The Mathematical Association of Victoria

2001

MATHEMATICS: SPECIALIST

Trial Examination 1

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ______________________________________

Directions to students

This examination has two parts:
Part I (multiple-choice questions) and **Part II** (short-answer questions).

Answer all questions in **Part I** on the multiple-choice answer sheet provided. There are **30 marks** available for this part.

Part II consists of six questions. Answer all questions in **Part II** in the spaces provided. There are **20 marks** available for this part.

There are **50 marks** available for this task.

A formula sheet is attached.

These questions have been produced to assist students in their preparation for the 2001 Specialist Mathematics Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority (VCAA) Assessing Panels. The Association gratefully acknowledges the permission of the VCAA to reproduce the formula sheet.

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Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigometric) functions

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$ $\qquad$ $\cot^2 x + 1 = \operatorname{cosec}^2 x$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$ $\qquad$ $\sin(x-y) = \sin x\cos y - \cos x\sin y$

$\cos(x+y) = \cos x\cos y - \sin x\sin y$ $\qquad$ $\cos(x-y) = \cos x\cos y + \sin x\sin y$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$ $\qquad$ $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin 2x = 2\sin x\cos x$ $\qquad$ $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis} n\theta$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e x) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \log_e x + c$, for $x > 0$
$\frac{d}{dx}(\sin ax) = a\cos ax$	$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$
$\frac{d}{dx}(\cos ax) = -a\sin ax$	$\int \cos ax\,dx = \frac{1}{a}\sin ax + c$
$\frac{d}{dx}(\tan ax) = a\sec^2 ax$	$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + c$
$\frac{d}{dx}\left(\mathrm{Sin}^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \mathrm{Sin}^{-1}\frac{x}{a} + c, a > 0$
$\frac{d}{dx}\left(\mathrm{Cos}^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \mathrm{Cos}^{-1}\frac{x}{a} + c, a > 0$
$\frac{d}{dx}\left(\mathrm{Tan}^{-1}x\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}\,dx = \mathrm{Tan}^{-1}\frac{x}{a} + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)\,dx \approx (b-a)f\left(\frac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)\,dx \approx \frac{1}{2}(b-a)\left(f(a) + f(b)\right)$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h\,f(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET

Multiple-Choice Answer Sheet

Student's Name ______________________________

Circle the letter that corresponds to each answer.

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**
23.	**A**	**B**	**C**	**D**	**E**
24.	**A**	**B**	**C**	**D**	**E**
25.	**A**	**B**	**C**	**D**	**E**
26.	**A**	**B**	**C**	**D**	**E**
27.	**A**	**B**	**C**	**D**	**E**
28.	**A**	**B**	**C**	**D**	**E**
29.	**A**	**B**	**C**	**D**	**E**
30.	**A**	**B**	**C**	**D**	**E**

Mathematical Association of Victoria
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SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Part I
MULTIPLE CHOICE QUESTION BOOK

Question 1

If $\overrightarrow{OA} = 2\underset{\sim}{i} - \underset{\sim}{j} - 3\underset{\sim}{k}$ and $\overrightarrow{OB} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$, then $\overrightarrow{AB}$ is equal to

A. $\underset{\sim}{i} + 3\underset{\sim}{j} + 3\underset{\sim}{k}$

B. $-\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$

C. 3

D. $\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$

E. $\underset{\sim}{i} + 3\underset{\sim}{j} - 3\underset{\sim}{k}$

Question 2

The set of points in the complex plane defined by $|z-1| = |z+3|$ is

A. the circle with centre (1, 0) and radius 3
B. the circle with centre (1 , –3) and radius 1
C. the point $z = -1$
D. the line $\text{Re}(z) = -1$
E. the line $\text{Re}(z) = 1$

Question 3

The implied domain of the function with rule $f(x) = 2 + \text{Sin}^{-1}(\frac{x}{2} + 1)$ is

A. $[-6, -2]$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C. $[-4, 0]$

D. $[-1, 0]$

E. $\left[0, \frac{1}{2}\right]$

Question 4

If $\cos x = -\frac{2}{\sqrt{5}}$ and $\frac{\pi}{2} < x < \pi$, then $\sin x =$

A. $\frac{\sqrt{21}}{\sqrt{5}}$

B. $-\frac{\sqrt{21}}{\sqrt{5}}$

C. $\frac{1}{5}$

D. $\frac{1}{\sqrt{5}}$

E. $-\frac{1}{\sqrt{5}}$

Question 5

Which of the following is a polar form of $-\sqrt{3} - i$?

A. $2cis\left(\frac{\pi}{6}\right)$

B. $2cis\left(\frac{5\pi}{6}\right)$

C. $2cis\left(-\frac{5\pi}{6}\right)$

D. $2cis\left(\frac{\pi}{3}\right)$

E. $2cis\left(-\frac{2\pi}{3}\right)$

Question 6

An antiderivative of $\frac{2}{1 + 9x^2}$ is

A. $2\text{Tan}^{-1}(3x)$

B. $\frac{2}{9}\text{Tan}^{-1}(3x)$

C. $\frac{2}{3}\text{Tan}^{-1}(3x)$

D. $\frac{2}{3}\text{Tan}^{-1}\left(\frac{x}{3}\right)$

E. $2\text{Tan}^{-1}\left(\frac{x}{3}\right)$

Question 7

With a suitable substitution $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(3x)e^{\cos(3x)}\,dx$ can be expressed as

A. $\frac{1}{3}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} e^u du$

B. $-\frac{1}{3}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} e^u du$

C. $-\frac{1}{3}\int_0^1 e^u du$

D. $-\frac{1}{3}\int_{-1}^0 e^u du$

E. $\frac{1}{3}\int_{-1}^0 e^u du$

Question 8

A particle of mass 2 kg is acted on by a resultant force of $(6\underset{\sim}{i} - 2\underset{\sim}{j})$ newtons. The magnitude of the particle's acceleration in ms^{-2} is

A. $\sqrt{10}$

B. 4

C. $2\sqrt{2}$

D. $4\sqrt{5}$

E. $2\sqrt{10}$

Question 9

If $h'(x) = 2x\sqrt{1-x^2}$ and $h(0) = 1$, then $h(x) =$

A. $\frac{2}{3}(1-x^2)^{\frac{3}{2}}$

B. $\frac{2}{3}(1-x^2)^{\frac{3}{2}} - \frac{1}{3}$

C. $\frac{2}{3}(1-x^2)^{\frac{3}{2}} + \frac{5}{3}$

D. $\frac{5}{3} - \frac{2}{3}(1-x^2)^{\frac{3}{2}}$

E. $\frac{1}{3} - \frac{2}{3}(1-x^2)^{\frac{3}{2}}$

Question 10

The position vector of a particle at time t is given by $\underset{\sim}{r}(t) = 2e^{-3t}\,\underset{\sim}{i} + \frac{5}{2}\sin(3t)\,\underset{\sim}{j}$. The speed of the particle at time $t = 0$ is

A. $\frac{3\sqrt{41}}{2}$

B. $\frac{\sqrt{241}}{2}$

C. 2

D. $\frac{13}{2}$

E. $\frac{3}{2}$

Question 11

If $f(x) = 2x^2 + x - 15$, then the graph of $\frac{1}{f(x)}$ has

A. x-intercepts at $x = -3$ and $x = \frac{5}{2}$

B. asymptotes at $x = -3$ and $x = \frac{5}{2}$

C. asymptotes at $x = -3$ and $x = \frac{2}{5}$

D. a local minimum at the point $\left(-\frac{1}{4}, -\frac{1}{4}\right)$

E. a local maximum at the point $\left(-\frac{1}{4}, -15\frac{1}{4}\right)$

Question 12

By a suitable substitution $\int \sin^3(2x)\,dx$ can be expressed as:

A. $-2\int (1-u)^2\ du$

B. $2\int (1-u^2)\ du$

C. $-\frac{1}{2}\int (u-1)^2\,du$

D. $-\frac{1}{2}\int (1-u^2)\,du$

E. $\frac{1}{2}\int (1-u)^2\,du$

Question 13

An approximation to $\int_{-2}^{2} \sqrt{2-x}\, dx$ using the trapezoidal rule with two equal intervals is

A. 9.6569
B. 5.4641
C. 5.3334
D. 4.8284
E. 3.4142

Question 14

An antiderivative of $\dfrac{3x}{2x^2+3}$ is

A. $\frac{1}{4}\log_e(2x^2+3)$

B. $\frac{3}{4}\log_e(2x^2+3)$

C. $3\log_e(2x^2+3)$

D. $\frac{3}{2}\operatorname{Tan}^{-1}\left(x\sqrt{\frac{3}{2}}\right)$

E. $\frac{4}{3}\log_e(2x^2+3)$

Question 15

A vat initially holds 40 L of water. A salt solution of concentration 3 g L^{-1} is poured into the vat at a rate of 5 L min^{-1}. The mixture is kept uniform by stirring and flows out of the container at a rate of 2 L min^{-1}. If Q grams is the amount of salt in the vat t minutes after the start of pouring, then a differential equation for Q is

A. $\dfrac{dQ}{dt} = 15 - \dfrac{Q}{40}$

B. $\dfrac{dQ}{dt} = 15 - \dfrac{2Q}{40+t}$

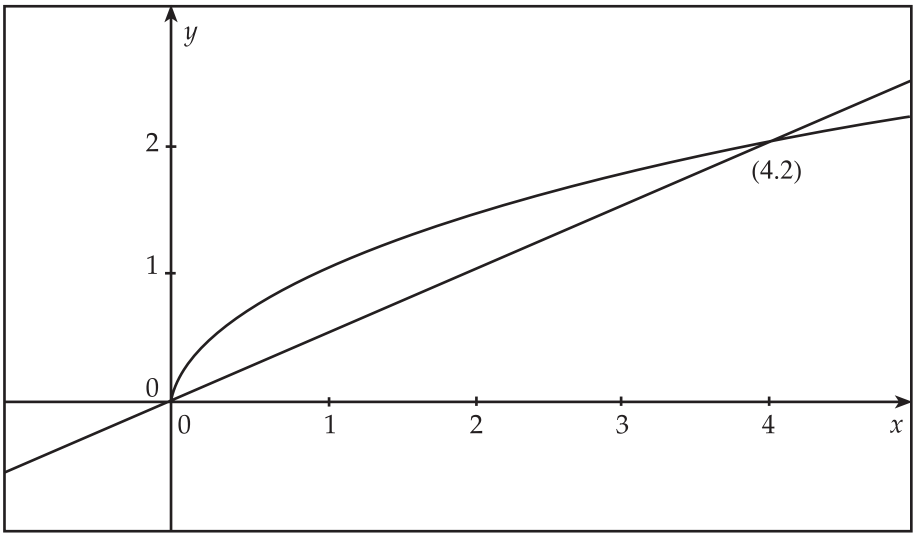
C. $\dfrac{dQ}{dt} = 15 - \dfrac{2Q}{40+3t}$

D. $\dfrac{dQ}{dt} = -\dfrac{Q}{3t}$

E. $\dfrac{dQ}{dt} = -\dfrac{3Q}{40}$

Question 16

The region bounded by the curve $y = \sqrt{x}$ and the straight line $y = \frac{1}{2}x$ as shown in the diagram below is rotated about the y-axis to form a solid of revolution. The volume of this solid in cubic units is given by



A. $\pi\int_0^2 (4y^2 - y^4)dy$

B. $\pi\int_0^4 (4y^2 - y^4)dy$

C. $\pi\int_0^2 (y^4 - 4y^2)dy$

D. $\pi\int_0^4 (y^4 - 4y^2)dy$

E. $\pi\int_0^2 (y^4 - 2y^2)dy$

Question 17

If $y = \log_e(\sin x)$, then

A. $\frac{2}{\sin 2x}\frac{dy}{dx} - \frac{d^2y}{dx^2} = 0$

B. $\frac{1}{\sin 2x}\frac{dy}{dx} + \frac{d^2y}{dx^2} = 0$

C. $\frac{2}{\sin 2x}\frac{dy}{dx} + \frac{d^2y}{dx^2} = 0$

D. $\frac{dy}{dx} + 2\frac{d^2y}{dx^2} = 0$

E. $\frac{1}{\sin x}\frac{dy}{dx} + \frac{d^2y}{dx^2} = \cos x$

Question 18

If $w = 2cis\left(\frac{3\pi}{4}\right)$ and $z = 3cis\left(\frac{\pi}{3}\right)$, then wz is equal to

A. $5cis\left(\frac{\pi^2}{4}\right)$

B. $6cis\left(\frac{\pi^2}{4}\right)$

C. $6cis\left(-\frac{11\pi}{12}\right)$

D. $5cis\left(-\frac{11\pi}{12}\right)$

E. $5cis\left(\frac{13\pi}{12}\right)$

Question 19

The position of a particle at time t is given by $\underset{\sim}{r}(t) = 3\cos(2t)\underset{\sim}{i} + 4\sin(2t)\underset{\sim}{j}, \quad t \geq 0$

The Cartesian equation of the path of the particle is

A. $x^2 + y^2 = 7, \ -\sqrt{7} \leq x \leq \sqrt{7}$

B. $x^2 + y^2 = 25, \ -5 \leq x \leq 5$

C. $3x + 4y = 1, \ x \geq 0$

D. $16x^2 + 9y^2 = 144, \ 0 \leq x \leq 3$

E. $16x^2 + 9y^2 = 144, \ -3 \leq x \leq 3$

Question 20

Euler's method, with a step size of 0.1 is used to solve the differential equation $\frac{dy}{dx} = 3x^2 - 1$ with $y = 3$ at $x = 1$. The value obtained for y at $x = 1.2$, correct to three decimal places, is

A. 3.200
B. 3.263
C. 3.463
D. 3.528
E. 3.532

Question 21

A projectile is launched with a speed of $4\sqrt{3}$ ms^{-1} at an angle of 30° to the horizontal. If $\underset{\sim}{i}$ is horizontally to the right and $\underset{\sim}{j}$ vertically upwards in the plane of motion of the projectile, then the initial velocity of the projectile in ms^{-1} is

A. $8\sqrt{3}\,\underset{\sim}{i} + 8\,\underset{\sim}{j}$

B. $8\sqrt{3}\,\underset{\sim}{i} + 2\,\underset{\sim}{j}$

C. $4\sqrt{3}\,\underset{\sim}{i} + 4\sqrt{3}\,\underset{\sim}{j}$

D. $6\,\underset{\sim}{i} + 2\sqrt{3}\,\underset{\sim}{j}$

E. $2\sqrt{3}\,\underset{\sim}{i} + 6\,\underset{\sim}{j}$

Question 22

If $\underset{\sim}{a} = 5\,\underset{\sim}{i} - 3\,\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j}$, the vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$ is

A. $\underset{\sim}{i} + \underset{\sim}{j}$

B. $6\,\underset{\sim}{i} - 2\,\underset{\sim}{j}$

C. $\frac{1}{17}(7\,\underset{\sim}{i} + 23\,\underset{\sim}{j})$

D. $4\,\underset{\sim}{i} - 4\,\underset{\sim}{j}$

E. $-4\,\underset{\sim}{i} + 4\,\underset{\sim}{j}$

Question 23

The acceleration of a body at time t seconds is given by $\frac{dv}{dt} = \frac{4}{v}$ cm s^{-2}, where v is the velocity of the body at time t. If the initial velocity of the body is –2 cm s^{-1}, the velocity of the body at time t is

A. $2\sqrt{2t + 1}$

B. $-2\sqrt{2t + 1}$

C. $4\log_e\left(-\frac{t}{2}\right)$

D. $2t + 4$

E. $8(t + 2)$

Question 24

The angle between the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$ is closest to

A. 0.75°
B. 42.83°
C. 71.68°
D. 86.69°
E. 87.20°

Question 25

If $y = \text{Cos}^{-1}\dfrac{3}{2x}$ and $x > \dfrac{3}{2}$, then $\dfrac{dy}{dx} =$

A. $\dfrac{3}{x\sqrt{4x^2 - 9}}$

B. $\dfrac{-3}{x\sqrt{4x^2 - 9}}$

C. $\dfrac{-2}{\sqrt{9 - 4x^2}}$

D. $\dfrac{2}{\sqrt{9 - 4x^2}}$

E. $\dfrac{-3}{\sqrt{1 - 4x^2}}$

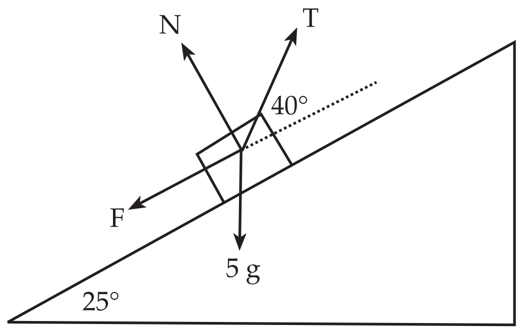
Question 26

A man of mass 75 kg is standing in a lift, which is moving with an upwards acceleration of 2.5 ms^{-2}. The magnitude of the force in newtons exerted by the floor of the lift on the man, is

A. $75(g + 2.5)$
B. $75(g - 2.5)$
C. $75g$
D. 187.5
E. $187.5g$

Question 27

The diagram shows all the forces acting on a block of mass 5 kg, which is being pulled up a rough inclined plane at constant speed. The plane is inclined at 25° to the horizontal and the coefficient of friction between the block and the plane is 0.2. The block is being pulled by a rope which makes an angle of 40° with the plane. T newtons is the tension in the rope, N newtons is the normal reaction of the plane on the block and F newtons is the frictional force.



Resolving forces parallel to the plane gives

A. $T\cos 40° = 5g\cos 25° - N$

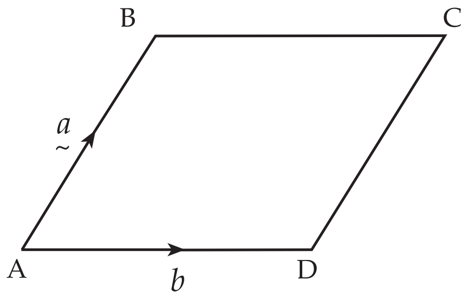
B. $T\cos 40° = 5g\sin 25°$

C. $T\cos 40° = 5g\sin 25° + 2g$

D. $T\cos 40° = 0.2N + 5g\cos 25°$

E. $T\cos 40° = 0.2N + 5g\sin 25°$

Question 28



To prove that the diagonals of the rhombus ABCD are perpendicular, it is sufficient to show:

A. $(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = 0$

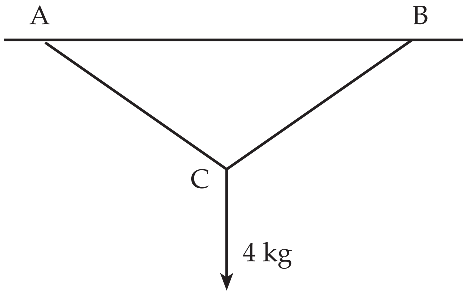
B. $(\underset{\sim}{a}+\underset{\sim}{b})-(\underset{\sim}{a}-\underset{\sim}{b}) = 0$

C. $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{AD} = \overrightarrow{BC}$

D. $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right|$

E. $\underset{\sim}{a}\cdot\underset{\sim}{b} = 0$

Question 29



A light inelastic string is attached to two points A and B, which are in a horizontal line. A particle of mass 4 kg is attached to the string at point C by means of a smooth ring and hangs in equilibrium. AC and BC each make an angle of θ with the horizontal. The tension in the string, measured in newtons is

A. $\dfrac{2}{\sin\theta}$

B. $\dfrac{2g}{\cos\theta}$

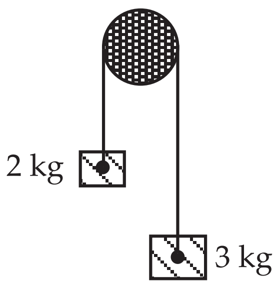
C. $\dfrac{g}{\cos\theta}$

D. $\dfrac{2g}{\sin\theta}$

E. $\dfrac{g}{2\sin\theta}$

Question 30

Two masses are connected by a light inelastic string that passes over a smooth pulley as shown. The acceleration due to gravity has magnitude g ms^{-2}. If a ms^{-2} is the magnitude of the acceleration of each mass, then a equals



A. $5g$

B. $\dfrac{5g}{2}$

C. g

D. $\dfrac{g}{2}$

E. $\dfrac{g}{5}$

SPECIALIST MATHEMATICS

PART II

Question 1

a. AB is the diameter of a circle whose centre is O. C is another distinct point on that circle.

Given that $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. Express $\overrightarrow{AC}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$

2 marks

b. Hence, show that the angle ACB is a right angle.

3 marks

Total marks: 5

Question 2

Two sets of traffic lights, on a straight road are separated by a distance of 1.6 kilometres. A car accelerates from rest from the first set of lights with an acceleration of 0.25 m/s^2 until it reaches a speed of 20 m/s. It maintains this speed until it decelerates to rest at the second set of lights, at 0.5 m/s^2.

a. Draw a velocity-time graph to represent this journey.

2 marks

b. Hence, find the time taken to complete the journey.

2 marks

Total marks: 4

Question 3

Two solutions of a polynomial equation $P(z) = 0$ are $z = 2$ and $z = \sqrt{3}i$.

a. Find, in expanded form, a polynomial of the lowest degree with these solutions.

1 mark

b. Find, in expanded form, a polynomial of the lowest degree with real coefficients that have these solutions.

2 marks

Total marks: 3

Question 4

Use calculus to evaluate $\int_{-\frac{1}{2}}^{\frac{3}{2}} x\sqrt{1+2x}\,dx$

4 marks

Question 5

Use trigonometric identities to find the exact values of $\sin\frac{\pi}{8}$ and $\cos\frac{15\pi}{8}$.

4 marks