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Student Name……

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

(FACTS, SKILLS AND APPLICATIONS TASK)

2001

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of Part I and Part II.
Part I consists of 30 multiple-choice questions and should be answered on the detachable answer sheet on page 20 of this exam. This section of the paper is worth 30 marks.
Part II consists of 6 short-answer questions, all of which should be answered in the spaces provided. Part II begins on page 14 of this exam. This section of the paper is worth 20 marks.
There is a total of 50 marks available.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Students may bring up to two A4 pages of pre-written notes into the exam.

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Part I

Question 1

The equations of the asymptotes to the curve with equation $y = \dfrac{3 - 2x^2}{x}$ are

A. $x = 0$ only
B. $x = 0$ and $y = 0$
C. $x = 0$ and $y = -2x$
D. $x = 0$ and $y = 2x$
E. $x = 0$ and $y = \pm\dfrac{\sqrt{6}}{2}$

Question 2

The hyperbola with equation $\dfrac{(x-3)^2}{1} - \dfrac{(y+2)^2}{9} = 1$ has its centre and its asymptotes given respectively by

A. $(-3, 2)$ $\quad y = -3x - 1, \quad y = 3x + 5$
B. $(3, -2)$ $\quad y = -3x - 1, \quad y = 3x + 5$
C. $(3, -2)$ $\quad y = 3x - 11, \quad y = -3x + 7$
D. $(1, 3)$ $\quad y = 3x - 11, \quad y = -3x - 7$
E. $(1, 3)$ $\quad y = 3x - 7, \quad y = -3x + 5$

Question 3

For the function f, with rule $f(x) = 2 + \text{Tan}^{-1}\dfrac{x}{2}$, the implied domain is

A. $(-\dfrac{\pi}{4}, \dfrac{\pi}{4})$
B. $(-\dfrac{\pi}{2}, \dfrac{\pi}{2})$
C. $(-\pi, \pi)$
D. $(2 - \dfrac{\pi}{2}, 2 + \dfrac{\pi}{2})$
E. R

Question 4

If $y = 2\text{Cos}^{-1}(3x)$ then $\frac{dy}{dx}$ is equal to

A. $\frac{-6\sin(3x)}{\cos^2(3x)}$

B. $\frac{6\sin(3x)}{\cos^2(3x)}$

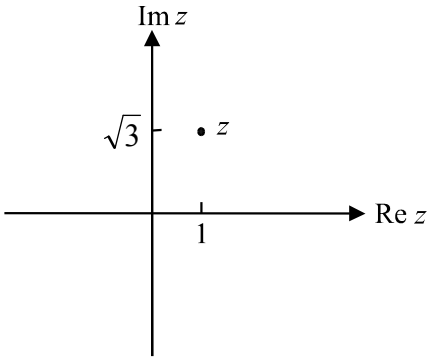
C. $\frac{-2}{\sqrt{1-3x^2}}$

D. $\frac{-2}{\sqrt{1-9x^2}}$

E. $\frac{-6}{\sqrt{1-9x^2}}$

Question 5

The Argand diagram below shows the complex number z.



This complex number z is given by

A. $2\text{cis}(\frac{\pi}{6})$

B. $2\text{cis}(\frac{\pi}{3})$

C. $\sqrt{10}\text{cis}(\frac{5\pi}{6})$

D. $\sqrt{10}\text{cis}(\frac{\pi}{3})$

E. $\sqrt{3}+i$

Question 6

If $v = 3 - 2i$, then $\frac{1}{\bar{v} - 2}$ is equal to

A. $\frac{1}{5} - \frac{2}{5}i$

B. $1 + \frac{i}{2}$

C. $1 - \frac{i}{2}$

D. $\frac{1}{5} + \frac{2}{5}i$

E. $\frac{1}{3} + \frac{2}{3}i$

Question 7

If $w = 1 - i$ then w^{10} is equal to

A. $\sqrt{2}\text{cis}(\frac{4\pi}{3})$

B. $10\sqrt{2}\text{cis}(\frac{-\pi}{2})$

C. $32\text{cis}(\frac{4\pi}{3})$

D. $32\text{cis}(\frac{-\pi}{2})$

E. $32\text{cis}(\frac{\pi}{2})$

Question 8

For any complex number z, where $z = x + y,$ and $x, y \in R$, it is true to say that $\arg\ z$ is equal to

A. $\text{Arg}\ (\bar{z})$

B. $\tan\frac{y}{x}$

C. $\sqrt{x^2 + y^2}$

D. $-\arg(\bar{z})$

E. $\arg(i\bar{z})$

Question 9

Which one of the following sets of points in the Complex plane forms a curve?

A. $\{z : |z| = |z - i|\}$

B. $\left\{z : \text{Arg}\, z = -\frac{\pi}{4}\right\}$

C. $\{z : |z + 1 - 2i| = |z - 2 + i|\}$

D. $\{z : \text{Im}\, z = 2\}$

E. $\{z : |z| = 1\}$

Question 10

An antiderivative of $\frac{9}{9 + x^2}$ is

A. $9 \log_e (9 + x^2)$

B. $\frac{2(x - 9)}{(9 + x^2)^2}$

C. $\text{Tan}^{-1} \frac{x}{3}$

D. $3\text{Tan}^{-1} \frac{x}{3}$

E. $\text{Sin}^{-1} \frac{x}{3}$

Question 11

$\int 6x^2 \sqrt{x^3 + 1}\, dx$ is equal to

A. $\frac{2(x^3 + 1)^{\frac{3}{2}}}{3} + c$

B. $\frac{4(x^3 + 1)^{\frac{3}{2}}}{3} + c$

C. $\frac{4(x^3 + 1)^{\frac{7}{2}}}{21} + c$

D. $\frac{3x^4}{2\sqrt{x^3 + 1}} + c$

E. $\frac{1}{x^3 + 1} + c$

Question 12

Using an appropriate substitution, $\int_{-2}^{0} x\sqrt{x+2}\,dx$ becomes

A. $\left[\frac{2u^{\frac{5}{2}}}{5}-\frac{4u^{\frac{3}{2}}}{3}\right]_{-2}^{0}$

B. $\left[\frac{2u^{\frac{5}{2}}}{5}-\frac{4u^{\frac{3}{2}}}{3}\right]_{0}^{2}$

C. $\left[\frac{1}{2u^{\frac{1}{2}}}+u\right]_{-2}^{0}$

D. $\left[\frac{3u^{\frac{1}{2}}}{2}-\frac{1}{u^{\frac{1}{2}}}\right]_{0}^{2}$

E. $\left[\frac{1}{3u^{\frac{1}{2}}}+\frac{1}{5u^{\frac{3}{2}}}\right]_{-2}^{0}$

Question 13

Given that $f'(x)=\frac{\sin^4 x}{\sec x}$ and $f(\pi)=0$, then $f(x)$ is equal to

A. $\frac{\sin^5 x}{5}$

B. $\frac{\sin^5 x}{5}-\frac{1}{5}$

C. $4\sin^3 x-1$

D. $\frac{-\sin^5 x}{5}$

E. $5\sin^5 x\sec x$

Question 14

$\int_{1}^{2}\log_e(x^2+1)\,dx$ is, correct to 2 decimal places, equal to

A. –1.17
B. –0.51
C. 0.51
D. 1.16
E. 1.17

Question 15

The approximate value of $\int_2^4 \frac{1}{x+1}\,dx$, obtained by using the trapezoidal rule with 2 equal intervals is

A. $\frac{37}{120}$
B. $\frac{7}{12}$
C. $\frac{8}{15}$
D. $\frac{17}{30}$
E. $\frac{31}{60}$

Question 16

Given that $\frac{dy}{dx} = 2e^{2x}$, $x = \log_e t$ and when $t = 1$, $y = 2$, then the function $y(t)$ is given by

A. t^2
B. $t^2 + 1$
C. $2t^2$
D. $2t$
E. $\frac{1}{t}$

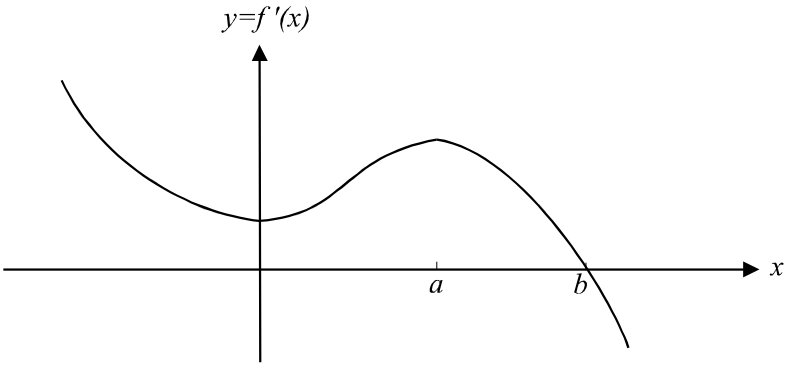
Question 17

The differential equation $\frac{dy}{dx} = 5x + 1$ where $y = \frac{1}{2}$ at $x = 1$ is solved using Euler's method with a step size of 0.1.
The value found for y at $x = 1.3$ is

A. 0.6
B. 0.83
C. 1.9
D. 2.45
E. 2.725

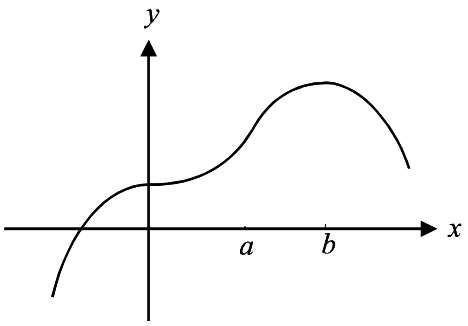
Question 18

The graph of $y = f'(x)$ is shown below.

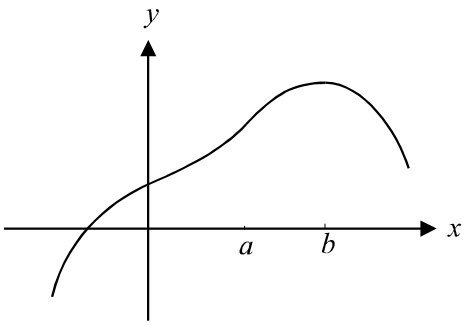


Which one of the following graphs could be the graph of $y = f(x)$?

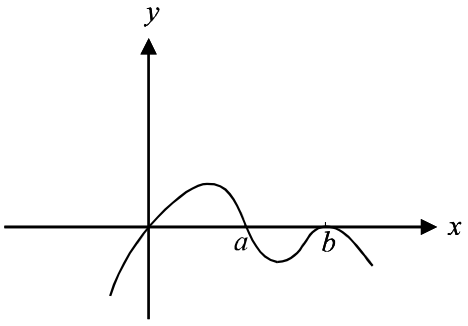
A.



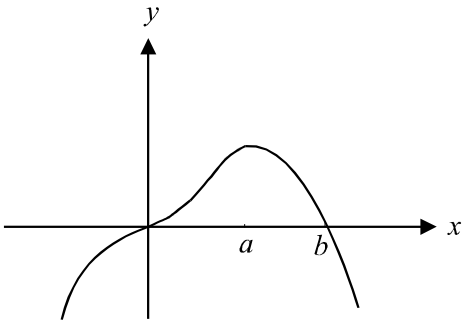
B.



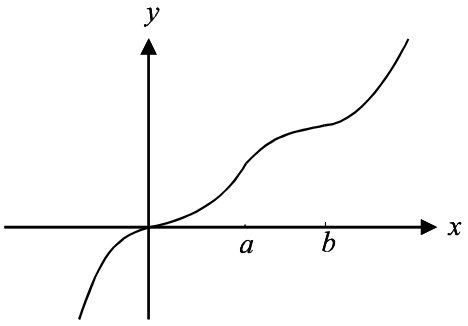
C.



D.



E.



Question 19

The region enclosed by the graphs of $y = x^2 + 2$, and $y = 6$ between $x = 0$ and $x = 2$, is rotated around the *x*-axis. The volume of the solid of revolution formed is given by

A. $\int_0^2 (-x^4 - 4x^2 + 32)dx$

B. $\int_0^2 (x^4 + 4x^2 - 32)dx$

C. $\pi\int_0^2 (4 - x^2)^2 dx$

D. $\pi\int_0^2 (-x^4 - 4x^2 + 32)dx$

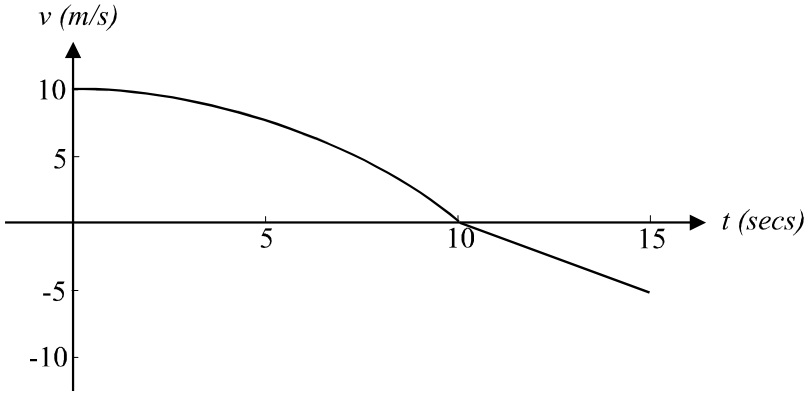
E. $\pi\int_0^2 (x^4 + 4x^2 - 32)dx$

Question 20

The velocity-time function of a particle is given by

$$v(t) = \begin{cases} \dfrac{-t^2}{10} + 10 & t \in [0,10] \\ -t + 10 & t \in (10,15] \end{cases}$$

The velocity –time graph is shown below.



The displacement of the particle over the interval $t \in [0,15]$ is given by

A. $v'(t)$

B. $\int_0^{10} (\frac{-t^2}{10} + 10)dt + \int_{10}^{15} (t - 10)\,dt$

C. $\int_0^{10} (\frac{-t^2}{10} + 10)dt - \int_{10}^{15} (t - 10)\,dt$

D. $\int_0^{10} (\frac{-t^2}{10} + 10)dt - \int_0^{15} (t + 10)\,dt$

E. $\int_0^{15} (\frac{-t^2}{10} + t)dt$

Question 21

The position vector of point P is $2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ and the position vector of point Q is $\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}$.

The magnitude of $\vec{PQ}$ is

A. 1
B. $\sqrt{14}$
C. $\sqrt{15}$
D. $\sqrt{74}$
E. $\sqrt{82}$

Question 22

A cattleman leaves a hut at point O and rides on a bearing of $N30°W$ for 12 km to a second hut at point P. If $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors of magnitude 1 km in the directions east and north respectively, then $\vec{OP}$ is given by

A. $3\underset{\sim}{i} + 9\underset{\sim}{j}$
B. $-6\underset{\sim}{i} + 6\sqrt{3}\underset{\sim}{j}$
C. $-6\sqrt{3}\underset{\sim}{i} + 6\underset{\sim}{j}$
D. $6\underset{\sim}{i} - 6\sqrt{3}\underset{\sim}{j}$
E. $6\sqrt{3}\underset{\sim}{i} - 6\underset{\sim}{j}$

Question 23

The angle between the vectors $3\underset{\sim}{i} + 2\underset{\sim}{j} - 6\underset{\sim}{k}$ and $\underset{\sim}{i} + \sqrt{2}\underset{\sim}{j} - 4\underset{\sim}{k}$, to the nearest minute is

A. 0°59'
B. 12°9'
C. 14°54'
D. 17°16'
E. 21°35'

Question 24

Let $\underset{\sim}{a} = 2\underset{\sim}{i} + 6\underset{\sim}{j} - 7\underset{\sim}{k}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$. The vector component of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$ is

A. 8
B. 89
C. $\frac{1}{3}(14\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k})$
D. $\frac{8}{3}(-\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k})$
E. $\frac{1}{89}(-137\underset{\sim}{i} + 34\underset{\sim}{j} - 10\underset{\sim}{k})$

Question 25

The position vector of a particle at time *t* is given by

$$\underset{\sim}{r} = \cos(2t)\underset{\sim}{i} + \cos t\,\underset{\sim}{j}, \qquad 0 \le t \le 2\pi$$

The path along which the particle moves has 2 endpoints. The Cartesian coordinates of those endpoints are

A. (1, 1) and (1, -1)
B. (1, -1) and (-1, -1)
C. (1, 1) and (-1, 0)
D. (-1, 0) and (1, -1)
E. (-1, 1) and (1, 0)

Question 26

A particle of mass 2kg sits on a smooth level surface and is at rest. A horizontal force of 100 N then acts on the particle. The number of seconds it will take for the particle to be moved 10 m is

A. $\frac{\sqrt{10}}{5}$
B. $\frac{\sqrt{20}}{10}$
C. 2
D. 5
E. 10

Question 27

An object of mass 2 kg is moving in a straight line. The velocity of the body is initially 12 m/s but increases to 15 m/s over a 3 second interval.
The change of momentum of the particle in kg m/s, in the direction of its established motion is

A. –6
B. –1
C. 2
D. 3
E. 6

Question 28

A mass of 20 kg is suspended in equilibrium by two strings of length 6 m and 8 m which are attached to a horizontal ceiling at 2 points which are 10 m apart.
The tension, in Newtons, in the shorter string is

A. $6\sqrt{2}g$
B. $\frac{6}{\sqrt{2}}g$
C. $10g$
D. $12g$
E. $16g$

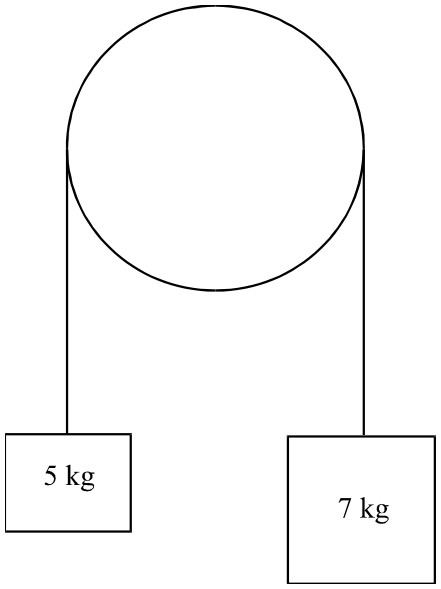
Question 29

A woman of mass 50 kg stands in a lift which is moving upwards with an acceleration of a m/s^2. The floor of the lift exerts a force of magnitude $75g$ Newtons on the woman.
The value of a is

A. $\frac{1}{2}$
B. $\frac{-g}{2}$
C. $\frac{g}{2}$
D. $\frac{75g-50}{50}$
E. $\frac{75g-50}{50g}$

Question 30

Two weights, one of mass 5 kg and one of mass 7 kg, are attached to either end of a light inextensible string which passes over a smooth pulley. The system is shown below.



The weights are held in place and then released.
If T is the tension force in the string, then the acceleration of the 7 kg mass can be given by

A. $\frac{T-5g}{7}$

B. $\frac{T-5}{5g}$

C. $\frac{7g-T}{7}$

D. $\frac{T-7}{7g}$

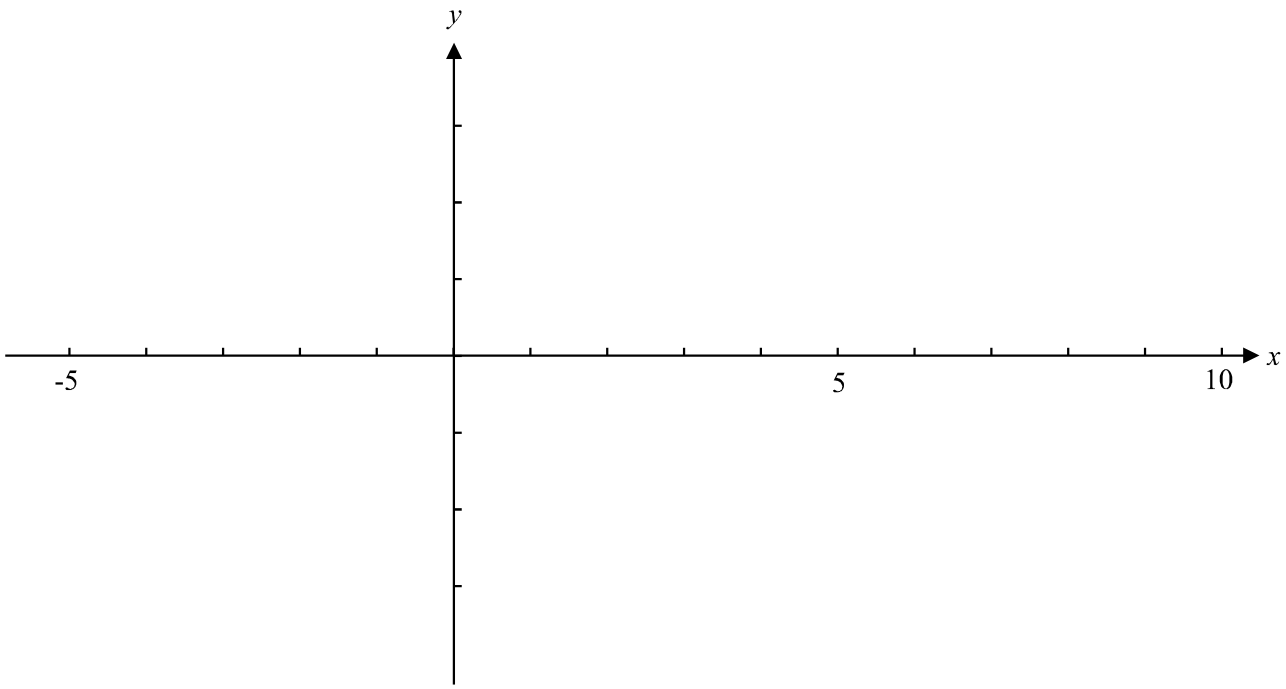
E. $\frac{-2g}{7}$

Part II

Question 1

Sketch the graph of $y = \dfrac{1}{2x^2 - 9x - 5}$ on the set of axes below.

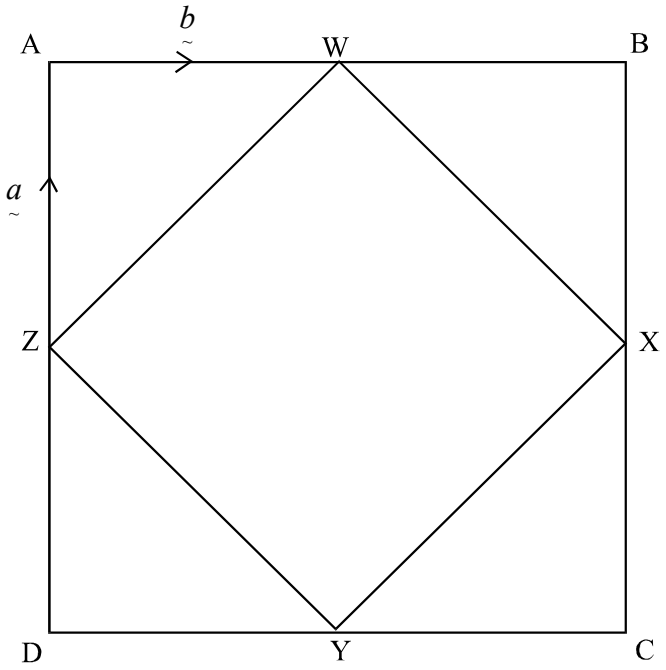
Indicate clearly any asymptotes. State, correct to 2 decimal places, the coordinates of any turning points or intercepts.



4 marks

Question 2

ABCD is a square. W, X, Y and Z are the midpoints of sides AB, BC, CD and DA respectively and $\vec{ZA} = \underset{\sim}{a}$ and $\vec{AW} = \underset{\sim}{b}$.



Show that WXYZ is a square.

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3 marks

Question 3

A toy box of mass 5 kg is resting on a rough level floor which has a coefficient of friction of $\frac{2}{g}$. A child applies a horizontal force of 8 Newtons to the box.
Explain whether or not the child will cause the toy box to move across the floor.

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3 marks

Question 4

a. Find the value of k for which $2i$ is a solution to the equation

$$z^4 + 2z^3 + 2kz^2 + 8z + 40 = 0 .$$

1 mark

b. Hence find the other solutions to the equation.

2 marks

Question 5

Use calculus to find the exact area of the region bounded by the graph of the equation $y = \sin^2 x \cos^3 x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{2}$.

3 marks

Question 6

A particle of mass1 kg is dropped from rest from the top of a tower and falls vertically, experiencing air resistance of $0.1v^2$. Now, v m/s represents the velocity of the particle, x m represents the position of the particle in relation to the top of the tower and g m/s^2 represents the acceleration due to gravity.

a. Taking the downwards direction as positive, show that

$$\frac{dx}{dv} = \frac{10v}{10g - v^2}$$

1 mark

b. Find the distance below the top of the tower for which the velocity of the particle is 1 m/s.
Express your answer correct to 2 significant figures.

3 marks
Total 20 marks

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2001

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:.....

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: (A) (■) (C) (D) (E)

The answer selected is B. Only one answer should be selected.

1.	A	B	C	D	E	11.	A	B	C	D	E	21.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E	22.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E	23.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E	24.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E	25.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E	26.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E	27.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E	28.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E	29.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E	30.	A	B	C	D	E