Resource material to assist in the implementation of VCE Specialist Mathematics Units 3 and 4

May 2000

Approaches to course development and the design of school assessed coursework

The study design provides details on areas of study, outcomes, assessment and advice for teachers to be used in developing mathematics courses of study. While Specialist Mathematics Units 3 and 4 is a fully prescribed course, there is sufficient flexibility in how teachers choose to implement this course to allow students to achieve outcomes in a number of ways. Teachers need to consider both the time they will allocate to each topic and the sequence of presentation for these topics. When developing such a sequence of topics, consideration needs to be given to the likely sequence of related material in the Mathematical Methods Units 3 and 4 course as this contains assumed knowledge for Specialist Mathematics Units 3 and 4.

The *Mathematics Assessment Guide Revised VCE 2000*, provides teachers with information on the nature and scope of tasks for coursework assessment and related criteria to assist teachers in assessing student work. Sample tasks and associated commentary outlined in this document provides further advice for possible implementation of these tasks. Careful consideration needs to be given to the scheduling of assessment tasks as their location in the course will influence the study patterns of students. In the schedule provided different timings for assessment tasks are considered, together with the effect that they might have on a student's approach to study, for example, the placement of a test before or after a term break will have an effect on the scope and nature of review and consolidation work carried out during the holiday break.

The sample assessment tasks contained in this advice draw on ideas and approaches found in former extended, test and examination common assessment tasks for Specialist Mathematics Units 3 and 4. These tasks, along with other tasks and activities that teachers have used to meet the previous work requirements for Specialist Mathematics Units 3 and 4, are a valuable resource that teachers can draw on to develop suitable tasks of their own for coursework assessment.

1. VCE Mathematics Units 3 and 4 Specialist Mathematics sample schedule for 2000

Semester 1

TERM 1		
Week	Date(s)	Task
1	25/1 – 28/1	School Starts[1]
2	31/1 – 4/2	Complex numbers
3	7/2 – 11/2	Complex numbers
4	14/2 – 18/2	Complex numbers
5	21/2 – 25/2	Complex regions
6	28/2 – 3/3	Coordinate geometry
7	6/3 – 10/3	Coordinate geometry[2]
8	13/3 –17/3	Analysis 1[3]
		Circular functions
9	20/3 – 24/3	Circular functions
10	27/3 – 31/3	Vectors
11	3/4 – 7/4	Vectors
Term 1 Break 10/4 25/4		

TERM 2		
Week	Date(s)	Task
1	26/4 – 28/4	Vectors
2	1/5 – 5/5	Vectors (including proof)
3	8/5 – 12/5	Calculus
4	15/5 – 19/5	Calculus
5	22/5 – 26/5	Calculus
6	29/5 – 2/6	Calculus
7	5/6 – 9/6	Analysis 2[4]
		Calculus
8	GAT and exams Unit 3/4 changeover	
9	19/6 – 23/6	Test 1[5]
		Calculus
Term 2 Break 26/6 7/7		

1. Set up course: explain expectations; assessment procedures.
2. Begin with complex numbers / coordinate geometry to match up with Mathematical Methods; could begin with vectors.
3. First analysis task: designed to test knowledge of one or both of complex numbers and coordinate geometry.
4. Second analysis task: designed to test one or both of vectors and calculus.
5. Test 1 could be here (the start of Unit 4) to fit in with examinations; alternatively, it could be delayed until early in Term 3 to encourage students to revise particular sections of their Specialist Mathematics work over the school break.

Semester 2

TERM 3		
Week	Date(s)	Task
1	10/7 – 14/7	Differential equations
2	17/7 – 21/7	Kinematics
3	24/7 – 28/7	Differential equations
4	31/7 – 4/8	Vector calculus
5	7/8 – 11/8	Application task[6]
6	14/8 – 18/8	
		Vector calculus
7	21/8 – 25/8	Mechanics
8	28/8 – 1/9	Mechanics
9	4/9 – 8/9	Mechanics
10	11/9 – 15/9	Test 2[7]
		Revision
Term 3 Break 18/9 24/9		

TERM 4		
Week	Date(s)	Task
1	2/10 – 6/10	**Exam Review**
2	9/10 –13/10	
3	16/10 – 20/10	
4	**Examinations**	
5		
6		
7		
8		
9		
10		

6. Application task could start earlier depending on course areas to be used. On the other hand, a task involving vector calculus would best occur at this time.
7. Test 2 could be here (the end of Term 3) to fit in with school examinations; alternatively, it could be delayed until early in Term 4 to encourage students to revise their Specialist Mathematics over the school break.

2. A sample application task

The suggested theme and possible starting points for the Specialist Mathematics application task in 2000 were published in the December 1999 edition of the *VCE Bulletin*. The suggested theme for 2000 is 'Paths in the Plane' and four starting points were described. The following sample application task uses Starting Point 1: *Minimum Path.*

Starting Point 1: Minimum path

In some 'real-life' applications, the 'best' path from one point to another in a plane must be found. In this context, 'best' might mean minimum time or minimum cost. The path taken may be described by the angle(s) it makes with some reference line. For this starting point, it is required to investigate the 'best' path subject to constraints related to the time of the journey or the cost of the path.

For example, in a biathlon, a contestant must race from one point to another in minimum time across two types of surfaces, with different speeds on each surface. As another example, a gas supplier wants to find the cheapest route from an offshore platform to a point on the shoreline, or to a point inland, where the cost of laying pipeline underwater is more expensive than on land.

Component 1: introduction via specific cases for the parameters or variables (e.g. using a direct route, or a route involving the shortest distance across the 'most expensive surface'); *Component 2*: investigation of the general case (consideration of the route from the initial point via a general point along the boundary of the surfaces); *Component 3*: determination of the effects on the best route of varying parameters in the problem (e.g. relative speeds or costs).

Key assessment features

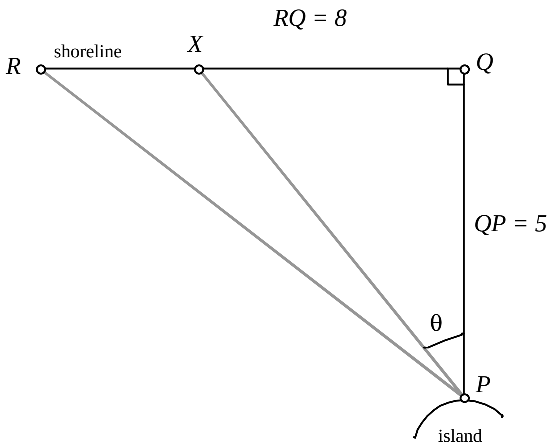
The key knowledge (and corresponding items from key skills) for Outcome 1 are listed under the theme.

Important aspects of mathematics to be considered in assessment of student work are:
- sketching graphs of functions, including circular and/or inverse circular functions, clearly identifying their key features such as domain, range and extreme points
- using calculus to find extreme points of functions
- obtaining approximate solutions to equations using technology.

Application task: Power problem

The mathematical techniques which might be required for this task include:
- functions – domain and range
- circular functions and their inverses
- calculus
- approximate methods of solutions of equations

The private owner of a small off-shore island wishes to run a power cable to a point *P* on the tip of the island. The nearest point *Q* on the shore is five kilometres from *P* and there is a power sub-station at point *R* eight kilometres along the (straight) shoreline from *Q*, as shown in the following diagram:



It costs \$5000 per kilometre to run the cable underwater and \$2500 per kilometre to run the cable along the shoreline.

Question 1

How much would it cost to run the cable

a. directly from R to P?

b. from R to Q to P?

To explore what happens if the owner tries a different route for the cable, let X be a point on RQ and let $\angle QPX = \theta$ as shown in the above diagram.

Question 2

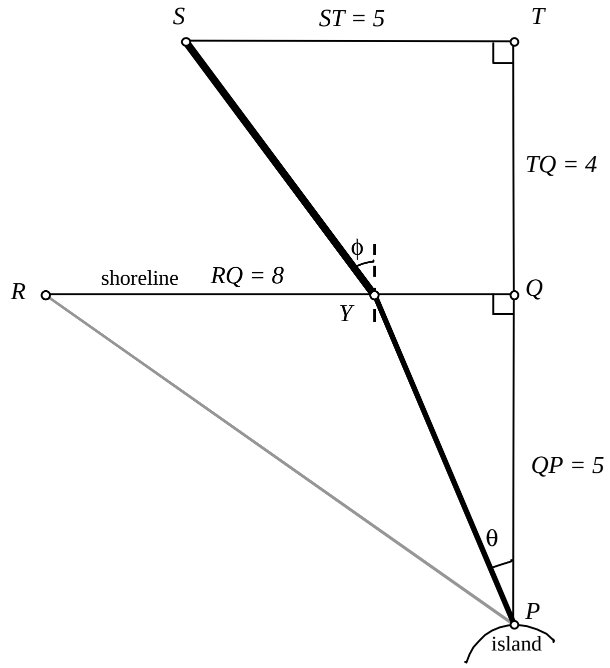
a. Show that $RX = 8 - 5 \tan \theta$ and find an expression in terms of θ for XP.

b. If C is the total cost in dollars of running the cable along the route RX followed by XP, find an expression for C in terms of θ and state the implied domain of the corresponding function.

c. Use a graphics calculator or computer graphing software to draw the graph of C versus θ and hence find approximately the value of θ that minimises C.

d. Use a calculus method to find exactly the value of θ that minimises C, and state the minimum cost and the location of X to the nearest metre along the shoreline.

A second power sub-station is located at S four kilometres back from the shoreline and five kilometres from the line through PQ as shown. Suppose it costs the same to run a cable from S directly to a point on the shoreline as it costs to run a cable along the shoreline. Although S is further from P than R, the island's owner wonders whether it will be cheaper to run the cable along the route SY followed by YP where Y is some point on the shoreline. Let the angle between SY and the vertical through Y be ϕ as shown.



Let the distance from Y to Q be y kilometres and let C_1 be the cost in dollars of running the cable from P to Y and C_2 be the cost in dollars of running the cable from Y to S, so that the total cost is given by $C = C_1 + C_2$.

Question 3

a. Write down expressions for C_1 and C_2 in terms of y, and find $\frac{dC_1}{dy}$ and $\frac{dC_2}{dy}$ in terms of y.

b. Express $\frac{dC_1}{dy}$ and $\frac{dC_2}{dy}$ in terms of θ and ϕ respectively.

c. **Hence** express $\frac{dC}{dy}$ in terms of θ and ϕ, and show that the minimum cost occurs when

$$a \sin\theta = b \sin\phi$$

where a and b are constants. Give an interpretation of these constants.

d. Show that θ and ϕ are also related by the equation $c \tan\theta + d \tan\phi = e$ where c, d and e are constants. To find the values of θ and ϕ corresponding to the minimum cost, we must solve the equations in **Question 3 c** and **d** simultaneously. Since they are non-linear, one approach is to use a suitable technology approach.

Question 4

a. Use the equation in **Question 3 b** to express ϕ in terms of θ and substitute this expression in the equation in **Question 3 c** to obtain an equation in θ.

Use a suitable technology approach to find approximately the value of θ that minimises C, and hence find

the corresponding value of ϕ;

ii. the location of Y to the nearest metre along the shoreline;

the best route for running the power cable to the island.

Question 5

Consider the possible effects of a change in the costs of running the cable underwater and/or on land.

Mapping of criteria and key skills for outcomes with these questions

When a task is developed, it would be designed with the clear intention to cover key knowledge and skills for each of the outcomes. The assessment criteria reflect this identification. To elaborate this process, the following table gives a detailed list of outcomes and related criteria for the components of this application task.

Question 1	**Criteria**
Outcome 1 Apply a range of analytical and numerical processes to obtain solutions (exact or approximate) to equations.	*Appropriate use of mathematical conventions, symbols and terminology.* *Accurate application of mathematical skills and techniques.*
Question 2	**Criteria**
Outcome 1 sketch graphs of specified functions and relations, clearly identifying their key features; use a variety of techniques to find derivatives and anti-derivatives and the use of anti-derivatives to evaluate definite integrals; apply a range of analytical and numerical processes to obtain solutions (exact or approximate) to equations.	*Appropriate use of mathematical conventions, symbols and terminology.* *Definition and explanation of key concepts.* *Accurate application of mathematical skills and techniques.*
Outcome 2 specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions related to a given context; give mathematical formulations of specific and general cases used to derive results for analysis within a given application context; communicate conclusions using both mathematical expression and expression in everyday language, in particular in relation to a given application context.	*Identification of important information, variables and constraints.* *Application of mathematical ideas and content from the specified areas of study.* *Analysis and interpretation of results.*
Outcome 3 distinguish between exact and approximate technological presentations of mathematical results, and interpret these results to a specified degree of accuracy; use appropriate domain and range specifications which illustrate key features of graphs of functions and relations; identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations.	*Appropriate selection and effective use of technology.* *Application of technology*

Question 3	Criteria
Outcome 1 use a variety of techniques to find derivatives and anti-derivatives and the use of anti-derivatives to evaluate definite integrals; apply a range of analytical and numerical processes to obtain solutions (exact or approximate) to equations.	*Appropriate use of mathematical conventions, symbols and terminology.* *Definition and explanation of key concepts.* *Accurate application of mathematical skills and techniques.*
Outcome 2 give mathematical formulations of specific and general cases used to derive results for analysis within a given application context; communicate conclusions using both mathematical expression and expression in everyday language, in particular in relation to a given application context.	*Identification of important information, variables and constraints.* *Application of mathematical ideas and content from the specified areas of study.* *Analysis and interpretation of results.*
Question 4	**Criteria**
Outcome 1 apply a range of analytical and numerical processes to obtain solutions (exact or approximate) to equations.	*Appropriate use of mathematical conventions, symbols and terminology.* *Definition and explanation of key concepts.* *Accurate application of mathematical skills and techniques.*
Outcome 2 specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions related to a given context; give mathematical formulations of specific and general cases used to derive results for analysis within a given application context; communicate conclusions using both mathematical expression and expression in everyday language, in particular in relation to a given application context.	*Identification of important information, variables and constraints.* *Application of mathematical ideas and content from the specified areas of study.* *Analysis and interpretation of results.*
Outcome 3 distinguish between exact and approximate technological presentations of mathematical results, and interpret these results to a specified degree of accuracy.	*Appropriate selection and effective use of technology.* *Application of technology*
Question 5	**Criteria**
Outcome 1 sketch graphs of specified functions and relations, clearly identifying their key features; use a variety of techniques to find derivatives and anti-derivatives and the use of anti-derivatives to evaluate definite integrals; apply a range of analytical and numerical processes to obtain solutions (exact or approximate) to equations.	*Appropriate use of mathematical conventions, symbols and terminology.* *Definition and explanation of key concepts.* *Accurate application of mathematical skills and techniques.*

Outcome 2 specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions related to a given context; give mathematical formulations of specific and general cases used to derive results for analysis within a given application context; communicate conclusions using both mathematical expression and expression in everyday language, in particular in relation to a given application context.	*Identification of important information, variables and constraints.* *Application of mathematical ideas and content from the specified areas of study.* *Analysis and interpretation of results.*
Outcome 3 distinguish between exact and approximate technological presentations of mathematical results, and interpret these results to a specified degree of accuracy; use appropriate domain and range specifications which illustrate key features of graphs of functions and relations; identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations.	*Appropriate selection and effective use of technology.* *Application of technology.*

Using the notions illustrated in this table, the following mark-sheet could be used to assist in the assessment of student work on this task. This score would need to be re-scaled to a mark in the range 0–20.

Application task

Theme:

Starting point/ context:		
Student:	**Date:**	
Outcome and criteria	Available	Score
Outcome 1	**15**	
Criterion 1: Appropriate use of mathematical conventions, symbols and terminology	3	
Criterion 2: Definition and explanation of key concepts	6	
Criterion 3: Accurate application of mathematical skills and techniques	6	

Outcome and criteria	Available	Score
Outcome 2	**20**	
Criterion 1: Identification of important information, variables and constraints	4	
Criterion 2: Application of mathematical ideas and content from the specified areas of study	8	
Criterion 3: Analysis and interpretation of results	8	
Outcome 3	**5**	
Criterion 1: Appropriate selection and effective use of technology	2	
Criterion 2: Application of technology	3	
Student Score	**40**	

Comments

3. Sample Test

The following sample test covers work related to *Mechanics* and *Vector Calculus*. In the schedule outlined earlier, this would be the second test, conducted towards the end of term 3.

Specialist Mathematics coursework assessment test 2

Part 1 multiple choice

Place the letter corresponding to the answer for each item in the following table.

Item	1	2	3	4	5	Total
Answer						

Teachers would select about five multiple-choice items (these may be similar to previous examination questions, or specifically devised by the teachers) using a suitable combination to cover Criteria 1, 2 and 3 for Outcome 1. For example, the following items from previous CATs or similar would be suitable.

Question 1 Equilibrium of a particle (Outcome 1 Criterion 2) – CAT 2, Part 1, 1996, Mechanics Module, Q.2

Question 2 Motion of a body (Outcome 1 Criterion 3) – CAT 2, Part 1, 1998, Mechanics Module, Q.1

Question 3 Vector calculus (differentiation) (Outcome 1 Criterion 3) – CAT 2, Part 1, 1998, Mechanics Module, Q.10

Question 4 Resultant force (Outcome 1 Criterion 2) – CAT 2, Part 1, 1999, Mechanics Module, Q.1

Question 5 Vector calculus (integration) (Outcome 1 Criterion 3) – CAT 2, Part 1, 1999, Mechanics Module, Q.10

Part 2 short answer

Select two or three short answer items, using a suitable combination of items, to cover Criteria 1, 2 and 3 for Outcome 1. For example, questions similar to the following items from previous examinations would be suitable.

Vector calculus (integration) (Outcome 1 Criteria 1 and 3) – CAT 2, Part 2, 1994, Mechanics Module, Q.9

Equilibrium of a particle (Outcome 1 Criteria 1, 2 and 3) – CAT 2, Part 2, 1995, Mechanics Module, Q.9

Motion of a body (Outcome 1 Criteria 2 and 3) – CAT 2, Part 2, 1998, Mechanics Module, Q.5

Question 1

A dot which moves across a computer screen has position vector $\underset{\sim}{r}$ at time t. The dot moves with a constant velocity given by $\dot{\underset{\sim}{r}} = \underset{\sim}{i}$, and initially its position vector is $2\underset{\sim}{i} + 4\underset{\sim}{j}$.

Find an expression for $\underset{\sim}{r}$ in terms of t.

[2 marks]

Question 2

A particle, P, of mass 6 kg hangs at the lower end of a taut string which is attached at its upper end to a fixed point O. The particle is held at rest by a force of magnitude 30 newtons which acts horizontally on the particle so that the string makes an angle of θ with the vertical.

a. Draw a diagram to show all the forces acting on the particle.
b. Find the magnitude of θ to the nearest degree.

[3 marks]

Question 3

A box of mass 3 kg lies on a level floor. The coefficient of friction between the floor and the box is 0.41. A child pulls on the box with a horizontal force of 10 N.

Show that the box does not move.

[2 marks]

Part 3 extended response

Select one or two extended response items, using a suitable combination of items, to cover Criteria 1, 2 and 3 for Outcome 1. For example, questions similar to the following items from previous examinations would be suitable.

Motion of a body (Outcome 1 Criteria 2 and 3; Outcome 3 Criteria 1 and 2) – CAT 3, 1996, Mechanics Module, Q.6a

Vector calculus (Outcome 1 Criteria 2 and 3) – CAT 3, 1996, Mechanics Module, Q.6b

These are reproduced below.

Question 1

A skier of mass 62.5 kilograms is being towed up a slope at a constant speed. The skier is being towed by a cable which makes an angle of θ to the line of the slope. The inclination of the slope to the horizontal is α where $\sin\alpha = 0.28$ and the coefficient of friction, μ, between the skis and the slope, is 0.05.

a. Draw a diagram to show the following forces which act on the skier:
   - the weight force of magnitude $62.5g$
   - the normal reaction of magnitude N
   - the friction force of magnitude F
   - the tension in the cable of magnitude T

b. By resolving these forces parallel to and perpendicular to the slope, show that T is given by

$$T = \frac{20.5g}{\cos\theta + 0.05\sin\theta}$$

c. Find the value of θ, to the nearest tenth of a degree, which minimises the tension in the cable.

(1 + 4 + 3 = 8 marks)

Question 2

Michael launches his model rocket from the origin where unit vector $\underset{\sim}{i}$ points east, unit vector $\underset{\sim}{j}$ points north and unit vector $\underset{\sim}{k}$ points vertically up. The rocket is projected so that its velocity $\underset{\sim}{v}(t)$ metres/second at time t seconds $(0 < t < 2)$ is given by

$$\underset{\sim}{v}(t) = 7\,\underset{\sim}{i} + 4\sqrt{2}\,\underset{\sim}{j} + (49.8t - 14.9t^2)\,\underset{\sim}{k}$$

a. Find the velocity of the rocket at $t = 2$ and hence find its speed at $t = 2$.
b. Find an expression for the acceleration of the rocket at time t, $0 < t < 2$.
c. Show that the rocket reaches a height of 59.9 metres at $t = 2$.
d. For $t > 2$, the rocket is subject only to acceleration due to gravity, that is, $-9.8\,\underset{\sim}{k}$ m/s^2. Find the maximum height reached by the rocket, correct to the nearest metre.
e. Find the position vector of the rocket when it is at its maximum height.

(2 + 1 + 3 + 2 + 2 = 10 marks)

Marking the test

A marking scheme for the sample test would be devised on the basis of Parts 1–3 being completed over an allocated time of about 45 minutes for a total mark allocation of 30 marks, which reflects the time: mark ratio for end-of-year examinations.

These test items were developed to cover key knowledge and skills for the relevant outcomes. The weightings for the criteria on this sample test satisfy the following allocation, which is illustrative only. Other allocations of weightings are possible. Items related to Outcome 3 have a smaller representation on this particular test.

Outcome 1			Outcome 3	
Criterion 1	**Criterion 2**	**Criterion 3**	**Criterion 1**	**Criterion 2**
5	**11**	**11**	**1**	**2**

The overall allocation for the **two** tests taken together should be weighted 15 marks on Outcome 1 and 5 marks on Outcome 2. In the sample test above, the weighting is 27 to 3. It follows that the first test would need to incorporate more items related to Outcome 3. Thus, for example, if Test 1 was also designed to be worth 30 marks (which ensures equal weighting of the two tests) the corresponding outcome weightings should be about 18 to 12.

Alternatively, this test could be modified to have a greater mark allocation for Outcome 3. This would require the selection of specific questions that involve student use of a graphics calculator.

Question

a. A particle has position vector $\underset{\sim}{r}(t) = (3 + 2\cos t)\,\underset{\sim}{i} + \sin t\,\underset{\sim}{j}$, express its position in parametric form *x(t)* and *y(t),* and sketch the graph of $\underset{\sim}{r}(t)$. (3 marks)

b. Find the point of intersection of $\underset{\sim}{r}(t)$ and the position vector with Cartesian equation $y = \dfrac{x}{\sqrt{5}}$.

c. On a graph mark this point of intersection **A** and the centre of the ellipse **B.**

d. State the position vector $\overrightarrow{OA}$ and $\overrightarrow{OB}$, and hence find $\overrightarrow{BA}$

(3 + 1 +1 + 3 marks)

The inclusion of this question, in place of question 1, would alter the allocation of marks. The weightings for the criteria on this sample test with this question would satisfy the following allocation, which is illustrative only, other allocations of weightings are possible.

Outcome 1			**Outcome 3**	
Criterion 1	**Criterion 2**	**Criterion 3**	**Criterion 1**	**Criterion 2**
5	**9**	**9**	**3**	**4**

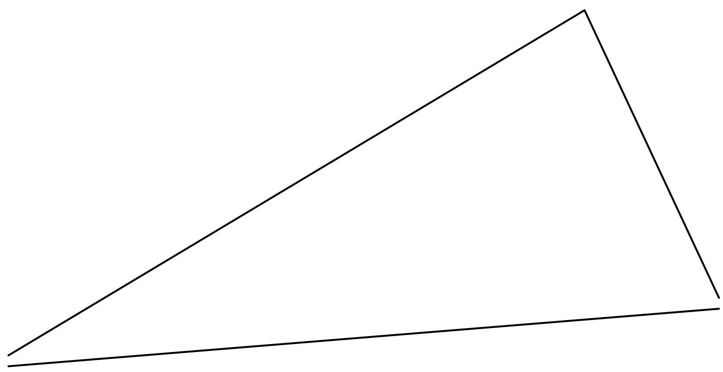
4. A short and focused investigative, challenging problem or modelling type analysis task vector proof

Introduction

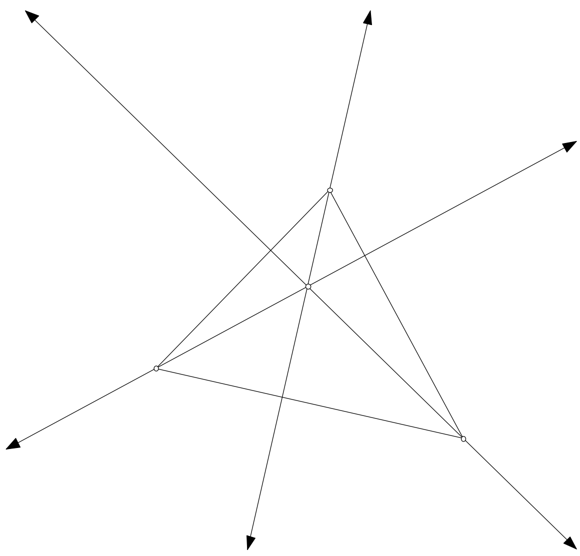
In this task you will explore certain geometric relations and apply vector proof techniques to establish geometric results involving triangles, lines and circles. In this task the use of dynamic geometry software will relate to Outcome 3 and enable students to explore the invariance of certain geometric properties as the sides or angles of triangles are varied. Students should also consider these constructions and their properties in relation to special case triangles. This task will involve: position and relative position vectors, vector sums and differences, scalar multiples, linear independence, dividing line segments in a given ratio and scalar products.

Part 1

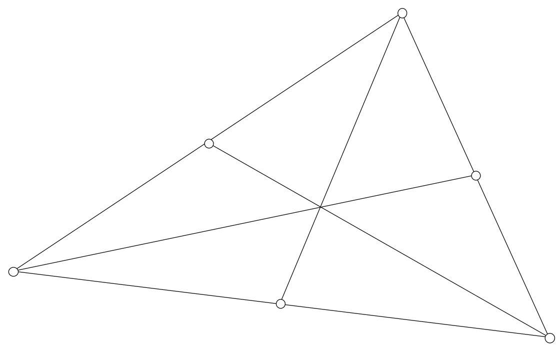
Draw a scalene triangle, like the one shown below, and make several copies of this triangle.



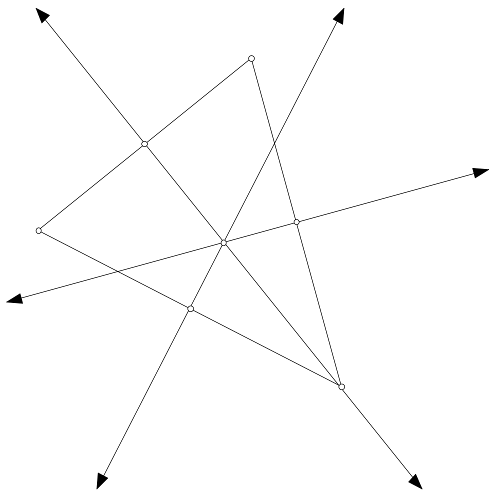
a. The *altitudes* of a triangle are those lines segments from each vertex of the triangle, which are perpendicular to the side opposite that vertex . On a copy of the triangle you have drawn, construct all three altitudes and show that they are concurrent. This point is called the *orthocentre* of the triangle. Locate this point for a selection of other triangles.



b. The *medians* of a triangle are those lines segments from each vertex of the triangle, which meet the side opposite that vertex at its midpoint. On a copy of the triangle you have drawn, construct all three medians and show that they are concurrent. This point is called the *centroid* of the triangle. Locate this point for a selection of other triangles.



c. The *perpendicular bisectors of the sides* of a triangle are those perpendicular line segments from the midpoint of each side of the triangle. On a copy of the triangle you have drawn, construct all three perpendicular bisectors and show that they are concurrent. This point is called the *circumcentre* of the triangle. Locate this point for a selection of other triangles.



d. For a selection of triangles show that these three points all lie on the same straight line.

Part 2

a. Use a vector method to prove that the medians of a triangle are concurrent. To develop such a proof , let **a**, **b**, and **c** be the position vectors of the points A, B and C (the corners of the triangle) respectively. Now consider the line segment AM, from the vertex A to the midpoint, M, of the line segment BC (this is one of the medians of the triangle). Let P be the point on AM which divides this median in the ratio $AP:PM$ = 2:1. Show that the position vector of P is given by:

$$\mathbf{p} = \frac{(\mathbf{a} + \mathbf{b} + \mathbf{c})}{3}$$

Show that when the other two medians are similarly divided in this ratio, the points corresponding to P also have the position vector:

$$\frac{(\mathbf{a} + \mathbf{b} + \mathbf{c})}{3}$$

b. Use a vector method to prove that the altitudes of a triangle are concurrent. To develop such a proof , let **a**, **b**, and **c** be the position vectors of the points A, B and C (the corners of the triangle) respectively. Find the position vector p for the point of intersection, P, of any two altitudes (this will involve the use of the scalar product and the sides of the triangle). Now show that the line segment connecting the vertex opposite the third side, and passing through the point P, is perpendicular to that third side. That is, show that:

$$(\mathbf{p} \quad \mathbf{c}) \, . \, (\mathbf{a} - \mathbf{b}) = 0$$

and interpret this result geometrically.

c. Use a vector method to prove that the *perpendicular bisectors* of the sides of a triangle are concurrent. This requires a similar approach to b. above, but also used the fact that the segments are *bisectors* and therefore also pass through the midpoints of the respective sides.

Part 3

Show that the orthocentre, P, circumcentre, Q, and centroid, R, of a triangle are collinear (all three points lie on the same straight line). This proof will involve midpoints, scalar products and linear (in)dependence of vectors. At some stage you will need to show that:

$$\mathbf{p} + 2\,\mathbf{q} - 3\,\mathbf{r} = \mathbf{0}$$

Mapping of criteria and key skills for outcomes with this analysis task

When this task was developed, the intention to cover key knowledge and skills for each of the Outcomes as listed below. The assessment criteria should reflect this identification. To satisfy Outcome 3 students would need to use appropriate computer software to explore the geometric properties of triangles.

Outcome 1

Key knowledge

- analytical, graphical and numerical techniques for setting up and solving equations involving specified functions and relations.

Key skills

- apply a range of analytical and numerical processes to obtain solutions (exact or approximate) to equations.

Outcome 2

Key knowledge

- specific and general formulations of concepts used to derive results for analysis within a given application context;
- the role of examples, counter-examples and general cases in developing mathematical analysis;
- the role of proof in establishing a general result;
- the use of inferences from analysis to draw valid conclusions related to a given application context.

Key skills

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions related to a given context;
- give mathematical formulations of specific and general cases used to derive results for analysis within a given application context;
- establish proofs for general case results;
- make inferences from analysis and use these to draw valid conclusions related to a given application context;
- communicate conclusions using both mathematical expression and expression in everyday language, in particular in relation to a given application context.

Outcome 3

Key knowledge

- the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- the appropriate selection of a technology application in a variety of mathematical contexts.

Key skills

- produce results using technology which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results using technology which support general analysis in problem-solving, investigative or modelling contexts;
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- specify the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to the nature of a particular mathematical task (investigative, problem solving or modelling).

Using these outcomes and the assessment criteria, the following mark-sheet could be used to assist in the assessment of student work on this task.

Sample Marking Sheet Analysis task

Analysis task: Vector Proofs		
Student:	**Date:**	
Outcome and criteria	Available	Score
Outcome 1	**7**	
Criterion 1: Appropriate use of mathematical conventions, symbols and terminology.	2	
Criterion 2: Definition and explanation of key concepts	2	
Criterion 3: Accurate application of mathematical skills and techniques	3	
Outcome 2	**8**	
Criterion 1: Identification of important information, variables and constraints	2	
Criterion 2: Application of mathematical ideas and content from the specified areas of study	3	
Criterion 3: Analysis and interpretation of results	3	
Outcome 3	**5**	
Criterion 1: Appropriate selection and effective use of technology	2	
Criterion 2: Application of technology	3	
Student Score	**20**	

5. An item response analysis for a collection of multiple-choice questions

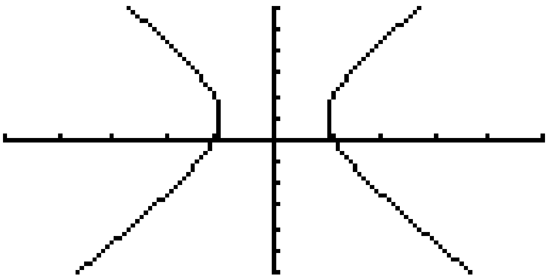
Specialist Mathematics Units 3 and 4 sample analysis task- item response analysis

In this task students are required to examine, analyse and discuss each of the multiple-choice options, indicating how the information provided enables them to distinguish between the correct and incorrect options. Students will use technology such a graphics calculators to do working and produce results for analysis. The questions presented may highlight a particular topic or collection of topics, for example non-calculus material involving complex numbers, vectors and coordinate geometry. Some multiple-choice questions are better suited for this kind of analysis task than others. Suitable questions would involve those similar to the following, taken from the 1998 Specialist Mathematics Common Assessment Task 2: Written examination (Facts, skills and applications), Part I are given below.

The level of detail in an item response analysis should be brief and mathematically focused, for example, in analysing responses to Question 6 student's need to point out that there are 3 solutions to the equation. Students need to show that they understand that the three solutions will be equally spaced around the circle of radius 2 (an angle of $2\pi/3$ between complex vectors). It is essential that students identify the simplest solution of $z^3 = 8i$ as $z = -2i$.

A. incorrect as there is only one value given and it is not a solution
B. incorrect as there is only one value given even though it is a solution
C. contains the simplest solution, and two other equally spaced solutions, and is therefore correct
D. incorrect as the three values given are not equally spaced, even though one value is a solution
E. incorrect because the initial value on which the equally spaced values are given is not itself a solution

In some questions students will likely use technology as part of their analysis, in particular for questions dealing with graphs in coordinate geometry. For example in doing an item response analysis for Question 9 Geometry Module, students could be asked to use technology to draw the graph of the hyperbola and use this information in determining the correct and incorrect options. A graph of the hyperbola (with un-numbered scale) is shown below. An appropriate view window needs to be selected, for example, x values for -5 to 5 with a scale of 1, and y values from -6 to 6 with a scale of 1.



Students need to recognise that this hyperbola is a translation of a simpler hyperbola whose asymptotes intersect at the origin. Two important facts need to be identified. First the asymptotes of the translated hyperbola will intersect at (0, 1). The second important fact is that the asymptotes of the simpler hyperbola $x^2 - y^2/4 = 1$ are $y = 2x$ and $y = -2x$. (Students need to justify this result by considering that when x and y are both large, $x^2 \approx y^2/4$.) If students simply use technology to draw the graphs of possible asymptotes and hence visually 'check' each option, this would not constitute a satisfactory analysis of the options. However, technology could be used to confirm the correct result.

A. incorrect as the asymptotes have the wrong gradient, although they intersect at (0, 1)
B. correct as the asymptotes have the correct gradients and they intersect at (0, 1)
C. incorrect as the asymptotes have the wrong gradient, even though they intersect at (0, 1)

D. incorrect as the asymptotes do not intersect at (0, 1) even though the gradients are correct
E. incorrect as the asymptotes do not intersect at (0, 1). Even though the gradients are correct, the two lines are incorrect translation of the asymptotes of the simpler hyperbola.

Question 5 from the Mechanics Module, involved the use of a vector triangle to represent three forces in equilibrium. Students need to understand the difference between equality of *magnitude* of two vectors and the conditions for two *vectors* to be equal. Essential for solving this problem is a recognition that the magnitudes of **S** and **T** are equal from the symmetry of the diagram. Also essential is a recognition that the vector sum **S** + **T** will equal and opposite to **W**.

A. incorrect as it confuses the equality of magnitudes of **S** and **T** with vector equality
B. incorrect since **S** is not in the same direction as **W**
C. correctly represents the relationship between the vector sum **S** + **T** and **W**
D. incorrectly represents the vector sum **S** + **T** which is not in the same direction as **W**
E. incorrect since it confuses a Pythagorean length relationship with a vector sum.

When constructing and item response analysis task, the selection of suitable questions need to match the assessment criteria and outcomes, and these should be reflected in student responses. For example, in meeting the outcomes, students need to identify the key mathematical concepts, skills and processes which they have used, as well as their use of technology (such as computation, solving equations, drawing graphs, producing tables) in their analysis.

Where the criteria for assessment relates to the use of technology, the selection of multiple choice questions should allow for the reasonable use of a graphing calculator to aid the analysis. Student could be encouraged to include screen dumps or hand drawn graphs which are used to support their analysis. The appropriate and effective use of technology will support mathematical analysis, in either generating results or checking solutions.

Since this is an analysis task, teachers need to select multiple choice items which provide opportunity for students to demonstrate achievement of Outcome 2. Some ‘Calculus’ related questions from the 1998 CAT Examination which could be used as a basis for developing suitable item response analysis are Questions: 18, 20, 22; for ‘Mechanics’, Questions 2 and 4, for ‘Geometry’, Questions 6, 7 and 8. Depending on the items selected, students could be asked to provide an item response analysis to about 6 to 10 questions.

Mapping of criteria and key skills of outcomes for this analysis task.

When an item response analysis task is developed the questions selected should cover key knowledge and skills for each of the outcomes. The assessment criteria should reflect this identification.

6. Summary of New and revised material

See March 1999 *VCE Bulletin* Supplement p.17.

Coordinate geometry

Sketch graphs of hyperbolas from the general Cartesian equation

Circular (trigonometric) functions

Compound and double angle formulas for tan

Algebra

Representation of relations and regions in the complex plane (previously in the Geometry Module)

Calculus

Deducing graphs of antiderivative functions

Evaluation of definite integrals numerically using technology

Numerical solution of differential equations by Euler’s method

Vectors in two and three dimensions

Linear dependence and independence

Sketch graphs of plane curves specified by a vector equation

Vector proofs (previously in the Geometry Module)

Mechanics

Revised so that this area of study, covered previously in the Mechanics module, no longer contains the topics ‘conservation of momentum’ and ‘simple harmonic motion’

7. Change to structure of examinations

All content is assessable in Specialist Mathematics Units 3 and 4. Examination 1, like the previous Specialist Mathematics CAT 2, will consist of two parts: Part I multiple-choice and Part II short-answer. There will be a slight change in the weightings of the multiple-choice and short-answer parts. In 2000, the first part of the examination will consist of 30 multiple-choice questions worth 30 marks; the second part of the examination will consist of about 6 short-answer questions worth 18-20 marks.

Examination 2, like the previous Specialist Mathematics CAT 3, will consist of a collection of extended-answer questions. In 2000, there will be 5 or 6 questions worth no more than 60 marks. As Specialist Mathematics units 3 and 4 is now a fully prescribed course, there will be only one section to the examination.

On the previous CATs, the Mechanics Module represented about 30% of the total marks available. The percentage of marks related to the Mechanics area of study will be reduced in response to the revision of course content.

Documents

All teachers should ensure they have a copy of the study design (accredited until 31 December 2003), the examination criteria (*VCE Bulletin*, No 148, December 1999, Supplement 2) and the sample examination material (*VCE Bulletin*, No 149, February 2000, Supplement 1).

The study design specifies the description of the task, the conditions under which the examination will be held and its contribution to the final assessment. The examination will be 'set by an examination panel using criteria published annually by the Board of Studies'. For 2000, these criteria appear in the VCE Bulletin, No 148, December 1999, Supplement 2.

New style of question

As has been foreshadowed for some time, the use by students of graphics calculators in end-of-year examinations will be expected from 2000. In particular, the study design states that in both Examination 1 and 2: 'Student access to an approved graphics calculator will be assumed by the setting panel'. In many cases, the graphics calculator may be used as a check on working. In some cases, the graphics calculator will provide the most effective or only feasible mode of response. Short-answer question 5 and the extended-answer question which follow exemplify this latter point.

Sample question material for Specialist Mathematics Examinations 1 and 2 was published in Bulletin Supplement 1 to the February 2000. Further advice on possible approaches to sample question material can be accessed from the Board of Studies website at http://www.bos.vic.edu.au

Students access to an approved graphics calculator will be assumed by the setting panel for each examination.

Students will need to be able to use analytical, numerical or graphical approaches to tackle questions, consistent with the Mathematics Study Design. Where a numerical answer to a question or part of a question is required, this may be able to be obtained using more than one of these approaches, however students should not assume that this will necessarily be the case. There may well be questions where no readily accessible or no analytical approach is available, in other instances it may be the case that only analytical approach will be suitable. The accuracy for numerical answers may be specified either correct to a required number of decimal places or correct to a number of significant figures. Students are expected to be familiar with both methods of specifying accuracy of an answer.