Year 2003

VCE

Specialist Mathematics

Trial Examination 1

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VICTORIAN CERTIFICATE OF EDUCATION
2003

SPECIALIST MATHEMATICS

Trial Written Examination 1 (Facts, skills and applications)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

Directions to students

Materials
There is a detachable formulae sheet of miscellaneous formulas at the end of the booklet.
You may bring into the examination up to (two A4 sheets) of pre-written notes.
You may use approved scientific and or graphics calculators, protractor, set square and aids for curve sketching.

The task.
Write your student number in the space provided above.
This examination consists of two parts.
Part I consists of 30 multiple choice questions, and comprises 16 pages. Mark your answer on the answer sheet provided. You should attempt all questions.
Part II consists of 6 short answer questions, totals 20 marks and comprises 9 pages.
Write your answers in the spaces provided in the separate question and answer booklet.
You need not give numerical answers as decimals unless instructed to do so. A decimal approximation, no matter how accurate will not be accepted if an exact answer is required to a question.
Calculus must be used to evaluate derivatives and definite integrals. A decimal value no matter how accurate, will not be rewarded unless the appropriate working is shown.
Unless otherwise stated, diagrams in this book are not drawn to scale.
All responses should be in English.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

Directions to students

Materials

Question book of 16 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the book.
You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve sketching
You should have at least one pencil and an eraser.

Instructions

Detach the formula sheet from the book during reading time.
Please ensure that your **name and student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
Answer **all** questions.
There is a total of 30 marks available for Part I.
All questions should be answered on the answer sheet provided for multiple-choice questions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book
(Part II). You may retain this question book.

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

cosine rule: $c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2 x+\sin^2 x=1$

$1+\tan^2 x=\sec^2 x$ $\quad\quad$ $\cot^2+1=\operatorname{cosec}^2 x$

$\sin(x+y)=\sin x\cos y+\cos x\sin y$ $\quad\quad$ $\sin(x-y)=\sin x\cos y-\cos x\sin y$

$\cos(x+y)=\cos x\cos y-\sin x\sin y$ $\quad\quad$ $\cos(x-y)=\cos x\cos y+\sin x\sin y$

$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$ $\quad\quad$ $\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$

$\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$

$\sin 2x=2\sin x\cos x$ $\quad\quad$ $\tan 2x=\frac{2\tan x}{1-\tan^2 x}$

Function	Sin^{-1}	Cos^{-1}	Tan^{-1}
Domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < Arg\, z \le \pi$

$z_1 z_2 = r_1 r_2 \,\mathrm{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\mathrm{cis}(\theta_1 - \theta_2)$

$z^n = r^n \mathrm{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$|\bar{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \bar{r}_1.\bar{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\bar{r}} = \dfrac{d\bar{r}}{dt} = \dfrac{dx}{dt}\bar{i} + \dfrac{dy}{dt}\bar{j} + \dfrac{dz}{dt}\bar{k}$

Mechanics

momentum: $\bar{p} = m\bar{v}$

equation of motion: $\bar{R} = m\bar{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u + v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e x + c,\ for\ x > 0$$

$$\frac{d}{dx}(\sin ax) = a\cos ax \qquad \int \sin ax dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax \qquad \int \cos ax dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}(\tan ax) = a\sec^2 ax \qquad \int \sec^2 ax dx = \frac{1}{a}\tan ax + c$$

$$\frac{d}{dx}(\mathrm{Sin}^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{dx}{\sqrt{a^2 - x^2}} = \mathrm{Sin}^{-1}\frac{x}{a} + c,\ a > 0$$

$$\frac{d}{dx}(\mathrm{Cos}^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{-1}{\sqrt{a^2 - x^2}} dx = \mathrm{Cos}^{-1}\frac{x}{a} + c,\ a > 0$$

$$\frac{d}{dx}(\mathrm{Tan}^{-1}x) = \frac{1}{1+x^2} \qquad \int \frac{a}{a^2 + x^2} dx = \mathrm{Tan}^{-1}\frac{x}{a} + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}(\frac{u}{v}) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)\, f(\frac{a+b}{2})$

trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$

Euler's method

If $\frac{dy}{dx} = f(x), x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

VCE SPECIALIST MATHEMATICS 2003
Trial Written Examination 1
ANSWER SHEET

NAME: _____

STUDENT NUMBER _____

SIGNATURE_____

Instructions

- Write your name in the space provided above.
- Write your student number in the space provided above. Sign your name.
- Use a **PENCIL** for **ALL** entries.
  If you make a mistake, **ERASE** it - **DO NOT** cross it out.
- Marks will **NOT** be deducted for incorrect answers.
- **NO MARK** will be given if more than **ONE** answer is completed for any question.
- All answers must be completed like **THIS** example.

A	**B**	C	D	E

1	A	B	C	D	E	**16**	A	B	C	D	E
2	A	B	C	D	E	**17**	A	B	C	D	E
3	A	B	C	D	E	**18**	A	B	C	D	E
4	A	B	C	D	E	**19**	A	B	C	D	E
5	A	B	C	D	E	**20**	A	B	C	D	E
6	A	B	C	D	E	**21**	A	B	C	D	E
7	A	B	C	D	E	**22**	A	B	C	D	E
8	A	B	C	D	E	**23**	A	B	C	D	E
9	A	B	C	D	E	**24**	A	B	C	D	E
10	A	B	C	D	E	**25**	A	B	C	D	E
11	A	B	C	D	E	**26**	A	B	C	D	E
12	A	B	C	D	E	**27**	A	B	C	D	E
13	A	B	C	D	E	**28**	A	B	C	D	E
14	A	B	C	D	E	**29**	A	B	C	D	E
15	A	B	C	D	E	**30**	A	B	C	D	E

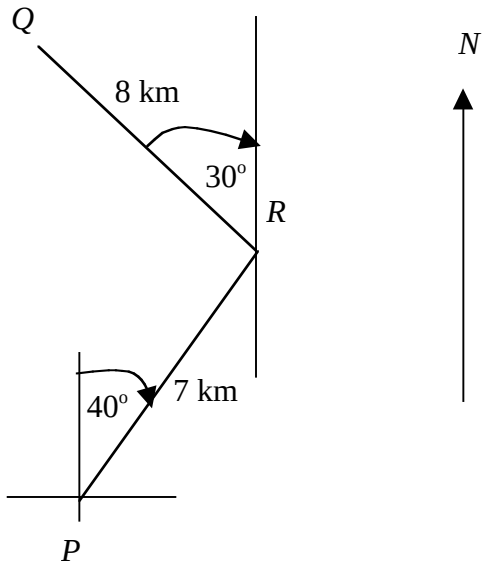
Please DO NOT fold, bend or staple this form

Instructions for Part I

Answer **all** questions in pencil, on the answer sheet provided for multiple-choice questions. A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No mark will be given for a question if more than one answer is completed for any question.

Question 1

A ship leaves a port *P*, and sails 7 km on a heading of N40°E to a position *R*. It then sails 8 km on a heading N30°W to a port *Q*. The magnitude of the displacement from *P* to *Q* is given by



A. $\sqrt{15-\cos(110^o)}$

B. $\sqrt{113+112\cos(110^o)}$

C. $\sqrt{113+112\sin(20^o)}$

D. $7\cos(40^o)+8\cos(30^o)$

E. 15

Question 2

The graph of $y = \dfrac{x^3 + 4x}{4x^2}$ has

A. a single asymptote at $x = 0$ and no turning points.

B. a single asymptote at $x = 0$ and two turning points at $x = \pm 2$.

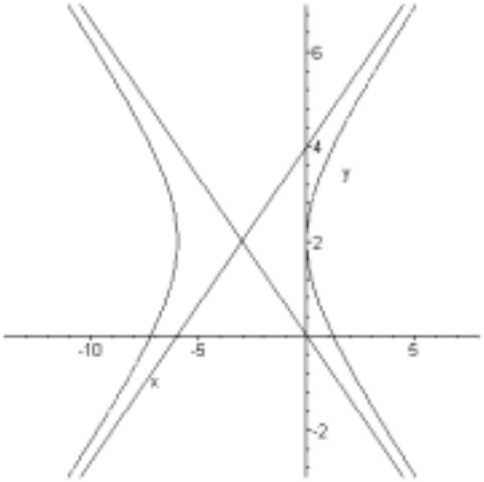
C. two asymptotes and two turning points.

D. no asymptotes and a point of inflexion at $x = 0$.

E. two asymptotes and no turning points

Question 3

The equation of the hyperbola shown is



A. $\dfrac{(x+3)^2}{9} - \dfrac{(y+2)^2}{4} = 1$

B. $\dfrac{(x+3)^2}{9} - \dfrac{(y-2)^2}{4} = 1$

C. $\dfrac{(y+2)^2}{4} - \dfrac{(x+3)^2}{9} = 1$

D. $\dfrac{(x-3)^2}{9} - \dfrac{(y+2)^2}{4} = 1$

E. $\dfrac{(y+2)^2}{4} - \dfrac{(x-3)^2}{9} = 1$

Question 4

If $y = \text{Cos}^{-1}\left(\frac{4}{3x}\right)$ and $x > 0$ then $\frac{dy}{dx}$ is equal to

A. $\frac{-12}{\sqrt{16-9x^2}}$

B. $\frac{-12}{\sqrt{9x^2-16}}$

C. $\frac{-3}{\sqrt{16-9x^2}}$

D. $\frac{\sqrt{16-9x^2}}{-12}$

E. $\frac{4}{x\sqrt{9x^2-16}}$

Question 5

If $\text{cosec}(x) = 4$ and $\frac{\pi}{2} < x < \pi$ then $\cot(x)$ equals

A. $\sqrt{15}$

B. $\sqrt{17}$

C. $-\frac{\sqrt{15}}{15}$

D. $-\sqrt{15}$

E. $-\frac{\sqrt{17}}{17}$

Question 6

The range of the function $y = a\,\text{Tan}^{-1}(bx) + c$ is

A. R

B. $[0, b\pi]$

C. $(0, b\pi)$

D. $\left(-\frac{a\pi}{2} + c, \frac{a\pi}{2} + c\right)$

E. $\left[-\frac{a\pi}{2} + c, \frac{a\pi}{2} + c\right]$

Question 7

If $z = x + iy$ where $x, y \in R$ and $z \in C$ then which of the following is **NOT** a real number

A. $z\bar{z}$

B. $z + \bar{z}$

C. $\text{Im}(z^2)$

D. $|z|$

E. $z - \bar{z}$

Question 8

If $u = a - bi$ where *a*, *b* are positive real numbers, then $\frac{1}{\bar{u}}$ in Cartesian form is

A. $\frac{1}{a} - \frac{1}{b}i$

B. $\frac{a - bi}{a^2 - b^2}$

C. $\frac{a + bi}{a^2 - b^2}$

D. $\frac{a - bi}{a^2 + b^2}$

E. $\frac{a + bi}{a^2 + b^2}$

Question 9

If $1-2i$ is a solution to the equation $z^3+az^2+bz-10$ where a and b are real then

A. $a=-4$ and $b=9$

B. $a=1$ and $b=-2$

C. $a=0$ and $b=1$

D. $a=5$ and $b=-3$

E. $a=1$ and $b=2$

Question 10

Which of the following could **NOT** represent the complex number $-\sqrt{3}-i$

A. $2cis\left(-150^0\right)$

B. $2cis\left(\frac{7\pi}{6}\right)$

C. $-2cis\left(\frac{\pi}{6}\right)$

D. $-2cis\left(\frac{5\pi}{6}\right)$

E. $2cis\left(-\frac{5\pi}{6}\right)$

Question 11

The value of $\int_1^3 \frac{x^2-3x}{4\tan\left(\frac{x}{2}\right)}dx$ is equal to

A. –0.0131

B. –1.3427

C. –57.2914

D. 0.7312

E. –0.7312

Question 12

If $z = -2a + ai$ where a is a real positive number, then which of the following represents $\text{Arg}(z)$

A. $\text{Tan}^{-1}2$

B. $\text{Tan}^{-1}\left(\frac{1}{2}\right)$

C. $\pi + \text{Tan}^{-1}\left(-\frac{1}{2}\right)$

D. $\text{Tan}^{-1}(-2)$

E. $\text{Tan}^{-1}\left(-\frac{1}{2}\right)$

Question 13

The derivative of $x\,\text{Tan}^{-1}(x)$ with respect to x is $\text{Tan}^{-1}(x) + \frac{x}{1+x^2}$

It follows that an antiderivative of $\text{Tan}^{-1}(x)$ is

A. $x\,\text{Tan}^{-1}(x) - \frac{1}{2}\log_e\left(1+x^2\right)$

B. $\int x\,\text{Tan}^{-1}(x)\,dx + \frac{1}{2}\log_e\left(1+x^2\right)$

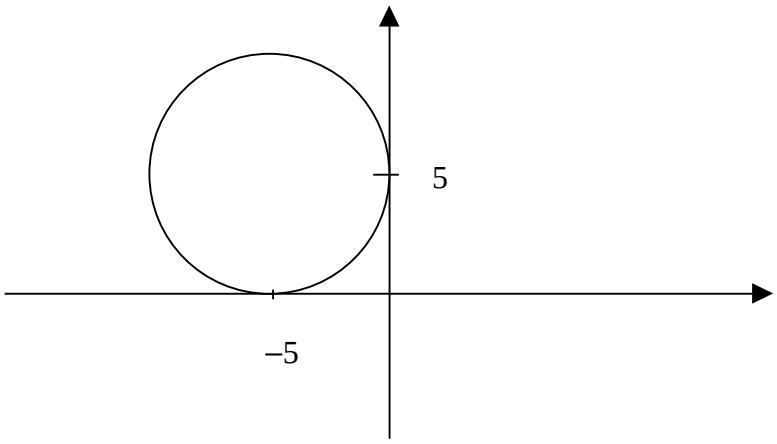
C. $x\,\text{Tan}^{-1}(x) + \frac{1}{2}\log_e\left(1+x^2\right)$

D. $\int x\,\text{Tan}^{-1}(x)\,dx + \frac{x}{1+x^2}$

E. $x\,\text{Tan}^{-1}(x) + \frac{x}{1+x^2}$

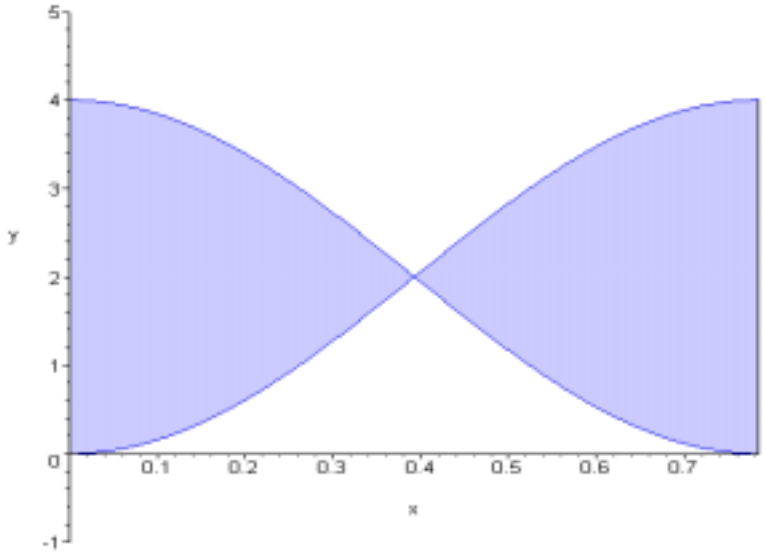
Question 14

The diagram shows a circle in the complex plane. The circle is specified by.



A. $\{z:(z+5+5i)(\bar{z}-5+5i)=25\}$
B. $\{z:(z+5-5i)(\bar{z}+5+5i)=25\}$
C. $\{z:(z-5-5i)(\bar{z}-5+5i)=25\}$
D. $\{z:|z+5+5i|=25\}$
E. $\{z:|z-5-5i|=5\}$

Question 15



The graphs of $y = 4\sin^2(2x)$ and $y = 4\cos^2(2x)$ are shown above, for $0 \le x \le \frac{\pi}{4}$

Given that the graphs intersect at $x = \frac{\pi}{8}$, then shaded area has a magnitude in square units of

A. 0.5

B. 1

C. 2

D. 4

E. 8

Question 16

If the length of the vector $-\bar{i} + t\bar{j} + \bar{k}$ is 4 then the possible value(s) of t are

A. $\pm\sqrt{2}$

B. ± 2

C. 2

D. $\pm\sqrt{14}$

E. $\sqrt{2}$

Question 17

The exact value of $\int_0^{\frac{4}{3}} \frac{dx}{\sqrt{64-9x^2}}$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{9}$

D. $\frac{\pi}{18}$

E. 0.5236

Question 18

If the vector $\frac{1}{2}\left(\bar{i}-\bar{j}+z\bar{k}\right)$ makes an angle of 135° with the positive z-axis, then the possible value(s) of z are

A. $\pm\sqrt{2}$

B. $-\sqrt{2}$

C. $\sqrt{2}$

D. $-\frac{\sqrt{2}}{2}$

E. $\frac{\sqrt{2}}{2}$

Question 19

If $\bar{r}(t) = \left(\sqrt{t} - 2\right)\bar{i} + 3t^2\bar{j} \quad t \geq 0$ then the Cartesian equation of the path is given by

A. $y = 3(x+2)^4 \quad x \geq -2$

B. $y = 3(x+2)^2 \quad x \geq 2$

C. $y = 3(x-2) \quad x \geq -2$

D. $y = 3(x-2)^2 \quad x \geq 2$

E. $y = 3x^2 \quad x \geq -2$

Question 20

A particle of mass 2 kg, is acted upon by two forces, one of magnitude *y* Newtons acting due south, the other of magnitude $6\sqrt{2}$ Newtons acting at a bearing N45°W. The magnitude of the acceleration is 5 m/s^2 . The number *y* is

A. 2

B. 4

C. 14

D. $10 - 6\sqrt{2}$

E. $6\sqrt{2} - 10$

Question 21

A particle starts from rest and moves in a straight line with velocity of $\frac{1}{\sqrt{4t+9}}$ m/s at a time *t* s. Its displacement from its starting point, in metres, when $t=\frac{7}{4}$ is

A. $\frac{1}{8}$

B. 1

C. $\frac{1}{2}$

D. 2

E. $2\log_e\left(\frac{4}{3}\right)$

Question 22

Two small boats *A* and *B* move so that a time *t* hours their position vectors are given by $\bar{r}_A(t)=\left(t^2-5t+6\right)\bar{i}+(2t-6)\bar{j}$ and $\bar{r}_B(t)=(2t-4)\bar{i}+\left(t^2-8t+15\right)\bar{j}$ where $\bar{i}$ and $\bar{j}$ are unit vectors of magnitude one kilometre in the directions of east and north respectively, it is true that

A. The two boats collide after 3 hours.

B. The two boats collide after 2 hours.

C. The two boats do not collide.

D. The two boats touch each other after 2 and 3 hours.

E. The two boats travel on straight line paths.

Question 23

If $\overrightarrow{PQ} = -\overrightarrow{PR}$ and $\left|\overrightarrow{RQ}\right| = 2$ then which of the following is **FALSE**

A. P , Q and R are collinear

B $\left|\overrightarrow{PQ}\right| = 1$

C. $\overrightarrow{PQ}.\overrightarrow{QR} = 0$

D. $\overrightarrow{PQ}$ is parallel to $\overrightarrow{RP}$

E. $\overrightarrow{PQ}.\overrightarrow{PR} = -1$

Question 24

Euler's method, with a step size of 0.5, is used to approximate the solution of the differential equation $x\dfrac{dx}{dy} = \sqrt{4x^2 + 9}$ with $x = 1$ $y = 2$. When $x = 2$ the value obtained for y, correct to four decimal places, is

A. 0.4433

B. 2.3155

C. 5.2170

D. 2.2565

E. 0.3486

Question 25

A particle moves so that $x = 1 + \dfrac{1}{t}$ while $y = \sqrt{12 + t^2}$ the value of $\dfrac{dy}{dx}$ when $t = 2$ is

A. -2

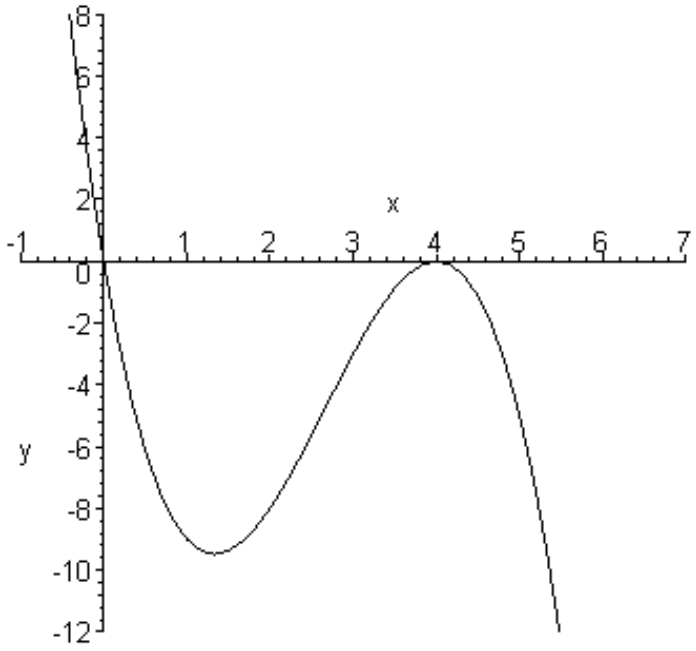
B. $-\dfrac{1}{8}$

C. $\dfrac{1}{8}$

D. $\dfrac{8}{3}$

E. 4

Question 26



The graph of $y = f'(x)$ is shown above.
Which of the following statements is true for the graph of $y = f(x)$?

A. The graph has a local maximum at $x = 4$ and a stationary point of inflexion at $x = 0$

B. The graph has a local minimum at $x = 4$ and a stationary point of inflexion at $x = 0$

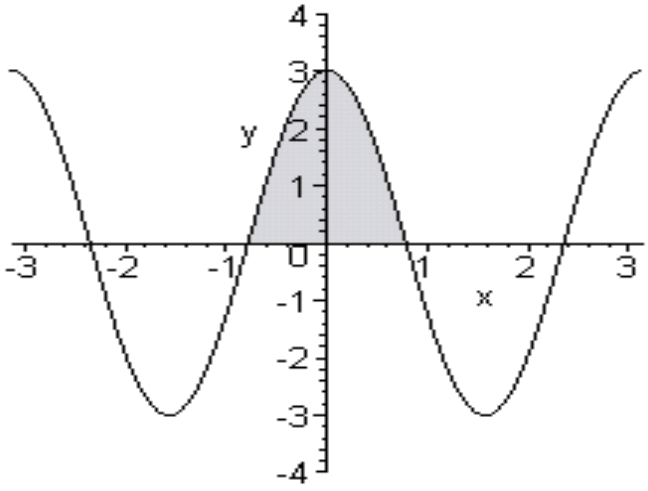
C. The graph has a local maximum at $x = 0$ and a stationary point of inflexion at $x = 4$

D. The graph has a local minimum at $x = 0$ and a stationary point of inflexion at $x = 4$

E. The graph has a local minimum at $x = 1$, a local maximum at $x = 4$, and a stationary point of inflexion at $x = 0$

Question 27

The shaded region is the area bounded by the *x*-axis and the curve $y = 3\cos(2x)$. This region is rotated about the *x*-axis to form a solid of revolution. The volume of the solid, in cubic units, is



A. 44.4132

B. $\frac{9\pi^2}{2}$

C. $18\pi^2$

D. 3

E. $\frac{9\pi^2}{4}$

Question 28

A particle of mass 2 kg, has an acceleration at a time t s, $t \geq 0$ given by
$\bar{a}(t) = \frac{3\pi}{2}\cos\left(\frac{\pi t}{2}\right)\bar{i} - \pi\sin\left(\frac{\pi t}{2}\right)\bar{j}$ m/s^2

If the initial velocity is $6\bar{j}$, then the momentum of the particle in kg m/s when $t = 1$ is

A. $6\bar{i}$

B. 10

C. $-6\bar{i} + 8\bar{j}$

D. $3\bar{i} + 4\bar{j}$

E. $6\bar{i} + 8\bar{j}$

Question 29

A tank contains 50 litres of water in which 5 grams of salt has been dissolved. A salt solution containing 3 grams per litre is poured into the tank at a rate of 4 litres per minute and the well stirred mixture leaves the tank at a rate of 2 litres per minute.
The differential equation for the amount of salt Q grams in the tank at a time t minutes, is given by
$\frac{dQ}{dt} = 12 - \frac{2Q}{(50+2t)}$

If $Q = Q(t) = 3(50+2t) + C(50+2t)^n$ is a solution, where C is an arbitrary constant, then the value of n is

A. -2

B. -1

C. 0

D. 1

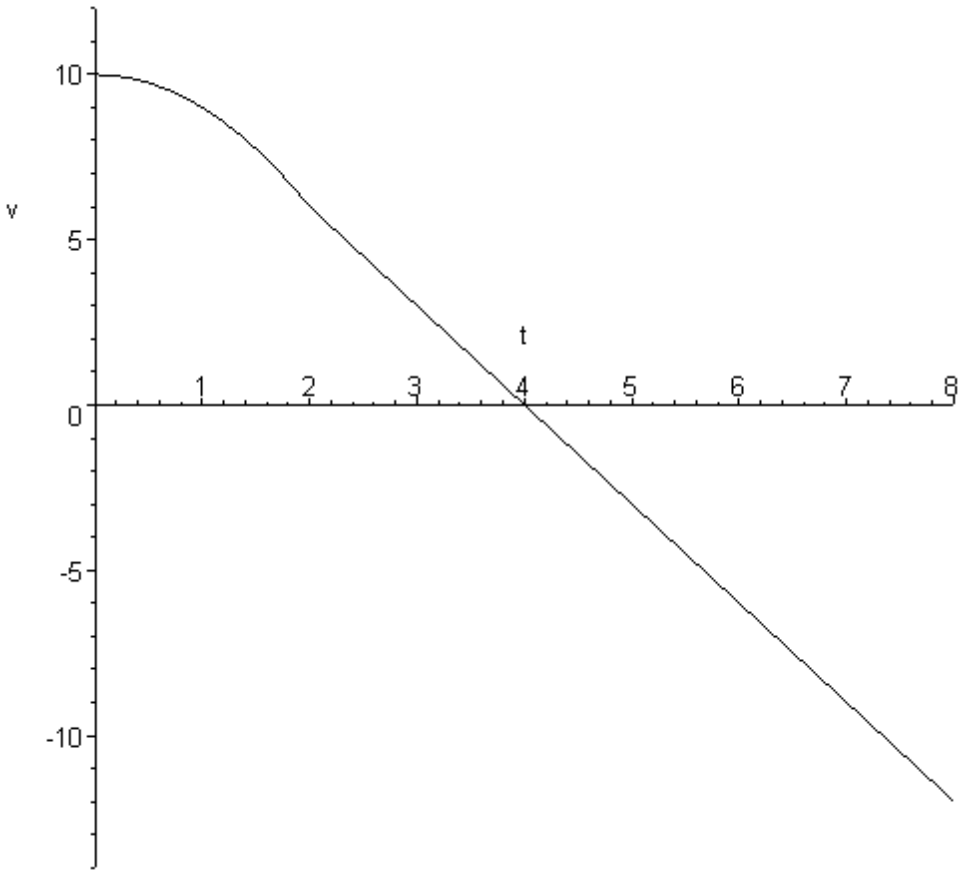
E. 2

Question 30

A particle moves in a straight line so that its velocity in m/s at a time *t* seconds is defined by

$$v(t) = \begin{cases} 10 - t^2 & 0 \le t \le 2 \\ 12 - 3t & 2 \le t \le 8 \end{cases}$$

Given the velocity time graph below, how far in metres is the particle from its starting point after 8 seconds have elapsed?



A. $-\frac{2}{3}$

B. $\frac{2}{3}$

C. 4

D. $47\frac{1}{3}$

E. 44

END OF PART I
MULTIPLE CHOICE QUESTION BOOK

STUDENT NUMBER

Figures

Words

VICTORIAN CERTIFICATE OF EDUCATION 2003

SPECIALIST MATHEMATICS

Trial Written Examination 1 (Facts, skills and applications)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

PART II

QUESTION AND ANSWER BOOK

Directions to students

This examination has two parts: Part I (multiple-choice questions) and Part II (short answer questions)
Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part. A detachable formula sheet for use in both parts is included.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
6	6	20

Directions to students

Materials

Question and answer book of 9 pages.
Working space is provided throughout the book.
You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve sketching

The task

Detach the formula sheet during reading time.
Please ensure that your **student number** in the space provided on the cover of this book.
The marks allotted to each question are indicated at the end of the question.
There is a total of 20 marks available for Part II.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions.
Where an exact answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
All written responses should be in English.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Instructions for Part II

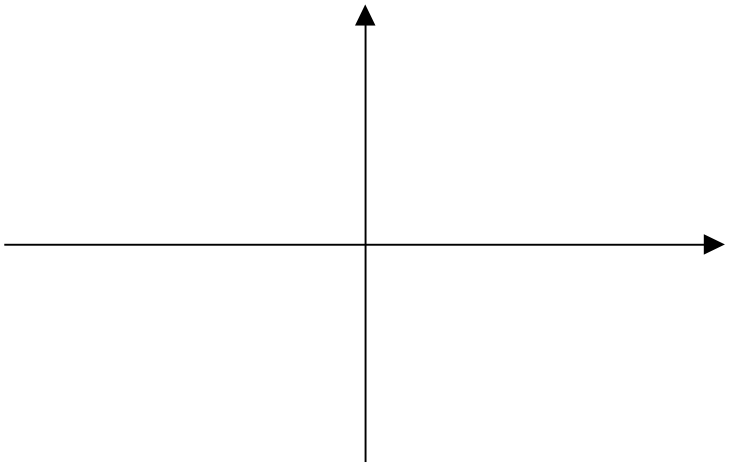
Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
Where an exact answer is required for a question, appropriate working must be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

A particle moves so that its position vector at a time t is given by

$\bar{r}(t) = 8\cos^3 2t\,\bar{i} + 8\sin^3 2t\,\bar{j} \quad t \geq 0$

i. sketch the path of the particle on the axes shown below, labeling the axial intercepts.



1 mark

Question 1 (continued)

ii. If the Cartesian equation of the path of the particle can be expressed in the form $x^n + y^n = a$ then find the values of n and a.

2 marks

iii. If the speed of the particle can be expressed in the form $c\sin bt$ then find the values of b and c.

2 marks

Question 2

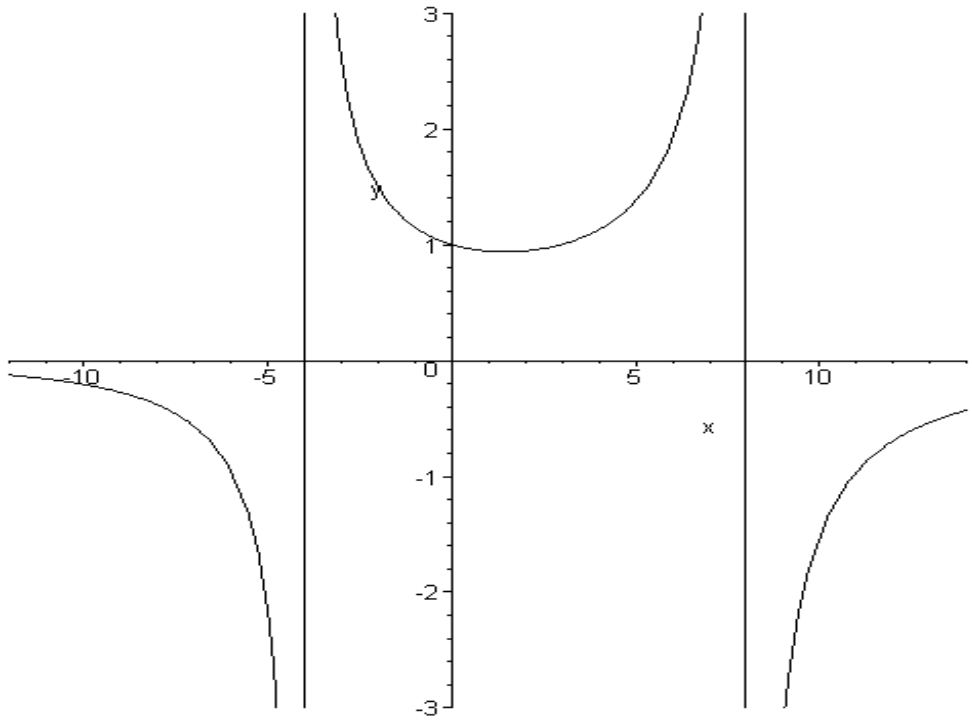
i. Show that $-2-3i$ is a solution of the equation $z^3+(2+3i)z^2+5z+10+15i=0$.

1 mark

ii. Find all solutions of the equation $z^3+(2+3i)z^2+5z+10+15i=0$.

2 marks

Question 3



i. The graph of $y = \dfrac{x + c}{-x^2 + bx + 32}$ is shown above, it has vertical asymptotes at $x = -4$ and $x = 8$. Show that $c = 32$ and $b = 4$.

_____

_____

_____

1 mark

Question 3 (continued)

ii. Express $y = \dfrac{x+32}{-x^2+4x+32}$ in partial fractions in the form $\dfrac{A}{8-x} + \dfrac{B}{x+4}$

1 mark

iii. If the area bounded by the graph, the co-ordinates axes and the line $x = 4$ can be expressed in the form $p\log_e 2$ find using calculus, the exact value of p, shading the area on the axes above.

2 marks

Question 4

Find using calculus the exact value of $\int_{\frac{6}{\pi}}^{\frac{12}{\pi}} \frac{4\cos\left(\frac{2}{x}\right)}{x^2}\,dx$

2 marks

Question 5

A particle of mass m kg is falling in the earth's gravitational field, with a speed of v m/s. The resistance to the motion is kv^n Newton's where k is a positive constant and n is positive integer.

i. Set up the differential equation and show that the acceleration a m/s^2 of the particle is given by $a = g - \frac{kv^n}{m}$

1 mark

Question 5 (continued)

ii. If $k = 0.5$, $m = 2$ and $n = 3$ find the distance in metres correct to three decimals that the particle moves from rest until its speed is 2 m/s.

_____

_____

_____

_____

_____

2 marks

Question 6

A boy of mass 35 kg is pulling a baby girl on a sled horizontally forward, on a smooth grass lawn. The girl and the sled have a mass of 20 kg. The boy exerts a horizontal force of 30 Newtons. He pulls on a string attached to the sled which makes an angle of 25^{o} with the ground. You may assume there are no frictional forces acting.

i. Redraw the diagram, marking in all the forces acting on the boy and the girl in the sled.

1 mark

ii. By resolving the forces, find the acceleration of the boy and girl, and the tension in the string in Newtons, giving your answer correct to two decimal places.

2 marks

End of 2003 Specialist Mathematics Trial Examination 1
Question and Answer Book

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