VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2001

SPECIALIST MATHEMATICS

Written examination 1
(Facts, skills and applications)

Monday 5 November 2001

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I

MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this book.

At the end of the examination
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
You may keep this question book.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

Materials
- Question book of 19 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.
- Up to four pages (two A4 sheets) of pre-written notes (typed or handwritten).
- An approved scientific and/or graphics calculator, ruler, protractor, set square and aids for curve sketching.
- At least one pencil and an eraser.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name and student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

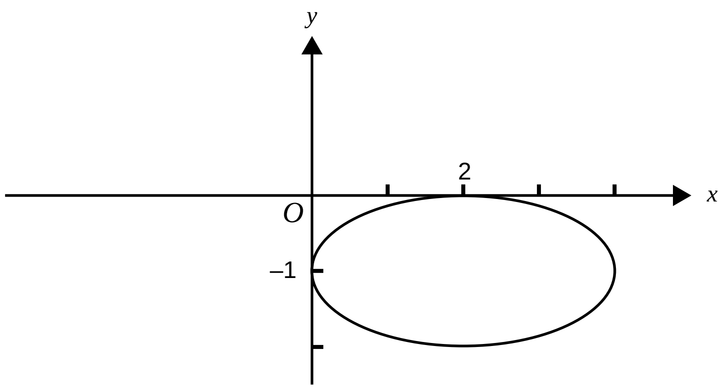
At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).

Instructions for Part 1

Answer **all** questions on the answer sheet provided for multiple-choice questions.

A correct answer scores 1; an incorrect answer scores 0. Marks will not be deducted for incorrect answers. No credit will be given if more than one answer is completed for any question.

Question 1



The equation for the ellipse shown is

A. $\dfrac{(x+2)^2}{4}+(y-1)^2=1$

B. $\dfrac{(x-2)^2}{4}+(y-1)^2=1$

C. $\dfrac{(x-2)^2}{2}+(y+1)^2=1$

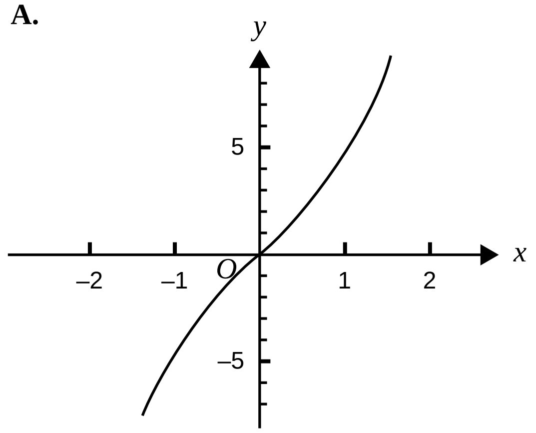
D. $\dfrac{(x-2)^2}{4}+\dfrac{(y+1)^2}{2}=1$

E. $\dfrac{(x-2)^2}{4}+(y+1)^2=1$

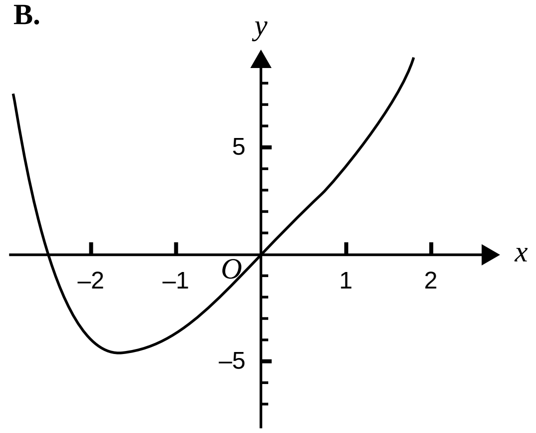
Question 2

Which one of the following shows part of the graph of $f: R \rightarrow R$ where $f(x) = \dfrac{x^3 + 16}{4x}$?

A.



B.



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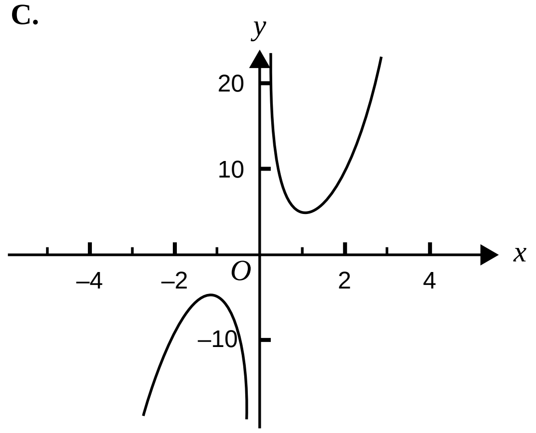

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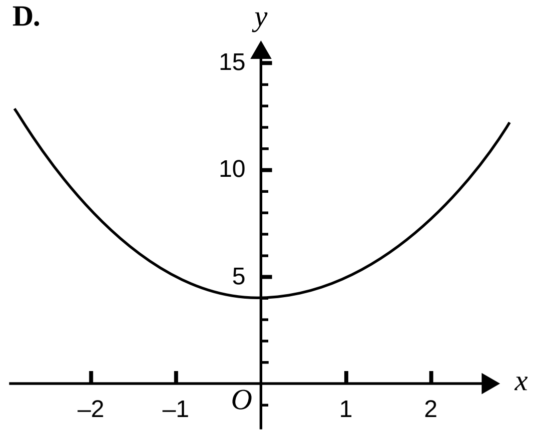

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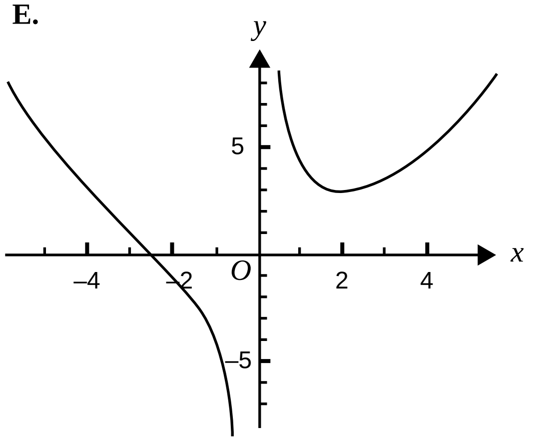


Question 3

If $y = \mathrm{Tan}^{-1}\left(\frac{1}{2x}\right)$, then $\frac{dy}{dx} =$

A. $\frac{1}{2(x^2+1)}$

B. $-\frac{1}{2(x^2+1)}$

C. $\frac{1}{2(4x^2+1)}$

D. $-\frac{2}{4x^2+1}$

E. $\frac{4x^2}{4x^2+1}$

Question 4

If $\cos(x) = -\frac{1}{10}$ and $\frac{\pi}{2} < x < \pi$, then cosec(x) equals

A. $\frac{10\sqrt{11}}{33}$

B. $\frac{10}{\sqrt{101}}$

C. $\frac{\sqrt{101}}{10}$

D. $-\frac{10}{3\sqrt{11}}$

E. –10

Question 5

Which one of the following is a polar form of $\sqrt{3}\,i-1$?

A. $2\ \mathrm{cis}\left(-\frac{4\pi}{3}\right)$

B. $2\ \mathrm{cis}\left(-\frac{\pi}{6}\right)$

C. $2\ \mathrm{cis}\left(\frac{\pi}{3}\right)$

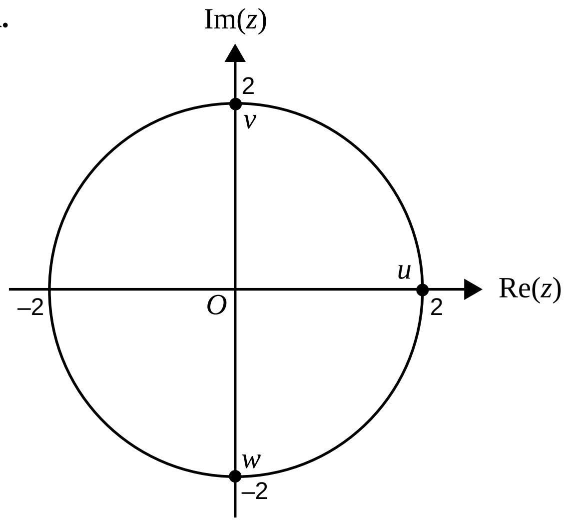
D. $4\ \mathrm{cis}\left(\frac{2\pi}{3}\right)$

E. $2\ \mathrm{cis}\left(\frac{5\pi}{6}\right)$

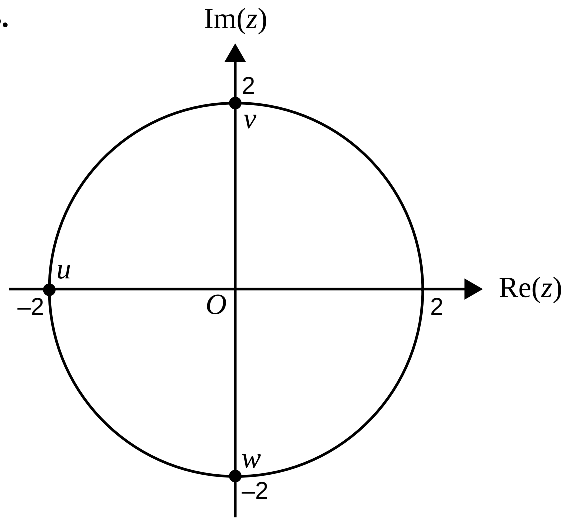
Question 6

The roots u, v and w, of the equation $z^3 - 8 = 0$, where $z \in C$, are shown on which one of the following Argand diagrams?

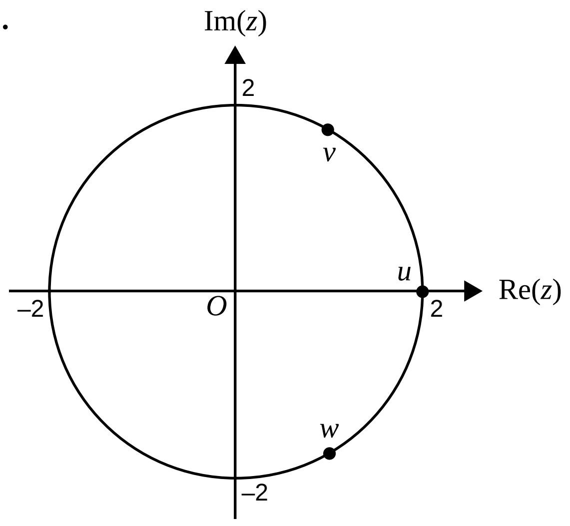
A.



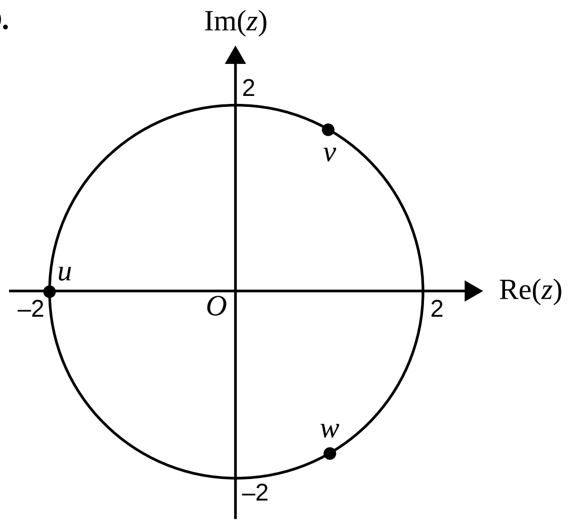
B.



C.



D.



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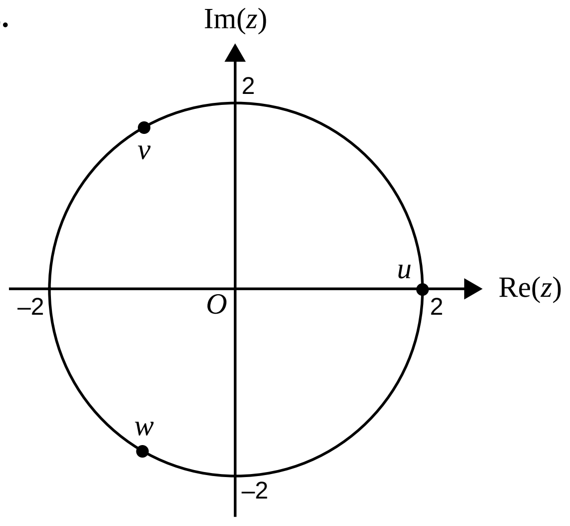


Question 7

$P(z)$ is a cubic polynomial with real coefficients.

If $z = 3i$ is a solution of $P(z) = 0$, then $P(z)$ could be

A. $z^3 + 9$

B. $z^3 - 9z$

C. $z^3 + 9z$

D. $z^3 - 27$

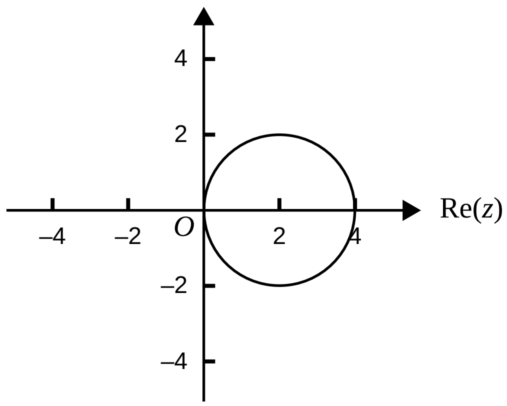
E. $z^3 + 27$

Working space

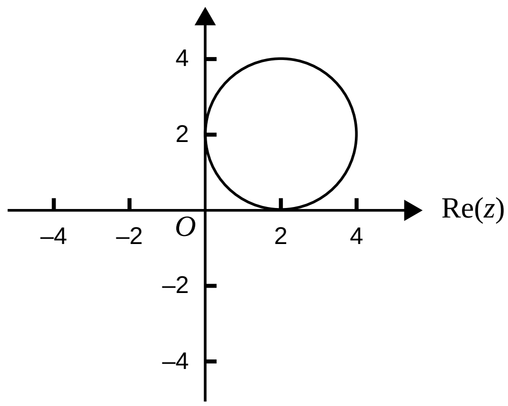
Question 8

Which one of the following shows the region of the complex plane specified by $\{z: (z-2)(\bar{z}-2) = 4, z \in C\}$?

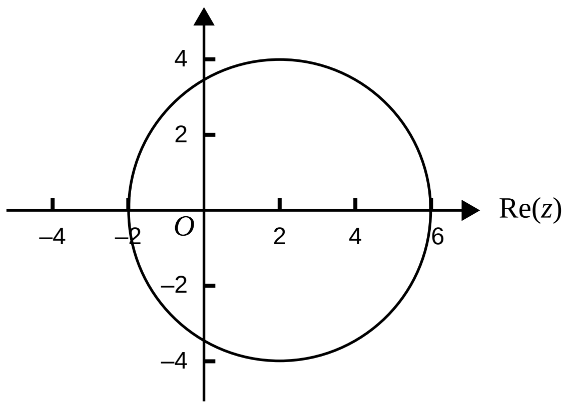
A.



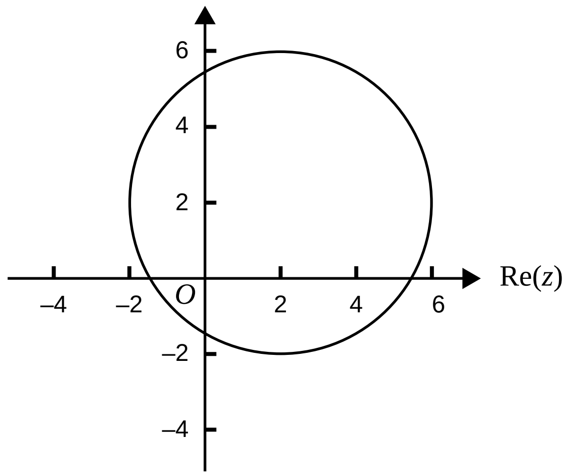
B.



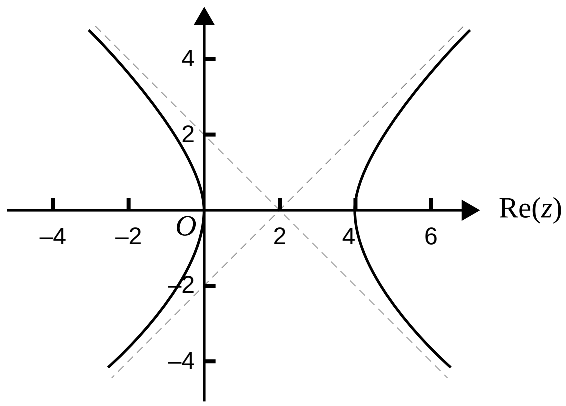
C.



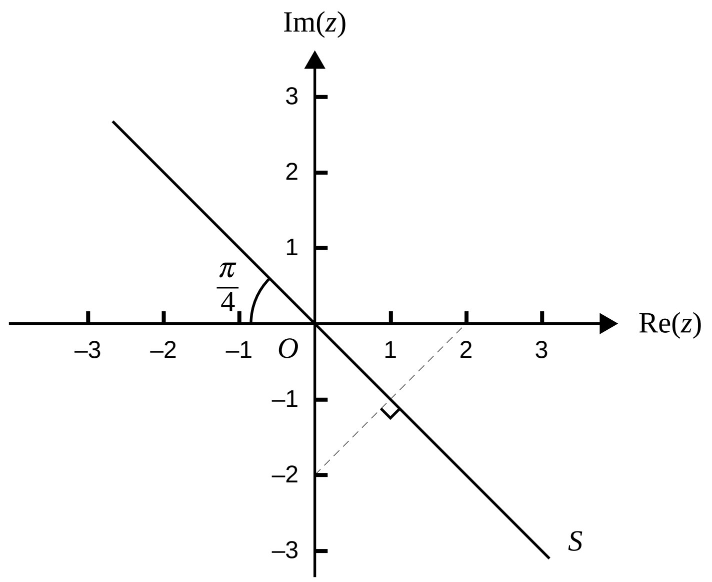
D.



E.



Question 9



Which one of the following, where $z \in C$, describes the line S in the diagram above?

A. $\text{Arg}(z) = \frac{\pi}{4}$

B. $\text{Arg}(z) = \frac{3\pi}{4}$

C. $\text{Re}(z) - \text{Im}(z) = 0$

D. $|z - 2| = |z - 2i|$

E. $|z - 2| = |z + 2i|$

Question 10

An antiderivative of $\dfrac{1}{x\ \log_e(2x)}$ is

A. $\log_e(2x)$

B. $\frac{1}{2}\log_e\left(\log_e(2x)\right)$

C. $\log_e\left(\log_e(2x)\right)$

D. $2\ \log_e\left(\log_e(2x)\right)$

E. $\log_e\left(x\ \log_e(2x)\right)$

Question 11

Using a suitable substitution, $\int_0^{\frac{\pi}{6}} \cos^2(2x)\cos(2x)\ dx$ can be expressed as

A. $\int_{\frac{1}{2}}^{1} u^2(1-2u^2)du$

B. $\frac{1}{2}\int_0^{\frac{\pi}{6}} (1-u^2)du$

C. $\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}} (1-u^2)du$

D. $\frac{1}{2}\int_0^{\frac{1}{2}} (1-u^2)du$

E. $2\int_0^{\frac{\sqrt{3}}{2}} (1-u^2)du$

Question 12

The exact value of $\int_{\sqrt{3}}^{3} \frac{3}{x^2+3}\,dx$ is

A. $3\log_e(2)$

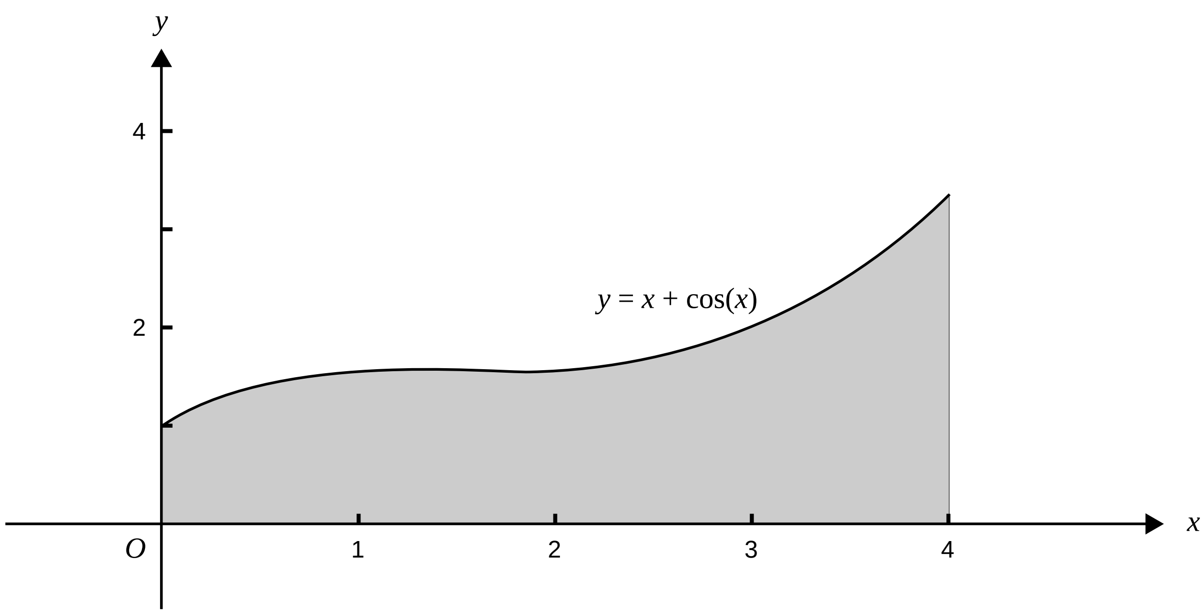
B. $\frac{\pi}{4}$

C. $\frac{\pi}{12}$

D. $\frac{\pi\sqrt{3}}{12}$

E. $\frac{\pi\sqrt{3}}{36}$

Question 13



The graph of $y = x + \cos(x)$, $0 \le x \le 4$, is shown above.

The midpoint rule with two equal intervals is used to approximate the area shaded.

The value obtained, correct to three decimal places, is

A. 6.335

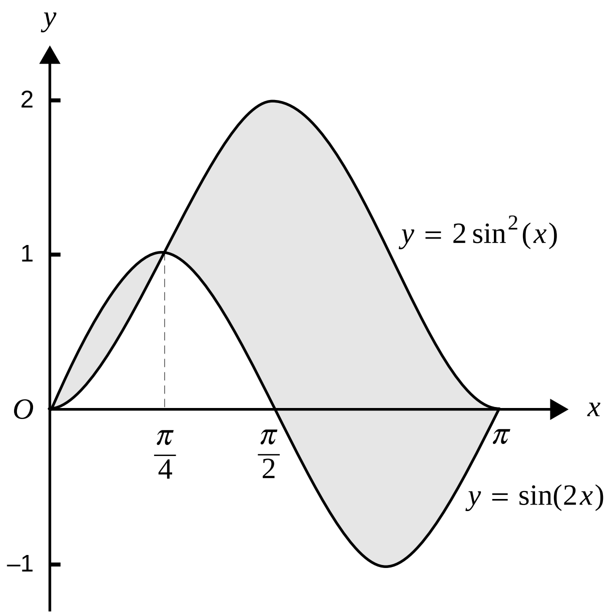
B. 7.101

C. 7.243

D. 7.514

E. 11.997

Question 14



The diagram above shows the curves with equations $y = 2\sin^2(x)$ and $y = \sin(2x)$, $0 \le x \le \pi$.
The total area of the shaded regions enclosed by the two curves for $0 \le x \le \pi$ is given by

A. $\displaystyle\int_0^{\pi} \left(\sin(2x) - 2\sin^2(x)\right)dx$

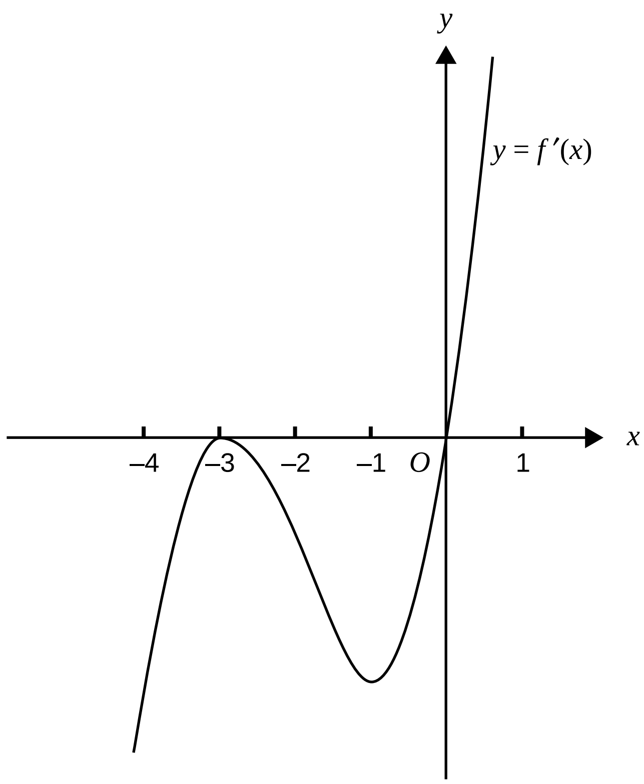
B. $\displaystyle\int_0^{\pi} \left(2\sin^2(x) - \sin(2x)\right)dx$

C. $\displaystyle\int_0^{\frac{\pi}{4}} \left(2\sin^2(x) - \sin(2x)\right)dx + \int_{\frac{\pi}{4}}^{\pi} \left(2\sin^2(x) - \sin(2x)\right)dx$

D. $\displaystyle\int_0^{\frac{\pi}{4}} \left(\sin(2x) - 2\sin^2(x)\right)dx - \int_{\frac{\pi}{4}}^{\pi} \left(\sin(2x) - 2\sin^2(x)\right)dx$

E. $\displaystyle\int_0^{\pi} 2\sin^2(x)\,dx - \int_0^{\frac{\pi}{2}} \sin(2x)dx - \int_{\frac{\pi}{2}}^{\pi} \sin(2x)dx$

Question 15



The graph of $y = f'(x)$ is shown above.

Which one of the following statements is true for the graph of $y = f(x)$?

A. The graph has a local maximum at $x = -3$ and a stationary point of inflexion at $x = 0$.

B. The graph has a local minimum at $x = -3$ and a stationary point of inflexion at $x = 0$.

C. The graph has a stationary point of inflexion at $x = -3$ and a local minimum at $x = 0$.

D. The graph has a stationary point of inflexion at $x = -3$ and a local maximum at $x = 0$.

E. The graph has a local maximum at $x = -3$, a local minimum at $x = -1$, and a stationary point of inflexion at $x = 0$.

Question 16

A particle travelling in a straight line has velocity v m/s at time t s.

Its acceleration is given by $\frac{dv}{dt} = -0.05(v^2 - 5)$.

Its velocity is 50 m/s initially and is reduced to 3 m/s.

Which one of the following is an expression for the time taken in seconds for this to occur?

A. $20 \int_{50}^{3} \frac{1}{v^2 - 5} \, dv$

B. $20 \int_{3}^{50} \frac{1}{v^2 - 5} \, dv$

C. $0.05 \int_{3}^{50} \frac{1}{v^2 - 5} \, dv$

D. $-0.05 \int_{50}^{3} (v^2 - 5) \, dv$

E. $-0.05 \int_{50}^{3} (v^2 - 5) \, dt$

Question 17

Euler's method, with a step size of 0.2, is used to approximate the solution of the differential equation $\left(\sqrt{1 + x^2}\right)\frac{dy}{dx} = 1$, with $y = 1$ at $x = 0$.

When $x = 0.4$, the value obtained for y, correct to four decimal places, is

A. 1.3280

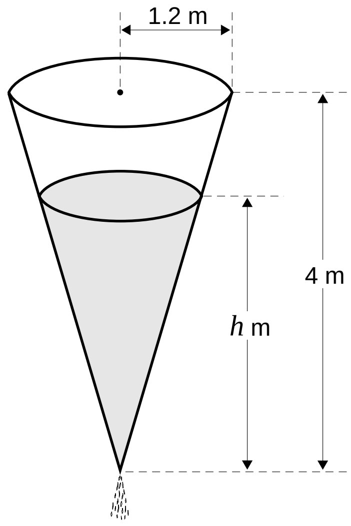
B. 1.3857

C. 1.3961

D. 1.4000

E. 1.4040

Question 18



An inverted cone, as shown in the diagram, is initially full of water. The water flows out through a hole at the bottom at the rate of $0.1\sqrt{h}$ m^3 per hour, where h m is the depth of water remaining after t hours. The volume V m^3 of water is given by $V = 0.03\pi h^3$.

At time t hours, $\frac{dh}{dt}$ is given by

A. $-0.9\pi h^{\frac{3}{2}}$

B. $0.9\pi h^{\frac{3}{2}}$

C. $-0.009\pi h^{\frac{5}{2}}$

D. $-\frac{1}{0.9\pi h^{\frac{3}{2}}}$

E. $\frac{1}{0.9\pi h^{\frac{3}{2}}}$

Question 19

A particle starts from rest and moves in a straight line with acceleration $6\sin(2t)$ m/s^2 at time t s.

Its displacement from its starting point, in metres, at time t is given by

A. $-24\sin(2t)$

B. $1.5\sin(2t) - 3t$

C. $3 - 3\cos(2t)$

D. $-1.5\sin(2t)$

E. $3t - 1.5\sin(2t)$

Question 20

A particle moves in a straight line so that at time t, $t \geq 0$, its velocity is v and its displacement from a fixed point on the line is x.

If $\frac{dv}{dx} = \frac{1}{v}$, then the particle moves with

A. constant acceleration and constant velocity.

B. constant acceleration and increasing velocity.

C. constant acceleration and decreasing velocity.

D. increasing acceleration and decreasing velocity.

E. decreasing acceleration and increasing velocity.

Question 21

A vector parallel to $\underset{\sim}{i} - 2\underset{\sim}{j} + 5\underset{\sim}{k}$ and with magnitude 6 is

A. $6(\underset{\sim}{i} - 2\underset{\sim}{j} + 5\underset{\sim}{k})$

B. $\frac{\sqrt{30}}{5}(\underset{\sim}{i} - 2\underset{\sim}{j} + 5\underset{\sim}{k})$

C. $-\frac{6}{\sqrt{30}}(\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k})$

D. $\frac{1}{5}(\underset{\sim}{i} - 2\underset{\sim}{j} + 5\underset{\sim}{k})$

E. $\frac{6}{\sqrt{22}}(\underset{\sim}{i} - 2\underset{\sim}{j} + 5\underset{\sim}{k})$

Question 22

The vector resolute of $3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ in the direction of $-\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$ is

A. $-\frac{7}{12}(-\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k})$

B. $-\frac{1}{2}(3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k})$

C. $\frac{7}{\sqrt{14}}(\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k})$

D. $-\frac{7}{\sqrt{14}}(3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k})$

E. $\frac{1}{2}(\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k})$

Question 23

If the vectors defined by $\underset{\sim}{a} = -m\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} - n\underset{\sim}{j} - 6\underset{\sim}{k}$ are perpendicular, then which one of the following could be true?

A. $m = -2$, $n = 2$

B. $m = -2$, $n = -2$

C. $m = -2$, $n = -4$

D. $m = 2$, $n = 2$

E. $m = 2$, $n = -2$

Question 24

Consider non-zero vectors $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$. If $\underset{\sim}{p} = 2\underset{\sim}{q} - 3\underset{\sim}{r}$, which one of the following statements **must** be true?

A. $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ are linearly dependent.

B. $\underset{\sim}{q}$ and $\underset{\sim}{r}$ are linearly dependent.

C. $\underset{\sim}{q}$ is parallel to $\underset{\sim}{r}$.

D. $\underset{\sim}{q}$ and $\underset{\sim}{r}$ are linearly independent.

E. $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ are linearly independent.

Question 25

The position vector of a particle at time t, $t \geq 0$, is given by $\underset{\sim}{r}(t) = (t - 3)\underset{\sim}{i} - (\sqrt{t})\underset{\sim}{j}$.

The particle is closest to the origin when t is

A. 0

B. 1.5

C. 2.5

D. 3

E. 3.5

Question 26

The displacement of a particle from the origin at time t, $t \geq 0$, is given by $\underset{\sim}{r}(t) = e^{-2t}\underset{\sim}{i} + (\sin(\pi t))\underset{\sim}{j} + 2\underset{\sim}{k}$.

The initial direction of motion of the particle is

A. $4\underset{\sim}{i}$

B. $\underset{\sim}{i} + 2\underset{\sim}{k}$

C. $-2\underset{\sim}{i} + \underset{\sim}{j}$

D. $-2\underset{\sim}{i} + \pi\underset{\sim}{j}$

E. $-2\underset{\sim}{i} + \pi\underset{\sim}{j} + 2\underset{\sim}{k}$

Question 27

A particle is subject to two forces, one of 4 newtons acting due east, the other of 2 newtons acting at a bearing of N30°E.

The magnitude of the resultant force, in newtons, acting on the particle is

A. $2\sqrt{3}$

B. $2\sqrt{7}$

C. 6

D. $5 + \sqrt{3}$

E. $\sqrt{20 + 8\sqrt{3}}$

Question 28

Initially, a particle has velocity $-6\underset{\sim}{i}$ m/s. A constant force acts on the particle so that after 2 s its velocity is $8\underset{\sim}{j}$ m/s.

The acceleration, in m/s^2, of the particle is

A. 7

B. $3\underset{\sim}{i} + 4\underset{\sim}{j}$

C. $-3\underset{\sim}{i} + 4\underset{\sim}{j}$

D. $3\underset{\sim}{i} - 4\underset{\sim}{j}$

E. $-3\underset{\sim}{i} - 4\underset{\sim}{j}$

Question 29

A box of mass m kilograms rests on rough, level ground. The box is pulled with a force of magnitude P newtons at an angle of 20° to the horizontal. There is a normal reaction of magnitude N newtons and the coefficient of friction between the box and the ground is μ.

If the box is on the point of sliding along the ground, which one of the following equations is correct?

A. $N = mg$

B. $N = mg\cos(20°)$

C. $P\sin(20°) = mg$

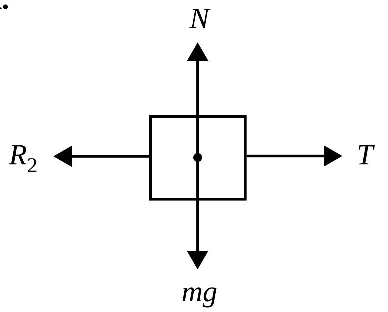
D. $P\cos(20°) = \mu N$

E. $P\sin(20°) = \mu N$

Question 30

A car pulls a caravan of mass m kg along a straight, level road. There are resistance forces of magnitude R_1 newtons on the car and R_2 newtons on the caravan. The tension in the towbar has magnitude T newtons and there is a normal reaction on the caravan of magnitude N newtons. Which one of the following diagrams shows the forces acting on the **caravan**?

A.



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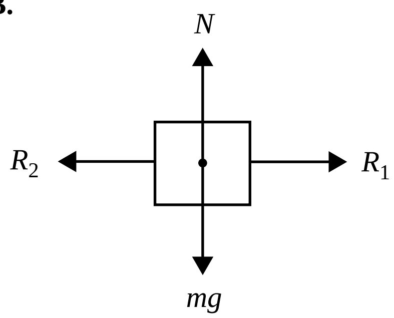
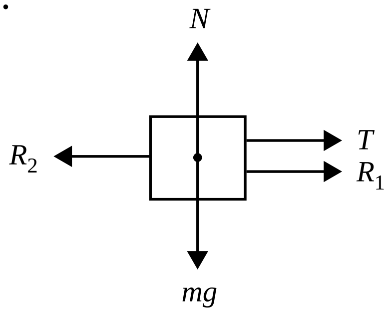
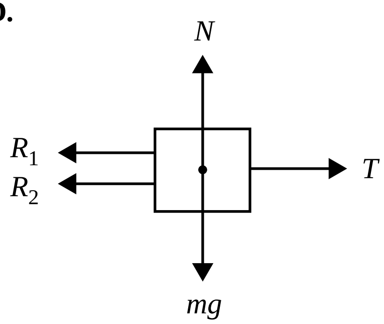


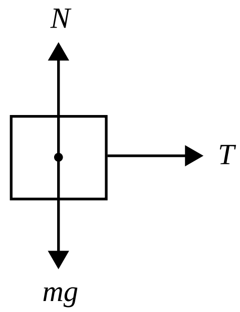
C.



D.



E.



END OF PART I MULTIPLE-CHOICE QUESTION BOOK

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter:
Words									

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2001

SPECIALIST MATHEMATICS

Written examination 1

(Facts, skills and applications)

Monday 5 November 2001

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART II

QUESTION AND ANSWER BOOK

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- Working space is provided throughout the book.
- Up to four pages (two A4 sheets) of pre-written notes (typed or handwritten).
- An approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve sketching.

Instructions
- Detach the formula sheet from the centre of the Part I question book during reading time.
- Write your **student number** in the space provided on the cover of this book.
- A decimal approximation will not be accepted if an exact answer is required to a question.
- Where an exact answer is required to a question, appropriate working must be shown.
- Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Instructions for Part II

Answer **all** questions in this part in the spaces provided.

Question 1

Find the **exact** solutions of $\sin(4x) = \cos(2x)$, for $0 \le x \le \pi$.

3 marks

Working space

Question 2

a. Express $\dfrac{2x-1}{x^2+6x+9}$ in partial fraction form.

2 marks

b. Hence write down an antiderivative of $\dfrac{2x-1}{x^2+6x+9}$, $x > -3$.

1 mark

c. Evaluate $\displaystyle\int_{-2}^{4} \frac{2x-1}{x^2+6x+9}\,dx$, correct to three significant figures.

1 mark

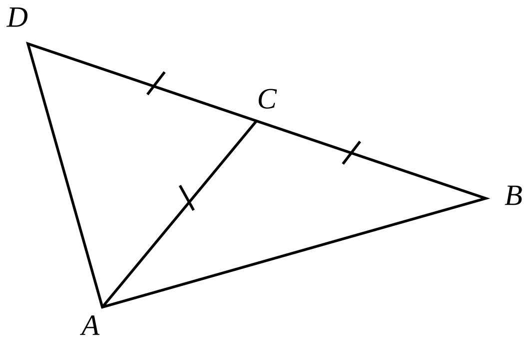
Question 3

Find the cube roots of $-2 + 2i$ in Cartesian form, giving the real and imaginary parts of each root correct to three decimal places.

3 marks

Working space

Question 4



In triangle ABD, $AC = CD = CB$. Let $\overrightarrow{AC} = \underset{\sim}{u}$ and $\overrightarrow{BC} = \underset{\sim}{v}$.

Use a vector method to prove that $\angle BAD$ is a right angle.

3 marks

Question 5

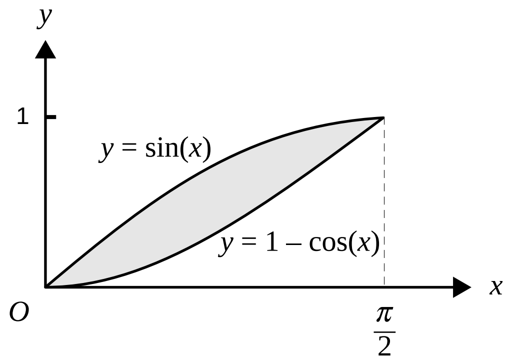
A tank initially contains 500 litres of salt solution of concentration 0.02 kg/litre. A solution of the same salt, but of concentration 0.05 kg/litre, flows into the tank at the rate of 5 litres/minute. The mixture in the tank is kept uniform by stirring and the mixture flows out at the rate of 3 litres/minute.

Let Q kg be the quantity of salt in the tank after t minutes. Set up (but do **not** attempt to solve) the differential equation for Q in terms of t, and specify the initial condition.

3 marks

Working space

Question 6



The shaded region is enclosed by the curves with equations $y = 1 - \cos(x)$ and $y = \sin(x)$, $0 \le x \le \frac{\pi}{2}$.
This region is rotated about the x-axis to form a solid of revolution.
Find the **exact** volume of this solid of revolution.

4 marks

END OF PART II QUESTION AND ANSWER BOOK

Click here for Formula Sheet

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
hyperbola:	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$

$\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$

$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$

$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$

$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$

$-\pi<\operatorname{Arg} z\le\pi$

$z_1z_2=r_1r_2\operatorname{cis}(\theta_1+\theta_2)$

$\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n=r^n\operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\text{Sin}^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \text{Sin}^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\text{Cos}^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \text{Cos}^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\text{Tan}^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \text{Tan}^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)f\left(\frac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)\left(f(a) + f(b)\right)$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h f(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \leq \mu N$

END OF FORMULA SHEET