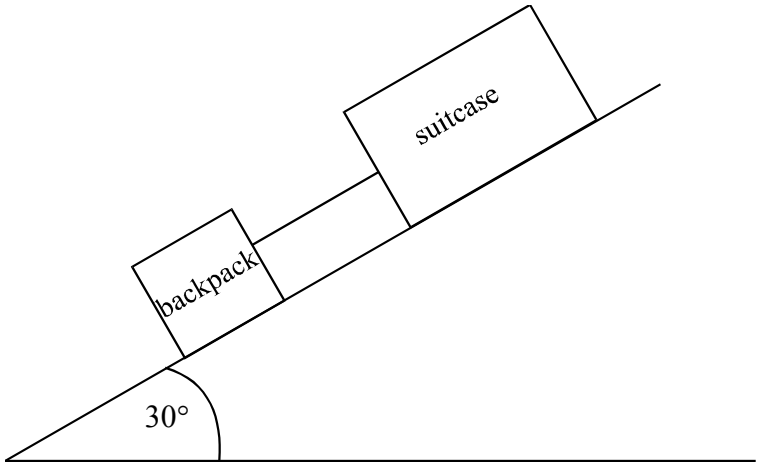
_____

1 mark

The man comes to a short ramp which has the same carpet as that in the other parts of the airport. The ramp is inclined at an angle of $30°$.

The man changes the position of the handle of his suitcase. He locks it into a position which is parallel to the slope of the ramp. The man exerts a pulling force of P newton on the handle.

e. On Figure 3 below, show the forces acting on the suitcase and the backpack now.



2 marks

Figure 3

f. If the man exerts a pulling force of 300 newton, find the acceleration of the suitcase and the backpack. Express your answer correct to 4 decimal places.

5 marks

ii. The ramp is 40 metres long and the man is stationary before walking up it. Assuming that he maintains a pulling force of 300 newton for the duration of the walk up the ramp, how long will it take him to go up the ramp? (Express your answer to the nearest tenth of a second.)

1 mark

Total 15 marks

Question 5

A special container is used to transport certain pathology samples which have been frozen. The temperature, C, in degrees Celsius, of the samples at time *t*, minutes, after having been placed in the special container, can be described by the differential equation

$$\frac{dC}{dt} = (C-3)(C+2) \qquad t > 0$$

a. Find an expression for *t* in terms of C given that when C = -3, $t = \frac{1}{5}\log_e 6$

4 marks

b. Express C in terms of *t*.

1 mark

c. Find how many degrees below $-2°$ Celsius the temperature of the pathology sample is at $t = 4$ minutes. Express your answer to 1 significant figure.

_____

_____

_____

_____

_____

1 mark

d. Graph the function obtained in part **b.** and use this to explain what happens to the temperature of the pathology sample once it is placed in the special container.

_____

_____

_____

_____

_____

2 marks

e. Show algebraically that the temperature of the pathology sample approaches but never reaches $-2°$ Celsius.

_____

_____

_____

_____

_____

_____

_____

2 marks

Total 10 marks