Victorian Certificate of Education 2000

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 30 October 2000: 11.45 am to 1.30 pm
Reading time: 11.45 am to 12 noon
Writing time: 12 noon to 1.30 pm
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this book.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
You may retain this question book.

Structure of book

Number of questions	*Number of questions to be answered*
30	30

Directions to students

Materials

Question book of 17 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the book.
You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.
An approved scientific and/or graphics calculator may be used.
You should have at least one pencil and an eraser.

The task

Detach the formula sheet from the centre of this book during reading time.
Please ensure that your **name and student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
There is a total of 30 marks available for the multiple-choice part of this examination.
All questions should be answered on the answer sheet provided for multiple-choice questions.
Unless otherwise indicated, the diagrams in this book are **not** necessarily drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
You may retain this question book.

Instructions for students

Answer **all** questions on the answer sheet provided for multiple-choice questions.

A correct answer scores 1; an incorrect answer scores 0. No credit will be given for a question if two or more letters are marked for that question. Marks will not be deducted for incorrect answers.
You should attempt every question.

Question 1

If $f(x) = 3x^2 + 8x + 5$, then the graph of $y = \frac{1}{f(x)}$ has

A. x-intercepts at $x = -1$ and $x = -\frac{3}{5}$

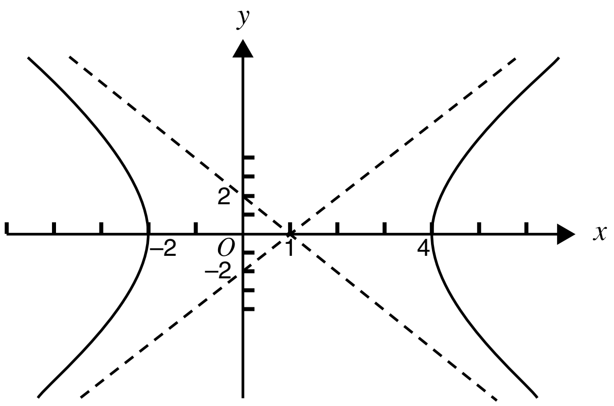
B. asymptotes at $x = -1$ and $x = -\frac{5}{3}$

C. asymptotes at $x = -1$ and $x = -\frac{3}{5}$

D. a local minimum at the point $\left(-\frac{4}{3}, -3\right)$

E. a local maximum at the point $\left(-\frac{4}{3}, -\frac{1}{3}\right)$

Question 2



The equation for the graph shown is

A. $\dfrac{(x-1)^2}{9}-\dfrac{y^2}{36}=1$

B. $\dfrac{(x+1)^2}{9}-\dfrac{y^2}{36}=1$

C. $\dfrac{(x-1)^2}{9}-\dfrac{y^2}{6}=1$

D. $(x+1)^2-\dfrac{y^2}{4}=1$

E. $\dfrac{(x-1)^2}{9}-\dfrac{y^2}{4}=1$

Question 3

The implied domain of the function with rule $f(x)=1+\text{Cos}^{-1}(3x)$ is

A. $[-3, 3]$

B. $\left[-\dfrac{1}{3}, \dfrac{1}{3}\right]$

C. $[-2, 4]$

D. $\left[-\dfrac{2}{3}, \dfrac{4}{3}\right]$

E. $[1, \pi+1]$

Question 4

If $y = \text{Sin}^{-1}\frac{5}{x}$ and $x > 5$, then $\frac{dy}{dx} =$

A. $\frac{1}{\sqrt{25 - x^2}}$

B. $-\frac{5}{\sqrt{x^2 - 25}}$

C. $\frac{5}{x\sqrt{x^2 - 25}}$

D. $-\frac{5}{x\sqrt{x^2 - 25}}$

E. $\frac{x}{\sqrt{x^2 - 25}}$

Question 5

If $w = 5 - 2i$, then $\frac{1}{2 - w} =$

A. $\frac{3}{5} + \frac{2}{5}i$

B. $-\frac{3}{5} - \frac{2}{5}i$

C. $-\frac{9}{2} + 2i$

D. $-\frac{3}{13} + \frac{2}{13}i$

E. $-\frac{3}{13} - \frac{2}{13}i$

Question 6

For any complex number z, the complex number $u = i^5 z$ is found by

A. reflecting z in the Im(z) axis

B. reflecting z in the Re(z) axis

C. reflecting z in the line Im(z) = Re(z)

D. rotating z through $\frac{\pi}{2}$ in a clockwise direction about the origin

E. rotating z through $\frac{\pi}{2}$ in an anti-clockwise direction about the origin

Question 7

Which one of the following is a polar form of $\sqrt{3}+i$?

A. $2\,\text{cis}\left(-\frac{11\pi}{6}\right)$

B. $2\,\text{cis}\left(-\frac{\pi}{6}\right)$

C. $2\,\text{cis}\left(\frac{\pi}{3}\right)$

D. $2\,\text{cis}\left(\frac{5\pi}{6}\right)$

E. $2\,\text{cis}\left(\frac{7\pi}{6}\right)$

Question 8

The set of points in the complex plane defined by $|z|=|z+2|$ is

A. the point $z = -1$

B. the line $\text{Re}(z) = -1$

C. the line $\text{Re}(z) = 1$

D. the circle with centre (2, 0) and radius 2

E. the circle with centre (–2, 0) and radius 2

Question 9

An antiderivative of $\dfrac{-1}{\sqrt{1-2x^2}}$ is

A. $-\sqrt{2}\ \text{Sin}^{-1}\left(\dfrac{x}{\sqrt{2}}\right)$

B. $\sqrt{2}\ \text{Cos}^{-1}\left(\dfrac{x}{\sqrt{2}}\right)$

C. $\sqrt{2}\ \text{Cos}^{-1}\left(\sqrt{2}x\right)$

D. $\dfrac{1}{\sqrt{2}}\text{Cos}^{-1}\left(\dfrac{x}{\sqrt{2}}\right)$

E. $\dfrac{1}{\sqrt{2}}\text{Cos}^{-1}\left(\sqrt{2}x\right)$

Question 10

The value, correct to four decimal places, of $\int_0^1 (\cos^2 x - \sin^2 x)dx$ is

A. –0.5943
B. –0.4546
C. 0.4546
D. 0.5943
E. 0.9942

Question 11

Using an appropriate substitution, $\int_{1}^{3} \frac{1}{x^2} e^{\frac{3}{x}} \, dx$ becomes

A. $-\frac{1}{3}\int_{-3}^{-\frac{1}{3}} e^u du$

B. $-\frac{1}{3}\int_{1}^{3} e^u du$

C. $\frac{1}{3}\int_{1}^{3} e^u du$

D. $-\int_{1}^{3} \frac{u^2}{9} e^u du$

E. $\int_{1}^{3} \frac{u^2}{9} e^u du$

Question 12

For $x > 4$, an antiderivative of $\frac{4}{x^2 - 4x}$ is

A. $\log_e(x^2 - 4x)$

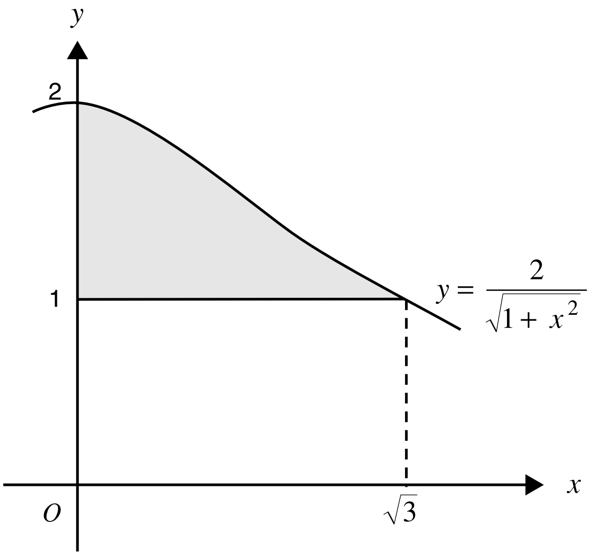
B. $4\log_e(x^2 - 4x)$

C. $-\frac{4}{x} - \log_e x$

D. $\log_e\left(\frac{x}{x-4}\right)$

E. $\log_e\left(\frac{x-4}{x}\right)$

Question 13



The shaded region is enclosed by the curve $y = \frac{2}{\sqrt{1+x^2}}$, the straight line $y = 1$ and the y-axis. This region is rotated about the x-axis to form a solid of revolution. The volume of this solid, in cubic units, is given by

A. $\pi\left[4\,\text{Tan}^{-1}x - 1\right]_0^{\sqrt{3}}$

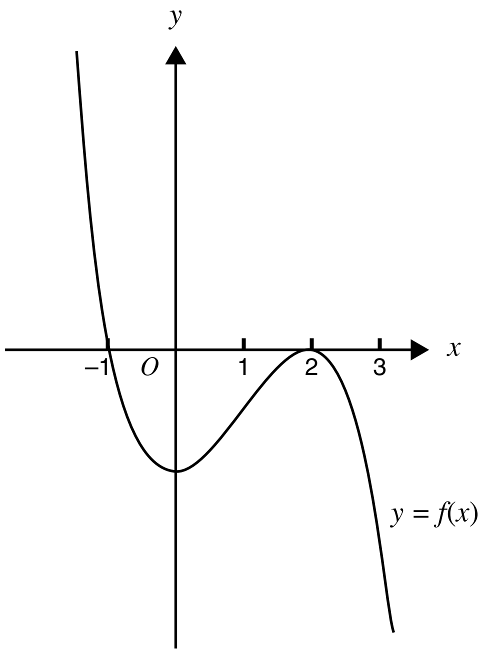
B. $\int_0^{\sqrt{3}} \left(\frac{2}{\sqrt{1+x^2}} - 1\right)dx$

C. $\pi\int_1^{2} \left(\frac{2}{\sqrt{1+x^2}} - 1\right)^2 dx$

D. $\pi\int_0^{\sqrt{3}} \left(\frac{2}{\sqrt{1+x^2}} - 1\right)^2 dx$

E. $\pi\int_0^{\sqrt{3}} \left(\frac{4}{1+x^2} - 1\right)dx$

Question 14



The graph of $y = f(x)$ is shown above. If $F(x)$ is an antiderivative of $f(x)$, the stationary points of the graph of $y = F(x)$ are

A. local minimum at $x = 0$, local maximum at $x = 2$

B. stationary points of inflexion at $x = 0$ and $x = 2$, local maximum at $x = -1$

C. stationary points of inflexion at $x = 0$ and $x = 2$, local minimum at $x = -1$

D. stationary point of inflexion at $x = 2$, local maximum at $x = -1$

E. stationary point of inflexion at $x = 2$, local minimum at $x = -1$

Question 15

If $y = \cos(2x - 3)$, then

A. $y + 4\dfrac{d^2y}{dx^2} = 0$

B. $y - 4\dfrac{d^2y}{dx^2} = 0$

C. $4y - \dfrac{d^2y}{dx^2} = 0$

D. $4y - 2\dfrac{dy}{dx} + \dfrac{d^2y}{dx^2} = 4\sin(2x - 3)$

E. $4y + 2\dfrac{dy}{dx} + \dfrac{d^2y}{dx^2} = 4\sin(2x - 3)$

Question 16

If $g'(x) = x\sqrt{x^2 + 1}$ and $g(0) = 1$, then $g(x) =$

A. $\frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$

B. $\frac{1}{3}(x^2 + 1)^{\frac{3}{2}} + \frac{2}{3}$

C. $\frac{1}{3}(x^2 + 1)^{\frac{3}{2}} + 1$

D. $\frac{2}{3}(x^2 + 1)^{\frac{3}{2}} + \frac{1}{3}$

E. $\frac{2}{3}(x^2 + 1)^{\frac{3}{2}} + \frac{2}{3}$

Question 17

Euler's method, with a step size of 0.1, is used to solve the differential equation $\frac{dy}{dx} = \log_e x$ with $y = 2$ at $x = 1$. The value obtained for y at $x = 1.2$, correct to four decimal places, is

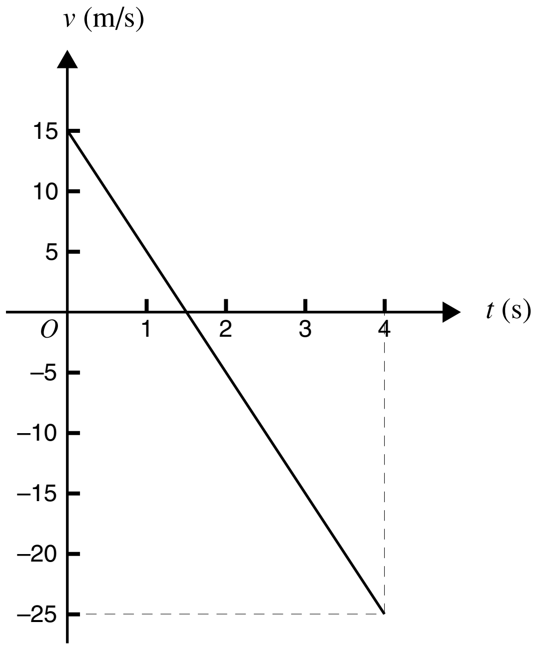
A. 2.0000
B. 2.0095
C. 2.0188
D. 2.0278
E. 2.1909

Question 18

A particle starts from rest at $t = 0$ and moves in a straight line in such a way that its acceleration, a, at time t is given by $a = 5e^{-0.1t}$. The velocity of the particle at $t = 1$, correct to two significant figures, is

A. –50
B. –4.8
C. –0.50
D. 4.8
E. 50

Question 19



This velocity-time graph represents the motion of a ball that is thrown vertically upwards from a high balcony and then falls to the ground below. The air resistance is negligible. The height in metres of the balcony above the ground is

A. 11.25

B. 15

C. 20

D. 25

E. 31.25

Question 20

In a trapezium *RSTU*, *RS* is parallel to and twice the length of *UT*.

If $\overrightarrow{UT} = \underset{\sim}{p}$ and $\overrightarrow{TS} = \underset{\sim}{q}$, then $\overrightarrow{UR} =$

A. $\underset{\sim}{q} - \underset{\sim}{p}$

B. $\underset{\sim}{p} - \underset{\sim}{q}$

C. $\underset{\sim}{p} + \underset{\sim}{q}$

D. $3\underset{\sim}{p} + \underset{\sim}{q}$

E. $\frac{1}{2}\underset{\sim}{p} + \underset{\sim}{q}$

Question 21

Which one of the following vectors has magnitude 14 and is parallel to $-3\underset{\sim}{i}+2\underset{\sim}{j}+6\underset{\sim}{k}$?

A. $6\underset{\sim}{i}-4\underset{\sim}{j}-12\underset{\sim}{k}$

B. $4\underset{\sim}{i}-12\underset{\sim}{j}+6\underset{\sim}{k}$

C. $\frac{2}{7}(3\underset{\sim}{i}-2\underset{\sim}{j}-6\underset{\sim}{k})$

D. $\frac{14}{5}(-3\underset{\sim}{i}+2\underset{\sim}{j}+6\underset{\sim}{k})$

E. $\frac{14}{\sqrt{31}}(-3\underset{\sim}{i}+2\underset{\sim}{j}+6\underset{\sim}{k})$

Question 22

If $\underset{\sim}{m}=-3\underset{\sim}{i}+4\underset{\sim}{j}$ and $\underset{\sim}{n}=-\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{k}$, then the scalar resolute of $\underset{\sim}{m}$ in the direction of $\underset{\sim}{n}$ is

A. $\frac{11}{9}$

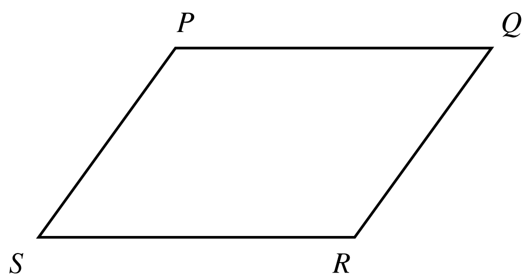
B. $\frac{11}{5}$

C. $\frac{11}{3}$

D. $\frac{11}{\sqrt{7}}$

E. 11

Question 23



To prove that quadrilateral *PQRS* is a rhombus, it is sufficient to show that

A. $\overrightarrow{PQ} = \overrightarrow{SR}$

B. $\overrightarrow{PQ} = \overrightarrow{SR}$ and $\overrightarrow{PS} = \overrightarrow{QR}$

C. $\overrightarrow{PQ} . \overrightarrow{PS} = 0$

D. $\overrightarrow{PR} . \overrightarrow{QS} = 0$

E. $\overrightarrow{PQ} = \overrightarrow{SR}$ and $|\overrightarrow{PQ}| = |\overrightarrow{PS}|$

Question 24

The position vector of a particle at time t is given by $\underset{\sim}{r} = (\sin^2 t)\underset{\sim}{i} - (2\cos^2 t)\underset{\sim}{j}, \quad 0 \le t \le \frac{\pi}{2}$.

The path along which the particle moves from (0, –2) to (1, 0) is

A. straight
B. circular
C. elliptical
D. parabolic
E. hyperbolic

Question 25

The position vector of a particle at time t is given by $\underset{\sim}{r} = (\frac{3}{2}\sin(2t))\underset{\sim}{i} + (2e^{-2t})\underset{\sim}{j}$.

The speed of the particle at time $t = 0$ is

A. $\frac{5}{4}$

B. 2

C. 4

D. 5

E. 25

Question 26

A body of mass 5 kg is travelling in a straight line. Its velocity decreases from 10 m/s to 6 m/s in a time of 2 s. The change of momentum of the particle in kg m/s, in the direction of its motion, is

A. –40

B. –20

C. –10

D. 20

E. 30

Question 27

A crate of mass 50 kg stands on level ground. A worker exerts a horizontal force of 100 N on the crate. If the coefficient of friction between the crate and the ground is 0.5, and F N is the magnitude of the frictional force, the crate

A. remains at rest since $F < 100$

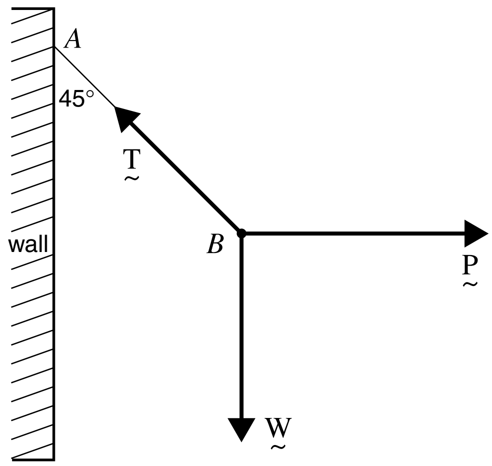
B. accelerates since $F < 100$

C. remains at rest since $F = 100$

D. accelerates since $F > 100$

E. remains at rest since $F > 100$

Question 28



The diagram shows a body, *B*, attached to the end of a light string. The string is fixed at its other end to a point, *A*, on a vertical wall. The force on *B* due to the tension in the string is $\underset{\sim}{T}$ and the weight force is $\underset{\sim}{W}$. When a horizontal force $\underset{\sim}{P}$ (directed away from the wall) is applied, *B* is held in equilibrium with the string inclined at 45° to the vertical as shown. Which one of the following vector equations is true?

A. $\underset{\sim}{P} = \underset{\sim}{W}$

B. $2\underset{\sim}{T} = \underset{\sim}{P} + \underset{\sim}{W}$

C. $\underset{\sim}{P} + \underset{\sim}{W} - \underset{\sim}{T} = \underset{\sim}{0}$

D. $\underset{\sim}{P} + \underset{\sim}{W} + \underset{\sim}{T} = \underset{\sim}{0}$

E. $\sqrt{2}\,\underset{\sim}{T} = \underset{\sim}{P} + \underset{\sim}{W}$

Question 29

A horizontal propulsion force of 900 N causes a car of mass 750 kg to accelerate along a straight, horizontal road. The total resistance to the car's motion is *cv* newtons per kilogram of its mass, where *v* m/s is the speed of the car at time *t* s and *c* is a constant. The equation of motion of the car can be expressed as

A. $750\dfrac{dv}{dt} = 900 - cv$

B. $\dfrac{dv}{dt} = 900 - cv$

C. $750\dfrac{dv}{dt} = 900 - 750cv$

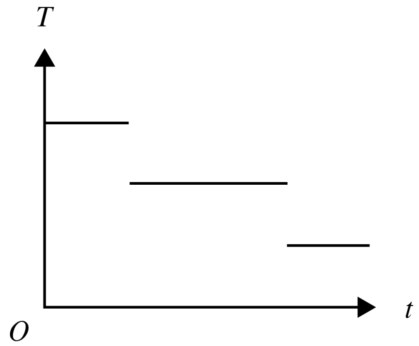
D. $\dfrac{dv}{dt} = 900 - 750cv$

E. $750\dfrac{dv}{dt} = 900 - 750cgv$

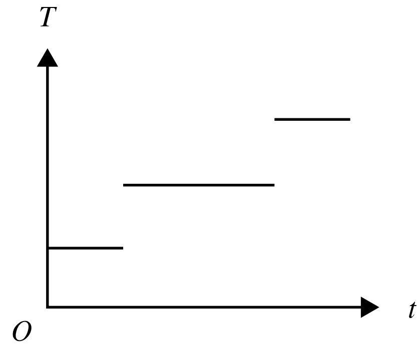
Question 30

A lift is pulled from the basement of a building to the top floor by a cable. The lift first moves with constant acceleration, then constant velocity and finally constant retardation. If T is the magnitude of the tension in the cable during the motion and t is time, which one of the following graphs could represent T versus t?

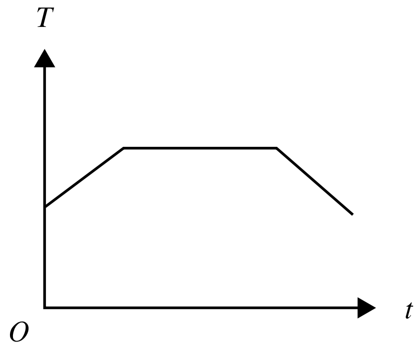
A.



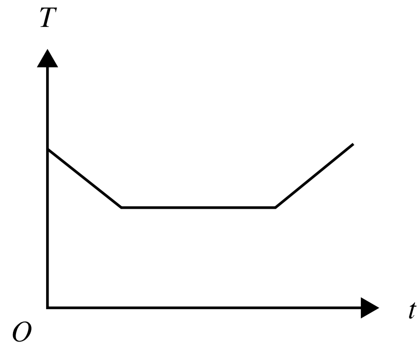
B.



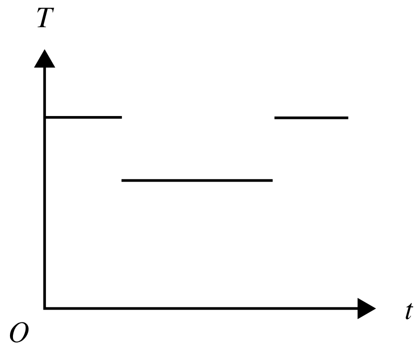
C.



D.



E.



END OF PART I MULTIPLE-CHOICE QUESTION BOOK

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

Victorian Certificate of Education 2000

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 30 October 2000: 11.45 am to 1.30 pm
Reading time: 11.45 am to 12 noon
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Total writing time: 1 hour 30 minutes

PART II

QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of the Part I question book.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Structure of book

Number of questions	*Number of questions to be answered*
6	6

Directions to students

Materials
Question book of 8 pages.
Working space is provided throughout the book.
You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve sketching.

The task
Detach the formula sheet from the centre of the Part I question book during reading time.
Please ensure that your **student number** is printed in the space provided on the cover of this book.
The marks allotted to each question are indicated at the end of the question.
There is a total of 20 marks available for the short-answer part of this examination.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions. A decimal approximation will not be accepted if an **exact** answer is required to a question.
Where an **exact** answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.
Unless otherwise indicated, the diagrams in this book are **not** necessarily drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.
All written responses should be in English.

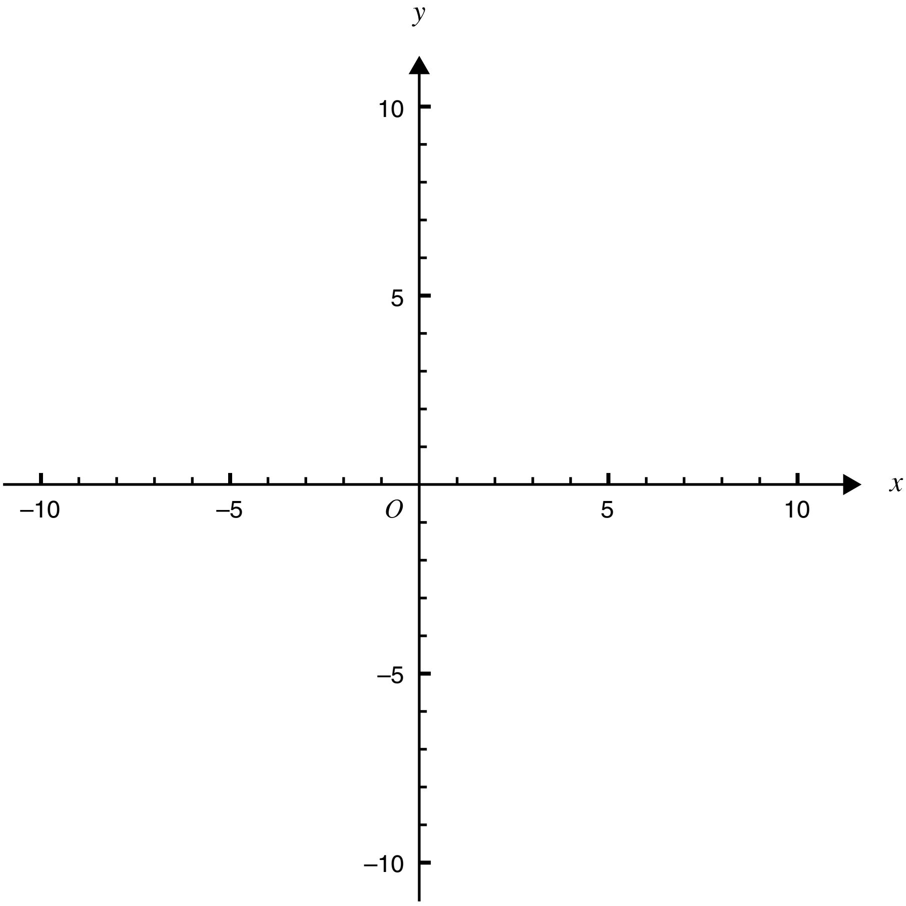
At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Instructions for students

Answer **all** questions in this part in the spaces provided in this question and answer book.

Question 1

Sketch the graph of the function with rule $f(x) = \dfrac{x^3 - 25}{5x}$ on the axes below. Clearly show any asymptotes, and give the coordinates of all intercepts and turning points correct to two decimal places.



4 marks

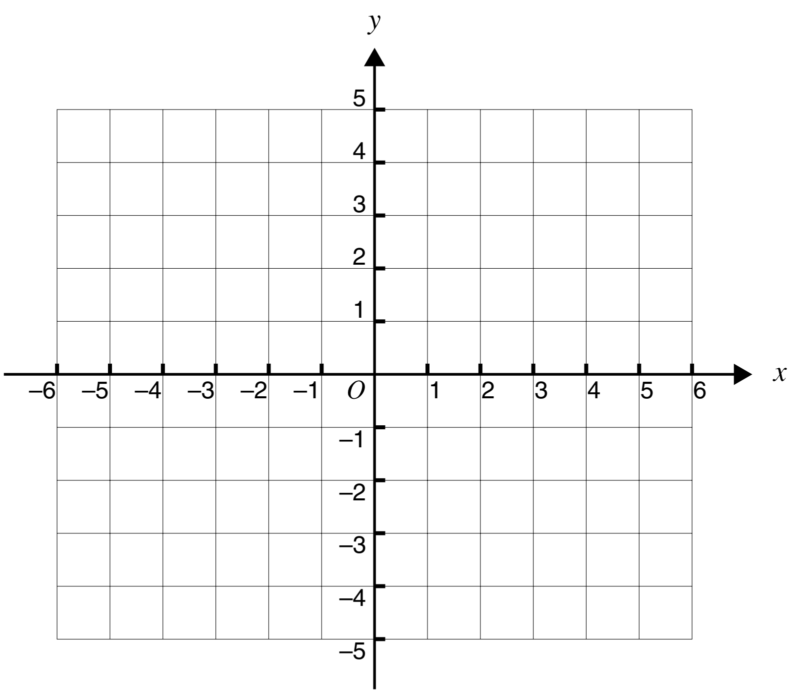
Question 2

a. The position vector of a particle at time t, $t \geq 0$, is given by $\underset{\sim}{r} = \sqrt{t}\,\underset{\sim}{i} - (t-2)\,\underset{\sim}{j}$.

Find the value(s) of t for which the particle is closest to the origin.

2 marks

b. Carefully sketch the path of the particle on the axes below.



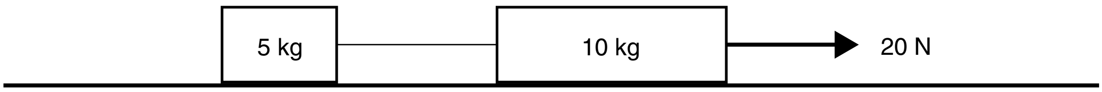
2 marks

Question 3

Use calculus to find the exact value of $\int_{\frac{\pi}{3}}^{\pi} \sin^3 \frac{x}{2} \, dx$.

3 marks

Question 4



Two blocks of mass 5 kg and 10 kg are attached to each other by a light, inextensible string. The blocks are pulled along a smooth horizontal surface by a horizontal force of magnitude 20 N.

a. Find the acceleration of the two blocks.

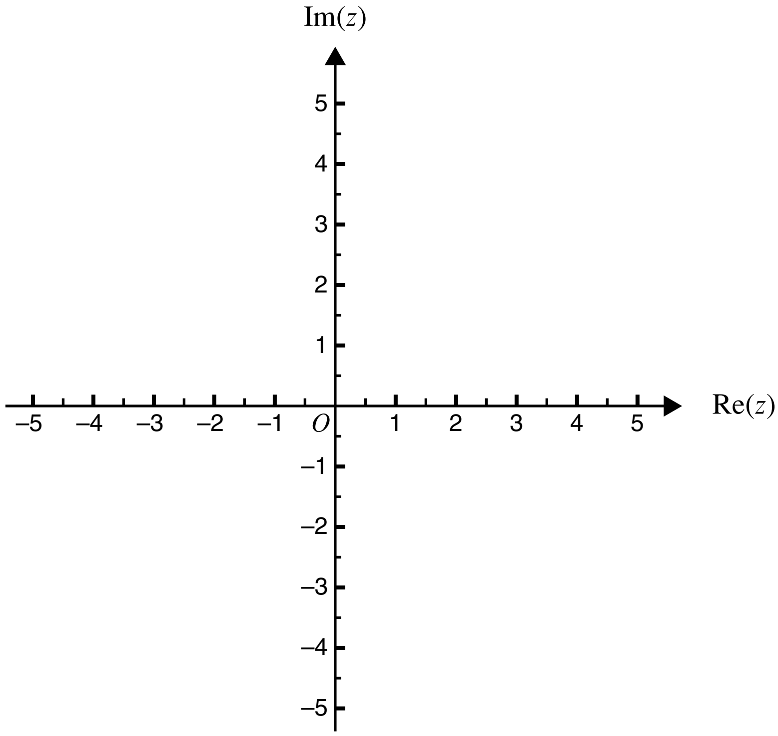
2 marks

b. Find the tension in the string.

1 mark

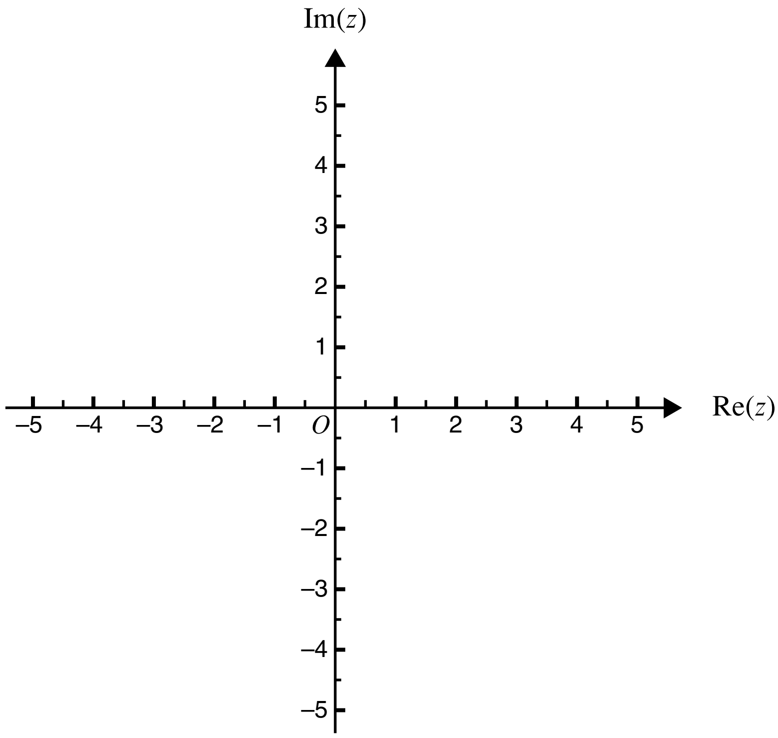
Question 5

a. Shade the region of the complex plane specified by $\{z : |z - i| \geq 2\}$.



1 mark

b. Shade the region of the complex plane specified by $\left\{z : \text{Arg}(z) \leq \dfrac{3\pi}{4}\right\}$.



2 marks

Question 6

a. Show that $2 - i$ is a solution of the equation $z^3 - (2 - i)z^2 + z - 2 + i = 0$.

1 mark

b. Find all the solutions of the equation $z^3 - (2 - i)z^2 + z - 2 + i = 0$.

2 marks

END OF PART II QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigometric) functions

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$\cot^2 x + 1 = \operatorname{cosec}^2 x$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin 2x = 2\sin x \cos x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r \operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis} n\theta$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e x + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin ax) = a\cos ax \qquad \int \sin ax \, dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax \qquad \int \cos ax \, dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}(\tan ax) = a\sec^2 ax \qquad \int \sec^2 ax \, dx = \frac{1}{a}\tan ax + c$$

$$\frac{d}{dx}\left(\operatorname{Sin}^{-1}x\right) = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{a^2-x^2}} dx = \operatorname{Sin}^{-1}\frac{x}{a} + c, a > 0$$

$$\frac{d}{dx}\left(\operatorname{Cos}^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{-1}{\sqrt{a^2-x^2}} dx = \operatorname{Cos}^{-1}\frac{x}{a} + c, a > 0$$

$$\frac{d}{dx}\left(\operatorname{Tan}^{-1}x\right) = \frac{1}{1+x^2} \qquad \int \frac{a}{a^2+x^2} dx = \operatorname{Tan}^{-1}\frac{x}{a} + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)f\left(\frac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)\left(f(a) + f(b)\right)$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h f(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \leq \mu N$

END OF FORMULA SHEET