The Mathematical Association of Victoria

Specialist Mathematics

2000 Written Examinations

Solutions

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2000 Specialist Mathematics Exam 1

Suggested Answers and Solutions

Part I (Multiple-choice) Answers

1. **B**	2. **A**	3. **B**	4. **D**	5. **E**
6. **E**	7. **A**	8. **B**	9. **E**	10. **C**
11. **C**	12. **E**	13. **E**	14. **D**	15. **D**
16. **B**	17. **B**	18. **D**	19. **C**	20. **A**
21. **A**	22. **C**	23. **E**	24. **A**	25. **D**
26. **B**	27. **C**	28. **D**	29. **C**	30 **A**

Solutions: Part 1

Question 1 [B]

$f(x) = 3x^2 + 8x + 5$

$= (3x + 5)(x + 1)$

$\frac{1}{f(x)}$ will have asymptotes when $f(x) = 0$,

for $x = \frac{-5}{3}$ and $x = -1$

Question 2 [A]

Consider first a hyperbola, with asymptotes passing through the origin, O.

It would cut the x–axis at $(-3, 0)$ and $(3, 0)$

i.e: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

For $y = 0$, $x = \pm a = \pm 3$ hence $a = 3$

Equations of asymptotes are given by: $y = \pm\frac{b}{a}x$

From the diagram, gradient of asymptotes is ± 2,

hence $\frac{b}{a} = 2$

$b = 6$

Equation becomes: $\frac{x^2}{9} - \frac{y^2}{36} = 1$

Translate the centre to $(1, 0)$ and so

$\frac{(x-1)^2}{9} - \frac{y^2}{36} = 1$

Question 3 [B]

Implied domain of

Cos^{-1} is $[-1, 1]$

that of $\text{Cos}^{-1} 3x$ is $\left[\frac{-1}{3}, \frac{1}{3}\right]$

that of $1 + \text{Cos}^{-1} 3x$ is also $\left[\frac{-1}{3}, \frac{1}{3}\right]$

Question 4 [D]

$y = \text{Sin}^{-1}\frac{5}{x}, \quad x > 5$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{5}{x}\right)^2}} \cdot \frac{-5}{x^2}$ using the Chain Rule

$= -\frac{5}{x\sqrt{x^2-25}}$

Question 5 [E]

$w = 5 - 2i$

$2 - w = 2i - 3$

$= \frac{1}{2-w} = \frac{1}{2i-3}$

$= \frac{1}{2i-3} \times \frac{2i+3}{2i+3}$

$= \frac{-(2i+3)}{13}$

Question 6 [E]

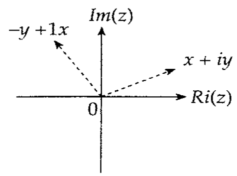
$u = i^5 z$

$= i(i)^4 z$

$= iz$

if $z = x + iy$

then $u = ix - y$



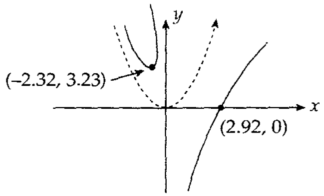
The effect is that of a rotation of $\frac{\pi}{2}$ in an anti-clockwise direction around the origin.

Exam 1 : Part 2

Question 1

$f(x) = \frac{x^3 - 25}{5x}$

$= \frac{x^2}{5} - \frac{5}{x}$



Question 2a

$\underset{\sim}{r} = \sqrt{t}\,\underset{\sim}{i} - (t-2)\,\underset{\sim}{j}$

The distance of the particles from the origin is given by:

$r^2 = t + (t-2)^2$

$= t^2 - 3t + 4$

distance will be a minimum when

$\frac{d(r^2)}{dt} = 0$

$2t - 3 = 0$

$t = 1.5$

Alternatively

$r = \sqrt{t^2 - 3t + 4}$

$\frac{dr}{dt} = \frac{2t-3}{2\sqrt{t^2-3t+4}} = 0$

$2t + 3 = 0$

$t = \frac{3}{2}$

Question 2b

$\cdot = x\,\underset{\sim}{i} + x\,\underset{\sim}{j}$

$= \sqrt{t},\ y = -(t-2)$

ence $y = -t + 2$

$- 2 = -t$

$- 2 = -(x)^2$

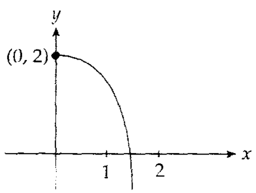
$y = 2 - x^2$ for $x \geq 0$, since $t \geq 0$

ıe value of the intercept is given by

$- x^2 = 0$

$x = \sqrt{2}$,

$x = 1.414$



Question 3

$\int_{\frac{\pi}{3}}^{\pi} \sin^3 \frac{x}{2}\, dx$

$\int_{\frac{\pi}{3}}^{\pi} \sin^2\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right) dx$

let $u = \cos\frac{x}{2},\ \frac{du}{dx} = -\frac{1}{2}\sin\frac{x}{2}$

$\int_{\frac{\sqrt{3}}{2}}^{0} \left(1-u^2\right) \cdot -2\frac{du}{dx} \cdot dx$

Since when $x = \frac{\pi}{3}$, $u = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

And when $x = \pi$, $u = \cos\frac{\pi}{2} = 0$

The integral can be rearranged as

$2\int_{0}^{\frac{\sqrt{3}}{2}} \left(1-u^2\right) du$

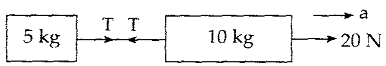
$2\left[u - \frac{u^3}{3}\right]_0^{\frac{\sqrt{3}}{2}}$

$2\left[\frac{\sqrt{3}}{2} - \frac{3\sqrt{3}}{24}\right]$

$2.\ \frac{9\sqrt{3}}{24}$

$\frac{3\sqrt{3}}{4}$

Question 4a



For the 10 kg mass

$20 - T = 10a$

For the 5 kg mass

$T = 5a$

$20 - 5a = 10a$

$15a = 20$

$a = \frac{4}{3}$

acceleration is $\frac{4}{3}$ m/s^2

Question 4b

$T = 5a$

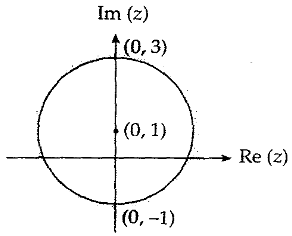
From 4a. $a = \frac{4}{3}$

$= \frac{20}{3}$

tension in string is $\frac{20}{3}$ N (or 6.67 N)

Question 5a

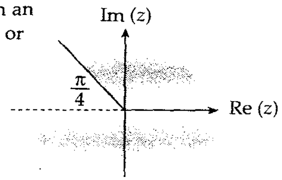
$\{z : |z - i| \geq 2\}$ defines all points on the circumference and outside the circle which has the centre at (0, 1) and with radius of 2 units.



Question 5b

$\left\{z: Arg(z) \leq \frac{3\pi}{4}\right\}$ *Note* $-\pi < Arg Z \leq \pi$

defines all points in the complex plane with an argument less than or equal to $\frac{3\pi}{4}$. Only those points are excluded with an argument greater than $\frac{3\pi}{4}$ and less than π, ie. The ray which gives Arg $Z = \frac{3\pi}{4}$ is a solid line and the negative x = axis is dotted.



Question 6a

$z = 2 - i$ Note $z - (2 - i) = 0$

$z^2 = 4 + i^2 - 4i$

$= 3 - 4i$

$z^3 = (3 - 4i)(2 - i)$

$= 6 + 4i^2 - 11i$

$= 2 - 11i$ Note $z^3 = (2 - i)\,z^2$

$z^3 - (2 - i)z^2 + z - 2 - i = 0$

Since the first two terms and the last two terms are equal to zero. (You could substitute for every term.)

An alternative

$z^3 - (2 - i)z^2 + z - 2 + i = 0$

$(2 - i)^3 - (2 - i)(2 - i)^2 + 2 - i - 2 + i$

$\Rightarrow (2 - i)^3 - (2 - i)^3 + (2 - i) - (2 - i) = 0$

Terms cancel out.

Question 6b

$z^3 - (2 - i)\ z^2 + z\ - 2 + i$ can be factorised

$(z - 2 + i)(z^2 + 1)$

so, if $z^3 - (2 - i)\,z^2 + z\ - z + i = 0$

$(z - 2 + i)(z^2 + 1) = 0$

Solutions are $z = 2 - i,\ z = \pm i$